Analytic properties of the sheath solution with warm ions

L. Kos[1], D. D Tskhakaya[3,4], N. Jelić[2,3], J. Duhovnik[1]

[1] *Association EURATOM-MHEST, Faculty of Mech. Eng., Univ. of Ljubljana, Slovenia*

[2] *Association EURATOM-ÖAW, Inst. for Theor. Phys. - Comput. Phys. TU Graz, Austria*

[3] *Association EURATOM-ÖAW, Inst. for Theoretical Physics, Univ. of Innsbruck, Austria*

[4] *Andronikashvili Institute of Physics, Georgian Academy of Sciences, Tbilisi, Georgia*

An analytic description of the boundary layer of a weakly ionized gas discharge as a whole single unit does not turn out well due to its mathematical difficulties. That is why the plasma-wall transition (PWT) layer is usually split into two sublayers: a quasineutral presheath (with the characteristic scale length L - the collision mean-free path) and the Debye sheath (with the scale-length λ_D - the electron Debye length). This subdivision, being valid only in the asymptotic two-scale limit, $\lambda_D/L \to 0$, allows to investigate the presheath and the sheath separately. Such a simplification calls for a detailed and in-depth study of the sublayers bearing in mind their further matching. In most publications on the weakly ionized PWT layer the neutral gas is assumed to be cold, which seems to be an over-simplification of the problem [1]. This paper presents kinetic theory of the sheath in the Tonks-Langmuir (T&L) model of the PWT layer with hot neutrals. Ion kinetics is governed by the ionization process due to electron-neutral particle collisions. The plasma consisting of Boltzmann distributed electrons and singly charged ions is in contact with a negative absorbing wall. Dependencies of the electric potential shape and its characteristics in the sheath on the neutrals' temperature are investigated for the first time.

The ions born at the neutrals' ionization acquire the negative velocity only due to the velocity spreading of the neutrals' velocity distribution function (VDF). The Debye sheath (DS) is collisionless. It means that there are no ions with the negative velocities. In the DS the space charge effect can no longer be neglected. The ion VDF in the DS can be found from the homogeneous kinetic equation. Hence the ion distribution function in the DS $\bar{f}_i(E)$ can depend only on the total ion energy $E = mv^2/2 + e\Phi$. The explicit form of this dependence can be found from the coincidence condition of $\bar{f}_i(E)$ at the sheath edge $x = x_s$ with the ion distribution function in the presheath. The result reads

$$\bar{f}_s(v,\Phi) = \bar{f}_s(v^2-\chi) = 2\bar{B}\frac{n_0}{\sqrt{2}c_s}\int_0^{-\Phi_s} d\Phi''\frac{dx_0(\Phi'')}{d\Phi''}\,\frac{\exp(\tau\sqrt{v^2-\chi-\Phi''})}{\sqrt{v^2-\chi-\Phi''}} \times\exp\left[\Phi_s+\Phi''\right]\mathrm{H}(v^2-\chi-\Phi'')\ . \tag{1}$$

Here $\Phi_s < 0$, $\Phi < 0$ and $\chi = \Phi_s - \Phi \geq 0$. *It should be mentioned that the necessary sheath edge condition* $\bar{f}_s(\Phi_s, v = 0) = 0$ is fulfilled and from Eq. (1) it follows that $\bar{f}_s(v,\Phi) = 0$ at $v^2 \leq \chi$. H(x) is Heaviside step function. Using the sheath scale ($\simeq \lambda_D$) and introducing corresponding coordinates we can write the Poisson equation in the form

$$\frac{d^2\chi}{d\xi^2} = n_i - n_e\ , \tag{2}$$

where $\xi = (z - z_r)/\varepsilon$ and $\chi = \Phi_s - \Phi$. z_r is an arbitrary reference point allowing suitable choice of origin for the sheath coordinate ξ. Further we'll follow again the procedure given in Ref. [2]. For the ion and electron densities we find

$$n_i(\chi) = \frac{1}{2}\int_0^\infty \frac{dy}{\sqrt{\chi+y}}\bar{f}_s(y)\ , \quad n_e(\chi) = \exp(\Phi_s-\chi)\ . \tag{3}$$

After integration from Eq. (2) we obtain

$$\xi - \xi_w = \int_{\chi_w}^{\chi} \frac{d\psi}{\sqrt{2[W(\psi)+\exp(\Phi_s-\psi)-\exp(\Phi_s)]}}\ , \tag{4}$$

where ξ_w is the wall coordinate, χ_w the relative potential at the wall, and

$$W(\psi) = \int_0^\infty dy\,\bar{f}_s(y)[\sqrt{\psi+y}-\sqrt{y}]\ . \tag{5}$$

The smallness of the relative potential in the vicinity of the sheath edge $\chi \ll 1$, allows us to find the analytic expression for the potential shape there. Obviously, the numerical solution of Eq. (4) must coincide with this analytic expression in the region indicated. We start with an expansion of the ion and electron densities [3]

$$\frac{d^2\chi}{d\xi^2} = \sum_\nu c_\nu \chi^{\nu/2}\ , \quad \text{where} \quad c_\nu = \begin{cases} a_{2n} - b_n\exp(\Phi_s) & \text{for } \nu = 2n\ , \\ a_{2n+1} & \text{for } \nu = 2n+1\ , \end{cases} \tag{6}$$

$$a_{2n} = \frac{(-1)^n}{2n!}\int_0^\infty \frac{dy}{\sqrt{y}}\frac{d^n\bar{f}_s(y)}{dy^n}\ , \quad a_{2n+1} = \frac{\pi(-1)^{n+1}}{2(n+\frac{1}{2})!}\left.\frac{d^n\bar{f}_s(y)}{dy^n}\right|_{y=0}\ , \quad b_n = \frac{(-1)^n}{n!}\ . \tag{7}$$

For coefficients c_ν we have: (i) according to the quasineutrality condition at the sheath edge $c_0 = \frac{1}{2}\int_0^\infty \frac{dy}{\sqrt{y}}\bar{f}_s(y) - \exp(\Phi_s) = 0$; (ii) due to boundary condition for the VDF at the

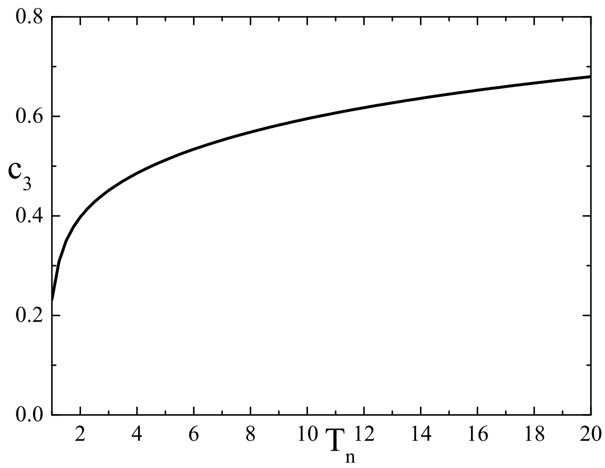

Figure 1: Coefficient $c_3(T_n)$.

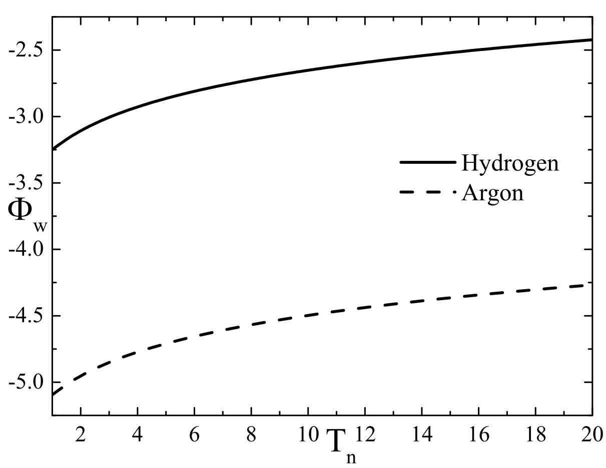

Figure 2: The floating wall potential for hydrogen and argon.

sheath edge $c_1 = -\bar{f}_s(0) = 0$; (iii) the Bohm criterion gives $c_2 = 0$. Neglecting higher order terms from Eq. (6) we find

$$\xi_{io} = \xi_0 - 2\sqrt{\frac{5}{c_3}}\frac{1}{\chi^{1/4}} \,. \tag{8}$$

The dependence of the coefficient c_3 on the temperature T_n is shown in Fig. 1. The integration constant ξ_0 and its relation to the ξ_w can be found by comparison of Eq. (8) with numerical solution of Eq. (4) [see Fig. 3]. It follows that potential drop in sheath for hydrogen at $T_{n1} = 3$, $\chi_w = \Phi_s(T_{n1}) - \Phi_w(T_{n1}) = 2.50$ and at $T_{n2} = 10$, $\chi_w = \Phi_s(T_{n2}) - \Phi_w(T_{n2}) = 2.262$. Hence the comparison of the numerical solution of Eq. (4) for small χ with Eq. (8) gives

$$\begin{aligned} \xi_0 &= \xi_w + 3.78 \quad \text{at} \quad T_n = 3 \quad \text{and} \\ \xi_0 &= \xi_w + 3.11 \quad \text{at} \quad T_n = 10 \,. \end{aligned} \tag{9}$$

For the floating wall potential we have

$$\Phi_w = \ln\left[\sqrt{2\pi}\sqrt{\frac{m_e}{Mi}}\int_0^{\Phi_s} d\Phi \frac{dx_0}{d\Phi}\exp(\Phi)\right] , \tag{10}$$

which is dependent on the gas used as shown in Fig. 2. It follows that potential drop in sheath for hydrogen at $T_{n1} = 3$, $\chi_w = \Phi_s(T_{n1}) - \Phi_w(T_{n1}) = 2.50$ and at $T_{n2} = 10$ $\chi_w = \Phi_s(T_{n2}) - \Phi_w(T_{n2}) = 2.262$.

To our knowledge this is the first attempt to study the influence of the ion source (neutral gas) temperature and fills the gap in the full understanding of the famous T&L model. This model forms the basis for all the recent investigations of the PWT layer.

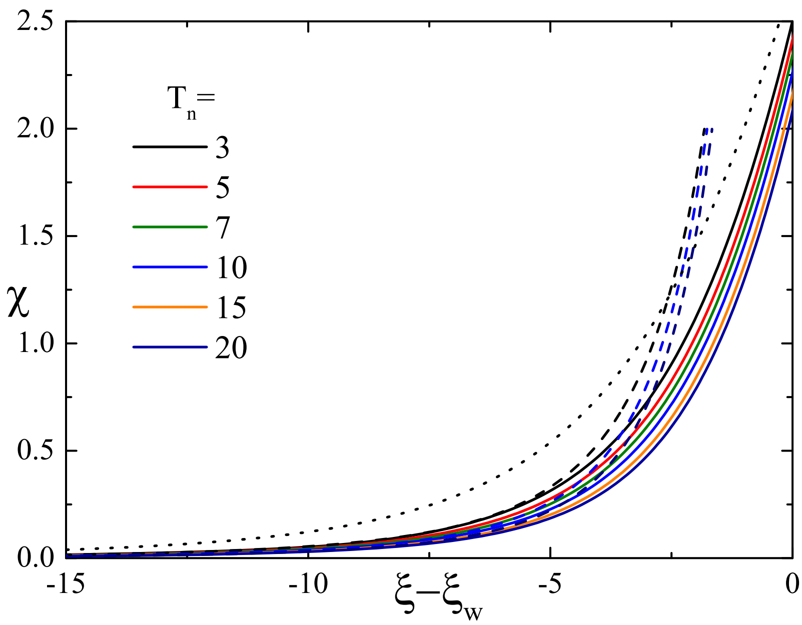

Figure 3: (Color online) Asymptotic ($\varepsilon = 0$) sheath potential variation $\chi(\xi - \xi_w)$ for various T_n in the hydrogen plasma. Dotted line shows sheath potential variation for cold ($T_n = 0$) case (see Ref. [2, Fig 3.]). Dashed lines show the plasma sided expansion $\xi_{io}(\chi)$ for $T_n = \{3, 10, 20\}$.

Acknowledgements

This work was supported by the European Commission under (i) the Contract of Association between EURATOM and the Austrian Academy of Sciences and by (ii) the Austrian Science Fund (FWF) under projects P9133 and P22345-N16. It was carried out within the framework of the European Fusion Development Agreement. The work was partially supported by the Georgian Science Foundation through the project No. 1-4/16 (GSNF/ST09_305_4-140). The views and opinions expressed herein do not necessarily reflect those of the European Commission.

References

[1] P. C. Stangeby, *The Plasma Boundary of Magnetic Fusion Devices* (IOP, Bristol, 2000).

[2] K.-U. Riemann, Phys. Plasmas **13**, 063508 (2006).

[3] K.-U. Riemann, J. Phys. D: Appl. Phys. **24**, 493 (1991).