Locality of non-linear interactions in gyrokinetic turbulence

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From a dynamical point of view, plasma physics, be it in astrophysical conditions or laboratory configurations, possesses an intrinsic nonlinear character. While a linear analysis will identify the possible instabilities that might develop at each dynamical scale and their respective intensities, it is the nonlinear couplings that are responsible for the transfer of information (energy, correlation functions, etc.) between the scales of the system. The nature of the transfers between scales leads to complications in the study of plasma physics, as strong nonlinear couplings tend to link the macroscopic and microscopic characteristics of the problem. Generally, the strength of such a link is assessed by looking at the scale locality of the dynamical system.

In the current short paper, we will present a way to measure the locality of interaction between different scales for a turbulent plasma state, in the gyrokinetic (GK) approximation. From the start, it is seen that the constraint imposed by the magnetic guide field on the charged flow creates an anisotropy in the system. The current work concentrates on the analysis of perpendicular spatial structures of electrostatic fluctuations generated by an ion-temperature gradient instability in toroidal axisymmetric flux tube geometry (x label refers to the magnetic flux surface, the y label identifies different field lines lying on the same flux surface) for a single ion species and adiabatic electrons. The total ion distribution function is split into an appropriately normalized Maxwellian part F_0 and a perturbed part f, the non-adiabatic contribution of the ion distribution function is given as $h = f + (Z\bar{\phi}_1/T_0)F_0$, where $\bar{\phi}_1$ is the gyro-averaged self-consistent electrostatic potential (linear in f) found from the gyrokinetic Poisson equation, T_0 is the ion background temperature (normalized to the electron temperature) and Z is the electric charge. Details of this work are presented elsewhere [1].

To understand the dynamics introduced by the nonlinear term, $N[f,f] = \frac{\partial\bar{\phi}_1}{\partial y}\frac{\partial h}{\partial x} - \frac{\partial\bar{\phi}_1}{\partial x}\frac{\partial h}{\partial y}$, that enters in the evolution equation of the ion distribution function ($\partial_t f$), the scale redistribution of free-energy (a GK ideal invariant, i.e. a global quantity that remains constant in time in the absence of source and sink effects) is usually investigated. The global free-energy contained in the system is defined as, $\mathscr{E} = \frac{1}{2}\int \mathrm{d}x\mathrm{d}y\mathrm{d}\Theta\, \frac{T_0}{F_0}\, hf$, where $\mathrm{d}\Theta = (\pi B_0 n_0)\mathrm{d}z\mathrm{d}v_\parallel \mathrm{d}\mu$. To analyze the excitation degree of perpendicular turbulent scales, an integral over the dΘ infinitesimal element and a Fourier decomposition of the remaining (x,y) space are performed. Each perpendicular scale of length can now be easily identified by the norm (k) of the wave-vector based in the k_x, k_y space (units of inverse ion Larmor radius). As result of the quadratic nonlinearity in f, the *triple-scale-transfer* that appears in the the free-energy evolution equation for a scale reads as,

$$T(k|p,q) = \iiint\limits_{|\mathbf{k}|=k,\ |\mathbf{p}|=p,\ |\mathbf{q}|=q} \mathrm{d}\mathbf{k}\mathrm{d}\mathbf{p}\mathrm{d}\mathbf{q} \left\{ \int \mathrm{d}\Theta \frac{T_0}{2F_0} \left[q_x p_y - q_y p_x \right] \left[\bar{\phi}_1(\mathbf{q}) h(\mathbf{p}) - \bar{\phi}_1(\mathbf{p}) h(\mathbf{q}) \right] h(\mathbf{k}) \delta(\mathbf{k}+\mathbf{p}+\mathbf{q}) \right\}, \quad (1)$$

where the symmetry in scales q and p is written explicitly [2]. The Dirac delta selects only interactions that occur between a triad of modes which obey the resonance condition, $\mathbf{k}+\mathbf{p}+\mathbf{q}=\mathbf{0}$ and the braces isolates the transfers that takes place for a single triad, known as the *triad-transfer*. For a triad, the free-energy conservation by the nonlinear interaction can be written as, $T(\mathbf{k}|\mathbf{p},\mathbf{q}) + T(\mathbf{p}|\mathbf{q},\mathbf{k}) + T(\mathbf{q}|\mathbf{k},\mathbf{p}) = 0$.

Taking a cutoff surface through the wavenumber space, identified by the k_c, we define the energy flux across that scale as,

$$\Pi(k_c) = -\int_0^{k_c} \mathrm{d}k\, T(k) = \int_{k_c}^{\infty} \mathrm{d}k\, T(k)$$
$$= \int_{k_c}^{\infty} \mathrm{d}k \iint_0^{\infty} \mathrm{d}p\, \mathrm{d}q\, T(k|p,q)\,. \quad (2)$$

As example, in Figure 1 we show the free energy flux normalized by the total dissipation rate $\mathscr{D}$ (equal to the total injection rate for a stationary state). As a particularity of GK turbulence, we observe that only a fraction (54% here) of the energy injected into the system contributes to the nonlinear cascade. This is due to the nontrivial dissipative character of GK turbulence, which acts at all scales. In fact, the scale flux plateau level is given by the sum of the positive part of the linear contribution, $\mathscr{L}^+$.

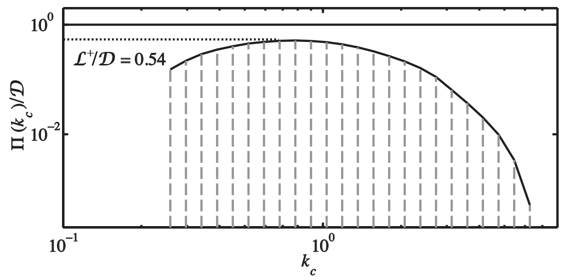

Figure 1: The free energy flux across the shell boundaries normalized by the total dissipation rate $\mathscr{D}$. The vertical dashed lines represent the shell boundaries, typically given as a power law in terms of the wavenumber, here $k_n = k_0 \times 2^{(n-1)/5}$. For the CBC simulation considered here, $k_0 = 0.258$.

To better understand the *triple-scale-transfers* contributing to the energy flux, we decompose the last two integrals over p and q in respect to k_c,

$$\Pi(k_c) = \int_{k_c}^{\infty} \mathrm{d}k \left[\underbrace{\int_0^{k_c} \mathrm{d}p \int_0^{k_c} \mathrm{d}q}_{I} + \underbrace{\int_0^{k_c} \mathrm{d}p \int_{k_c}^{\infty} \mathrm{d}q}_{II} + \underbrace{\int_{k_c}^{\infty} \mathrm{d}p \int_0^{k_c} \mathrm{d}q}_{III} + \underbrace{\int_{k_c}^{\infty} \mathrm{d}p \int_{k_c}^{\infty} \mathrm{d}q}_{IV} \right] T(k|p,q)\,. \quad (3)$$

The first term (I) contains the contribution of triads which have both legs across the surface. For the second term (II), only p is across the cutoff surface, while for the third term (III) only the q leg of the triad penetrates the surface. These two terms are equal in contributions since the *triple-scale-transfer* ($T(k|p,q)$) is symmetric in p and q. The last term (IV) is always zero due to the conservation of interactions. These terms contributions, in respect to p and q, are represented in Figure 2.

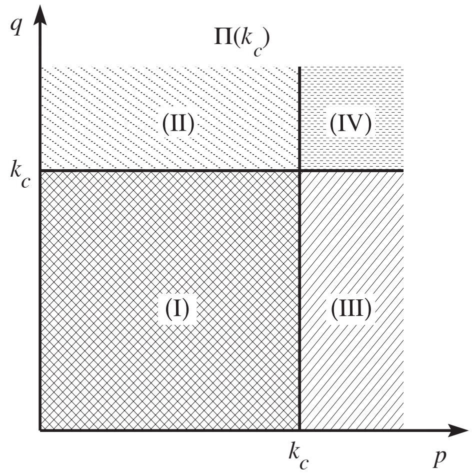

Figure 2: The contributions to the free energy flux across k_c, from p and q.

Starting from the definition of the flux, the infrared (IR) locality function is then defined by taking a second probe wavenumber surface (k_p) in such a way ($k_p \leq k_c$) that it limits the selection of triads that contribute to the energy flux through k_c, [3]. Conceptually, the definition can be seen as being obtained from Eq. 3, by replacing the limits of the integrals inside the square bracket from k_c to k_p and it reads as,

$$\Pi_{\mathrm{ir}}(k_p|k_c) = \int_{k_c}^{\infty} \mathrm{d}k \left[\int_0^{k_p} \mathrm{d}p \int_0^{k_p} \mathrm{d}q + 2\int_0^{k_p} \mathrm{d}p \int_{k_p}^{\infty} \mathrm{d}q \right] T(k|p,q) \; . \tag{4}$$

A similar definition is made for the ultraviolet (UV) locality functions, $k_c \leq k_p$,

$$\Pi_{\mathrm{uv}}(k_p|k_c) = \int_0^{k_c} \mathrm{d}k \left[\int_{k_p}^{\infty} \mathrm{d}p \int_{k_p}^{\infty} \mathrm{d}q + 2\int_0^{k_p} \mathrm{d}p \int_{k_p}^{\infty} \mathrm{d}q \right] T(k|p,q) \; . \tag{5}$$

which measures the contribution to the flux through k_c from triads of modes with at least one wavenumber greater than k_p, therefore providing information regarding the locality makeup of a scale k_c in relation with smaller and smaller scales. The idea of locality can be seen as the disparity between scales contributing to a nonlinear interaction. For a given energy flux through a scale, the degree to which each scale contributes to the mentioned flux represents an assertion of locality, [4]. For the interaction to be local, the contribution of highly separated scales should be small and decrease fast with the increase in separation between k_c and k_p.

Looking at the plot of $\Pi_{\mathrm{ir}}(k_p|k_c)/\Pi(k_c)$ as a function of k_p/k_c and $\Pi_{\mathrm{uv}}(k_p|k_c)/\Pi(k_c)$ as a function of k_c/k_p will reveal information related to the locality characteristic of the non-linear terms. The collapse of the locality functions dependence on k_p for different values of k_c represents a clear sign of self-similarity of the nonlinear interactions, which implies a dominance of the nonlinear terms in regard to the linear ones. Moreover, if the mentioned collapse exhibits a slope (in a log-log scale), then a state of asymptotic locality can be inferred, i.e. the nonlinear interactions saturate dynamically to the same level, no mater how large the turbulence level becomes. From our simulations, Figure 3, none of these two behaviors can be clearly observed.

Theoretically, a $(k_p/k_c)^{\pm 5/6}$ exponent for the IR and UV locality functions can be determined for an infinitely long inertial range, using scaling arguments similar to [5] and the fields scalings

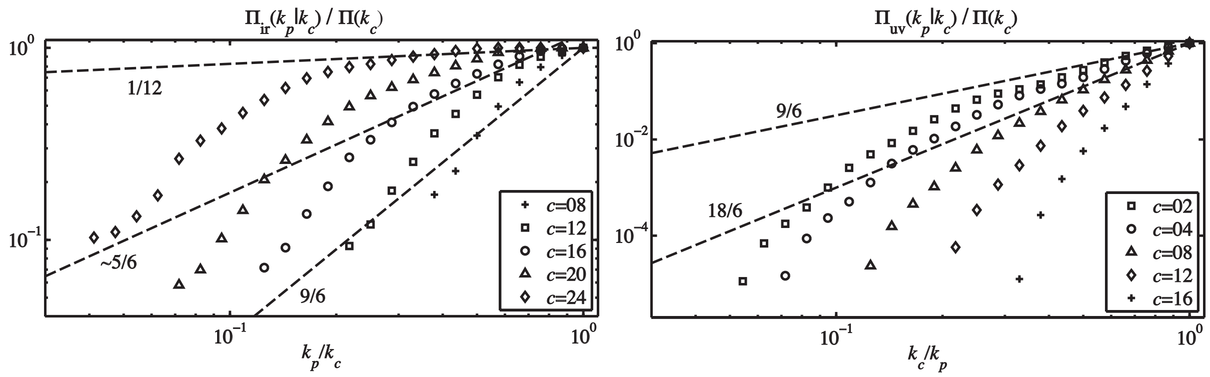

Figure 3: The IR and UV locality functions, displayed for selected cutoff wavenumbers identified by the boundary index c. Dashed lines equal or proportional to different power laws of the abscissae are displayed for reference.

provided by [6]. Although an asymptotic $5/6$ scaling of the IR and UV locality functions seems plausible and would indicate a more local interaction compared to magnetohydrodynamic turbulence ($2/3$) but more non-local compared to fluid turbulence ($4/3$), these values can not be clearly identified from our simulations. First, we need to consider that the theoretical $5/6$ exponent is found in the limit of an infinite inertial range, an ansatz not verified in any range for our GK turbulence simulation. In spite of the local energy cascade [6, 7], due to dissipation, the interaction of a given scale with smaller ones will be strongly damped, increasing the scaling of the UV locality functions. The same scale will itself be damped compared to the larger scales, decreasing the IR locality exponent. An effective non-local IR contribution signifies a dependence of GK turbulence on the type of instability driving it, while a stronger local UV depicts an insensitivity of GK's large scales on the small scales and therefore the type of collision mechanism employed.

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