Coupling axisymmetric plasma nonlinear evolution with three-dimensional conducting structures

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Abstract

We introduce a mathematical and numerical formulation able to couple a nonlinear axisymmetric plasma evolution (in the perturbed equilibrium limit) with a three-dimensional volumetric description of external conductors. The proposed formulation is applied to some ITER configurations.

Introduction

Due to electromagnetic forces and heat loads, the plasma disruptions have a serious impact on the operational lifetime of several components and in extreme cases seriously damage the integrity of fusion devices themselves. Consequently, disruptions play a fundamental role in the design of key elements (e.g. the vessel) of new experimental devices like ITER [1].

The analysis of causes and effects of disruptions is typically based on experimental data [2], collected in present-day devices. In order to extrapolate the available data to next-generation devices, reliable and complete computational tools are needed. Unfortunately, disruptions are complex events, requiring in principle extremely detailed models. Presently, several modelling approaches are available for the analysis of disruptions, e.g.:

- axisymmetric nonlinear models of plasma and conductors [3];
- 3D nonlinear models of plasmas, with an axisymmetric description of the structures [4];
- simplified plasma models (in terms of current-driven filaments) with a detailed 3D geometrical description of the structures [5];
- linearized plasma modelling, with detailed 3D structures [6, 7].

However, none of them is fully satisfactory, due to limitations and ranges of applicability. Consequently, the analysis of disruptions is one of the top priorities in modelling advances. In this paper, we present a formulation able to couple a nonlinear axisymmetric plasma model (perturbed equilibrium approach) with a three-dimensional volumetric description of surrounding structures. This formulation, in the stream of [6, 7], uses a coupling surface to describe the electromagnetic interaction between the plasma and the conductors.

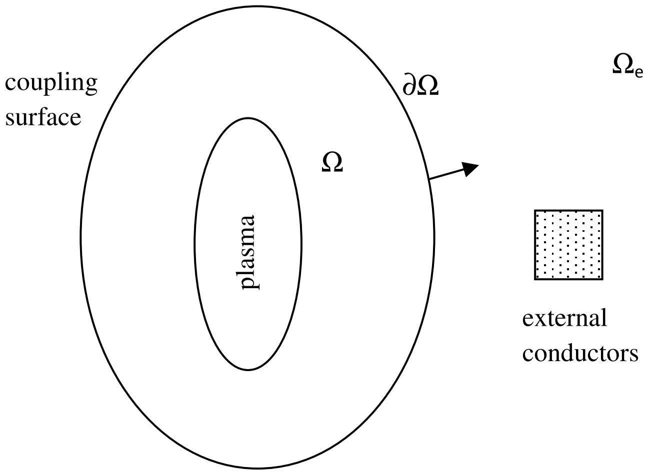


Fig. 1. Reference geometry

Formulation

With reference to Fig. 1, the mathematical model is:

$$L\,\psi = \mu_0\, j_\varphi(\psi) \text{ in } \Omega$$
$$\text{Conductors equations in } \Omega_e \qquad (1)$$

where the equations in Ω_e will be specified in the following, ψ is the magnetic flux per radian, L is the Grad-Shafranov operator and $j_\varphi(\psi)$ is the toroidal current density in the plasma, depending nonlinearly on ψ. We can mathematically close the problem as follows:

$$L\,\psi = \mu_0\, j_\varphi(\psi) \text{ in } \Omega$$
$$\psi\big|_{\partial\Omega} = \hat{\psi} \qquad (2)$$

where the unknown quantity $\hat{\psi}$ can be expressed as: $\hat{\psi} = \hat{\psi}_p + \hat{\psi}_e$; the suffix "*p*" (resp. "*e*") indicates the contribution of the (plasma) currents inside Ω (resp. external currents). We give a weak form of (2) in Ω:

$$-\int_\Omega \frac{1}{r}\nabla\psi\cdot\nabla w\, d\Omega + \int_{\partial\Omega}\frac{1}{r}\frac{\partial\psi}{\partial n} w\, dS = \int_\Omega \mu_0 j_\varphi(\psi)\, w\, d\Omega \qquad (3)$$

where w is a suitable test function. Giving a 2D finite elements discretization of $\Omega\cup\Omega_e$, and calling λ_i the hat function related to the i-th node of the mesh, we expand ψ in Ω as:

$$\psi = \sum_{i\in N_i}\psi_i\lambda_i + \sum_{j\in N_b}\hat{\psi}_j\lambda_j \qquad (4)$$

where N_i (resp. N_b) is the set of indices of the nodes inside Ω (resp. on ∂Ω). Using the Galerkin method, (3) becomes:

$$\underline{\underline{A}}\,\underline{\psi} = \underline{f}(\underline{\psi}) - \underline{\underline{\hat{A}}}\,\underline{\hat{\psi}} \qquad (5)$$

$$A_{i,j} = -\int_{\Omega} \frac{1}{r} \nabla\lambda_i \cdot \nabla\lambda_j \, d\Omega \quad , \quad i, j \in N_i$$
$$\hat{A}_{i,j} = -\int_{\Omega} \frac{1}{r} \nabla\lambda_i \cdot \nabla\lambda_j \, d\Omega \quad , \quad i \in N_i, j \in N_b \tag{6}$$
$$f_j(\underline{\psi}) = \int_{\Omega} \mu_0 j_\varphi \left(\sum_{i \in N_i} \psi_i \lambda_i \right) \lambda_j \, d\Omega \quad , \quad j \in N_i$$

For what concerns ψ_p, this quantity is solution of:

$$L\,\psi_p = \mu_0 \, j_\varphi(\psi) \text{ in } \Omega$$
$$L\,\psi_p = 0 \text{ in } \Omega_e \tag{7}$$
$$\psi_p \text{ regular at infinity}$$

whose numerical solution can be written as:

$$\underline{\underline{\tilde{A}}}\,\underline{\tilde{\psi}}_p = \underline{\tilde{f}}(\underline{\psi}) \tag{8}$$

where quantities with "~" have the same definitions as above, but extended over $\Omega \cup \Omega_e$. Inverting (8), we get:

$$\underline{\hat{\psi}}_p = \underline{\underline{E}}_2 \, \underline{\underline{\tilde{A}}}^{-1} \underline{\underline{E}}_1 \, \underline{f}(\underline{\psi}) = \underline{\underline{K}} \, \underline{f}(\underline{\psi}), \qquad \underline{\underline{K}} = \underline{\underline{E}}_2 \, \underline{\underline{\tilde{A}}}^{-1} \underline{\underline{E}}_1 \tag{9}$$

where $\underline{\underline{E}}_1$ and $\underline{\underline{E}}_2$ are suitable matrices containing 0's and 1's. Substituting in (5):

$$\underline{\underline{A}}\,\underline{\psi} = \underline{f}(\underline{\psi}) - \underline{\underline{\hat{A}}}\,\underline{\underline{K}}\,\underline{f}(\underline{\psi}) - \underline{\underline{\hat{A}}}\,\underline{\hat{\psi}}_e \tag{10}$$

The flux due to external currents in 3D conductors is computed with the same approach used in the CarMa code [6, 7]. Calling $\underline{I}$ the 3D currents, we have (matrices are defined in [6, 7]):

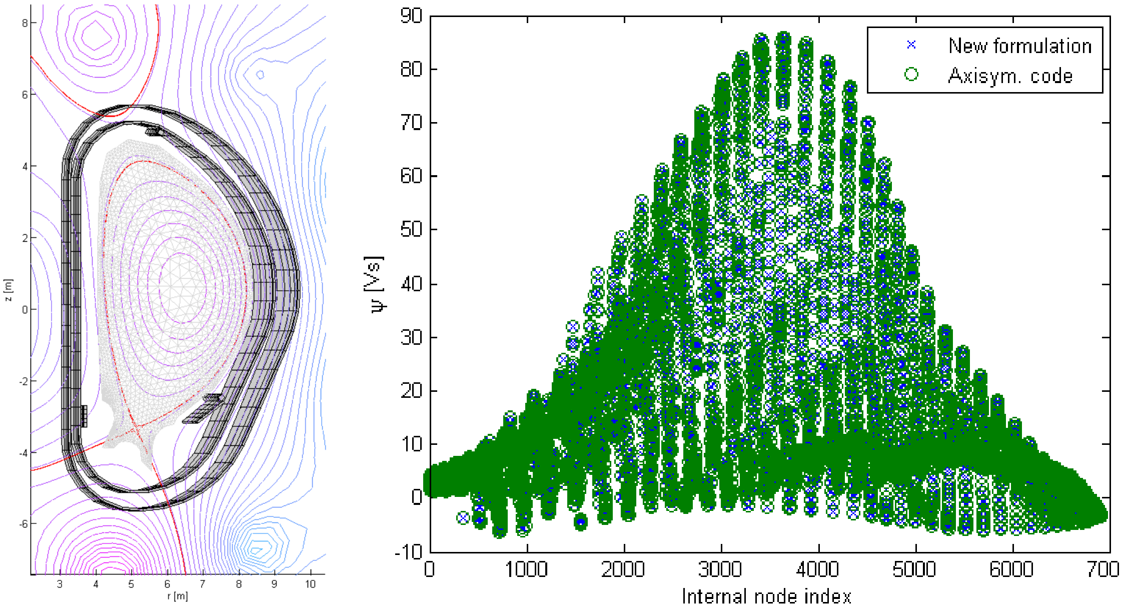
$$\underline{\underline{L}} \frac{d\underline{I}}{dt} + \underline{\underline{R}}\,\underline{I} + \underline{\underline{M}} \frac{d\underline{I}_{eq}}{dt} = \underline{\underline{F}}\,\underline{V}$$
$$\underline{\psi}_e = \underline{\underline{C}}\,\underline{I} \tag{11}$$
$$\underline{I}_{eq} = \underline{\underline{S}}\,\underline{f}(\underline{\psi})$$

where $\underline{V}$ are the voltages fed to electrodes and $\underline{I}_{eq}$ are equivalent toroidal currents that provide the same poloidal magnetic field as plasma outside Ω [7]. We assume no plasma toroidal flux variation. The various matrices introduced in (11) are defined in [6, 7]. Using an implicit Euler scheme to discretize time derivative:

$$\left(\underline{\underline{L}} + \Delta t\,\underline{\underline{R}}\right)\underline{I} + \underline{\underline{M}}\,\underline{I}_{eq} = \Delta t\,\underline{\underline{F}}\,\underline{V} + \underline{c}_{prev}$$
$$\underline{\psi}_e = \underline{\underline{C}}\,\underline{I} \tag{12}$$
$$\underline{I}_{eq} = \underline{\underline{S}}\,\underline{f}(\underline{\psi})$$

where $\underline{c}_{prev}$ is a known term depending on the previous time step. Combining (10) and (12), we get, with suitable definitions, the following nonlinear system of equations:

$$\underline{\underline{A}}\,\underline{\psi} + \underline{\underline{H}}_1\,\underline{f}(\underline{\psi}) + \underline{\underline{H}}_2\,\underline{f}(\underline{\psi}) + \underline{\underline{H}}_3\,\underline{V} + \underline{d}_{prev} = 0 \tag{13}$$



(a) (b)

Fig. 2. (a) Plasma configuration, 2D mesh in the plasma region and 3D conducting structures; (b) Comparison of magnetic flux in the nodes of the 2D finite elements mesh in the plasma region

Results

We consider an ITER configuration, with the following parameters: plasma current I_p=15MA, current centroid (6.29 m, 0.57 m), poloidal beta β_p=0.75, internal inductance l_i=0.79, elongation 1.878, triangularity 0.497. The plasma is circumvented by a conducting structure, which is discretized with a three-dimensional mesh mimicking an axisymmetric structure, in order to compare the results with available axisymmetric codes. Fig. 2a shows the plasma configuration, the 2D mesh in the plasma region and the 3D conducting structures considered. To test the procedure, we consider a current variation in one of the external conductors, giving rise to a significant variation of plasma configuration (around 15 cm of radial displacement and 3 cm of vertical displacement of the current centroid). Fig. 2b reports the comparison of the present formulation with an axisymmetric nonlinear code, showing excellent agreement.

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