Calculations of collisionless $\alpha$-particle losses for quasi-helically symmetric toroidal stellarators in real space coordinates

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Introduction
Quasi-helically symmetric stellarators represent one way to improve the particle and energy confinement in stellarators. In [1] the possibility of existence of quasi-helically symmetric (QHS) toroidal stellarators has been shown in magnetic coordinates. This idea has been practically realized in HSX [2]. In [3] it has been shown that for HSX the effective ripple is strongly reduced compared to the standard classical stellarator but it is larger than for the ideal QHS zero-beta variant. Here computations of collisionless $\alpha$-particle losses are carried out for the two configurations adapted to reactor plasma parameters with plasma average radius $a = 1.6m$ and $B = 5T$. Calculations are performed for the magnetic field produced by the coil system of HSX whereas the magnetic field for QHS consists of a decomposition in toroidal harmonic functions. The code [4] is applied for direct computations of particle losses solving the guiding center drift equations in real-space coordinates, whereas in [5] magnetic coordinates are used. In addition, simplified criteria [6,7] are used to analyze possible reasons for the difference in the confinement properties of HSX and QHS.

Computational procedure and initial conditions
In the approach [4], a sample of 1000 particles (trapped plus untrapped) is followed with random starting points as well as random values of pitch angles on an initial magnetic surface. Every particle orbit is followed until the particle reaches the boundary surface of the confinement region where it is recorded as lost. Out of the general sample of $\alpha$-particles, trapped particles cause the dominant contribution to collisionless particle losses. In the calculations all classes of trapped particles are taken into account, i.e., particles trapped not only within one magnetic field ripple but also trapped within several magnetic field ripples. Calculations are performed for the life time of 3.5 MeV $\alpha$-particles which are started on two magnetic surfaces with $r/a \approx 0.25$ and and 0.5 with $r$ being the magnetic surface average radius. For HSX the magnetic field is calculated from currents in the coils using the Biot-Savart law code. The magnetic field for QHS is presented as decomposition in toroidal harmonic functions containing associated Legendre functions. The decomposition coefficients are obtained by minimizing the magnetic field component that is normal to the boundary magnetic surface of the VMEC equilibrium in [1]. An ambipolar radial electric field is not taken into account because it has only negligible effect on $\alpha$-particle motion. To accelerate computations, a Lagrange polynomial interpolation of the magnetic field is used for both configurations.

Computational results
Fig. 1 presents the collisionless time evolution of trapped $\alpha$-particle fractions. A decrease of these fractions corresponds to an increase of the corresponding losses.

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Fig. 1. Collisionless evolution of trapped $\alpha$-particle fraction for QHS (1, red points), HSX (2, blue points) and HSXm (3, black points) adapted to reactor parameters. Particles are started on the magnetic surfaces corresponding to $r/a \approx 0.25$ (left) and $r/a \approx 0.5$ (right).

Results for QHS with starting values at $r/a \approx 0.25$ are in good agreement with results computed for magnetic coordinates in [5], whereas the results in Fig. 1 for $r/a \approx 0.5$ are somewhat worse compared to results in [5]. One clearly observes that $\alpha$-particle losses for HSX are essentially larger than for QHS. At the end of the time interval at 1 sec almost all trapped particles are lost. Since the HSX magnetic system is constructed in accordance with the criteria of quasi-helical symmetry it is important to analyze the possible reasons for such a difference. To demonstrate the magnetic field ripples, Fig. 2 shows the distribution of $B$ along field lines of QHS and HSX for $r/a \approx 0.5$. In contrast to QHS, HSX has many additional small ripples (and local minima of $B$). For the integration interval close to 6 field periods there are 30 local minima of $B$ in HSX compared to only 5 in QHS. The number of these additional local minima of $B$ in HSX correlates with the number of twisted coils.

Fig. 2. Distribution of $B/B_0$ along the magnetic field line in QHS (left) and HSX (right) for the magnetic surface $r/a \approx 0.5$. Numbered local minima of $B$ are also shown by points and $n$ represents the number of integration steps along the field line with 1280 steps per magnetic field period.

A set of simplified parameters, $\eta$, $v_\theta$ and $\gamma_c$, have been introduced in [6,7] to characterize the trapped particle motion in every magnetic field ripple. According to [6,7], $\eta$ represents the normalized bounce averaged drift velocity across a magnetic surface (for the standard stellarator the amplitude of $\eta$ is approximately 0.5), $v_\theta$ represents the normalized poloidal drift velocity and $\gamma_c$ allows to assess the angle between the $J_\parallel$ contours and the magnetic surface (for $|\gamma_c|=1$ the $J_\parallel$ contour is perpendicular to the magnetic surface and a corresponding trapped particle is quickly lost). Caused by the additional ripples, pronounced differences between HSX and QHS can be observed in the quantities $\eta$, $v_\theta$ and $\gamma_c$ which are presented in Fig. 3.
Fig. 3. Parameters $\eta$, $v_\theta$ and $\gamma_c$ in QHS (left) and HSX (right) for the magnetic surface $r/a \approx 0.5$. Every curve represents the dependence on the pitch angle $\gamma$ and corresponds to a certain number of a minimum of $B$.

To demonstrate the influence of a reduction of the small additional ripples in HSX, the number of twisted coils in HSX is artificially increased from 48 to 96. Of course, a correct way to obtain a new extended system of twisted coils would require a new design process where the pertinent codes would have to be used. However, to accelerate the study a simplified way is used here although this can lead to some violation of quasi-helical symmetry. In addition to the existing 48 base coils, a set of 48 additional twisted coils is constructed numerically with conductor coordinates chosen to be the average of the corresponding conductor coordinates for a pair of adjacent base coils. These new coils are placed between the base coils and together with them form the twisted coil system of 96 coils which further is named HSXm.

Results of computations for HSXm analogous to those in Figs. 2 and 3 are shown in Fig. 4. The number of local minima of $B$ decreases from 30 to 10 and results for $\eta$, $v_\theta$ and $\gamma_c$ strongly improve and show similar behavior than those for QHS. The corresponding results for col-
collisionless direct losses of $\alpha$-particles for HSXm are shown in Fig. 1 (curve 3). An essential improvement of collisionless $\alpha$-particle confinement can be observed and the corresponding curves for the evolution of the trapped particle fraction are rather close to those for QHS.

Fig. 4. The same as Figs. 2 and 3 for HSXm

**Summary**
The results show that the collisionless $\alpha$-particle losses for QHS are negligible for particles started at $r/a \approx 0.25$. For particles started at $r/a \approx 0.5$ roughly a quarter would be lost. This is in agreement with the corresponding results in [5] for $r/a \approx 0.25$ and somewhat exceeds those results for $r/a \approx 0.5$. For HSX the particle losses are essentially larger than for QHS. The reason is connected with the presence of additional small magnetic field ripples in HSX. The amplitude of these ripples can be decreased by increasing the number of twisted coils. For such a system with 96 coils instead of 48 coils, the particle losses are decreased to a level comparable to that in QHS.

**References**