On the linear O-X mode-coupling in warm magnetized plasmas in toroidal magnetic traps

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1. Introduction

In this paper we study the conversion of ordinary (O) waves to extraordinary (X) plasma waves near the plasma cut-offs in the electron cyclotron resonance frequency range in a toroidal geometry. This process plays an important role in excitation of the electron Bernstein waves, which in turn provide an effective way for high-frequency heating and diagnostics of overdense plasma in spherical tokamaks and optimized stellarators [1].

The O-X mode-conversion occurs when turning points of both modes becomes close to each other; for plane waves in cold plasma this is possible in the mode-coupling region defined by the following conditions [2]:

$$N_\perp << N_\parallel, \ |\varepsilon_\parallel| << 1, \ |\varepsilon_+ - N_\parallel^2| << 1, \qquad (1)$$

where $N_\perp = ck_\perp/\omega$ and $N_\parallel = ck_\parallel/\omega$ are wave refractive indexes transverse and parallel to the magnetic field, $\varepsilon_\parallel$ and ε_+ are components of the dielectric tensor in Stix representation [3],

$$\varepsilon_\parallel = 1 - X, \quad \varepsilon_+ = 1 - X/(1+Y), \quad X = \omega_{pe}^2/\omega^2, \quad Y = \omega_{ce}/\omega,$$

ω, ω_{ce} and ω_{pe} are, correspondingly, the wave, electron cyclotron and plasma frequencies. With these conditions one can get an approximate solution to the dispersion relation for the left-polarized waves in a vicinity of the mode-region in the following form

$$N_\perp^2 \approx 2\varepsilon_\parallel(\varepsilon_+ - N_\parallel^2)/\varepsilon_+ .$$

According to usual notation, this solution describes the O wave for $\varepsilon_\parallel > 0$ and $\varepsilon_+ > N_\parallel^2$, and the X wave for $\varepsilon_\parallel < 0$ and $\varepsilon_+ < N_\parallel^2$. Propagation regions for the O and X waves are separated by the evanescent region defined as $\varepsilon_\parallel(\varepsilon_+ - N_\parallel^2) < 0$; $N_\parallel$ may be considered as being constant due to the toroidal symmetry (neglecting the poloidal field). Typical view of this region in a toroidal device is shown in fig. 1. The evanescent region is formed by two cut-off surfaces, $\varepsilon_\parallel = 0$ and $\varepsilon_+ - N_\parallel^2 = 0$, each of them may be the locus of turning points of either the O-mode or the X-mode depending on which surface corresponds to more dense plasma. In a toroidal geometry plasma density and ambient magnetic field vary along different directions resulting in misalignment of the cut-off surfaces. The first surface, $\varepsilon_\parallel = 0$, is a flux surface with the critical plasma density, $\omega_{pe} = \omega$; however due to variation of the magnetic field, the second surface, $\varepsilon_+ - N_\parallel^2 = 0$, does not correspond to a flux surface. For a certain range of $N_\parallel$ these surfaces may cross (fig.1, left), then the mode conversion actually occurs as a tunnelling of the electromagnetic radiation through the non-slab evanescent region in the vicinity of the intersection line. After toroidal curvature is neglected, propagation of the O and X waves may be treated as a two-dimensionally inhomogeneous problem [4,5]. In this paper we concentrate on the effects of the poloidal curvature and finite electron temperature.

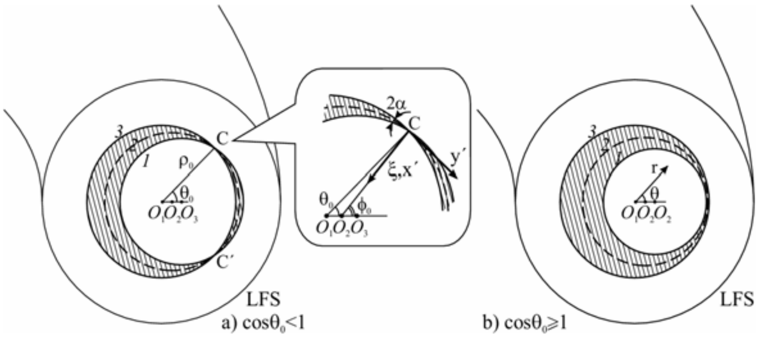

Fig. 1. O-X mode-coupling region in a tokamak geometry: (a) efficient coupling near the intersection points C and C' of the cutoff surfaces , (b) inefficient coupling for non-intersecting cut-off surfaces. The evanescent region for the left-polarized waves formed by two cut-off surfaces is shown in dashing; 1 – contours of $\varepsilon_{\|}=0$ which are circles with center O_1 at the magnetic axis; 2 – contours of $\tilde{\rho}=0$ which are circles with center O_2 shifted by $O_1O_2=\Delta\rho/2$ from the magnetic axis; 3 – contours of $\varepsilon_+ - N_{\|}^2=0$ which are approximately circles with center O_3 shifted by $O_1O_3=\Delta\rho$ from the magnetic axis.

2. Effects of poloidal curvature

Let us assume the simplest model for a toroidal magnetic configuration – the surfaces of constant pressure are formed by concentric circular tori, and the magnetic field strength is inversely proportional to a distance from the axis of symmetry, $B=B_0[1+(r/L_B)\cos\theta]^{-1}$, where θ is a poloidal angle counted from the equatorial plan and from the low-field-side. The considered geometry in a more symmetrical way, it is convenient to introduce new polar coordinates (ρ,ϕ) with the origin O_2 exactly between the centers O_1 and O_3 of two cut-off surfaces shown in fig. 1. Also it is convenient to shift the coordinate $\tilde{\rho}=\rho-\rho_0+\frac{1}{2}\Delta\rho\cos\theta_0$ such that $\tilde{\rho}$ is small inside the O-X coupling region, ρ_0 is the radius of a flux surface with critical density $X=1$. Finally one obtains

$$\varepsilon_{\|}\approx g_{\|}[\tilde{\rho}+\Delta\rho(\cos\phi-\cos\phi_0)/2],\ \ \varepsilon_+-N_{\|}^2\approx g_+[\tilde{\rho}-\Delta\rho(\cos\phi-\cos\phi_0)/2]$$

where $g_{\|}=|n_e'(\rho_0)/n_e(\rho_0)|=1/L_n$, $g_+=g_{\|}/(1+Y)$, $\Delta\rho=\rho_0 Y(L_n/L_B)/(1+Y)$, ϕ_0 is the new poloidal coordinate of the intersection of the cut-off surfaces defined from $\rho_0\sin(\phi_0-\theta_0)=\Delta\rho\sin\phi_0$. Thus, parameters $\Delta\rho$ and ϕ_0 fully characterize the geometry of the mode-coupling region. Fortunately, such spatial variation of dielectric tensor allows analytical solution of the reference wave equations in the mode-coupling region [6]. Wave field distribution in the beam propagating towards the plasma center may be found as

$$A^+(\xi,\phi)=\sum_{n=0}^{\infty}A_n\Phi_n(\phi)D_{i\nu_n}\left(\sqrt{2}e^{i\pi/4}\xi\right), \tag{2}$$

where $\xi=-\tilde{\rho}/L_\nabla$, $L_\nabla^2=N_{\|}/k_0\sqrt{2g_+g_{\|}}$, $D_{i\nu_n}$ are the parabolic cylinder functions, Φ_n are poloidal modes found the following equation with boundary condition $\Phi_n(0)=-\Phi_n(2\pi)$ [6]

$$\left(-d^2/d\phi^2+\delta^2(\cos\phi-\cos\phi_0)^2-\delta\sin\phi\right)\Phi_n=\lambda_n\Phi_n,\ \delta=\rho_0\Delta\rho/2L_\nabla^2,\ \lambda_n=2\rho_0^2\nu_n/2L_\nabla^2.$$

The obtained solutions in the cylindrical geometry suggest the "natural" coordinates – in the WKB region each poloidal mode propagates in radial direction and preserves its poloidal structure on surfaces $\rho = \text{const}$. Indeed, as it follows from properties of parabolic cylinder functions in the limit $|\xi| >> 1$, Eq. (2) may be rewritten as

$$A^+(\rho,\phi) = A_\phi^+(\phi)\exp\left(-i(\rho-\rho_0)^2/2L_\nabla^2\right),$$

i.e. the modulation over ϕ is conserved. Conservation of a poloidal structure results in modulation over the Cartesian coordinate y' across the beam propagation direction in the poloidal plane. However, just this coordinate seems to be most natural for specification of an incident beam. Let us consider for definiteness, an incident beam $A_y^+(y')$ specified on the plane tangent to the surface $\rho = \rho_i$ at point $\phi = \phi_i$:

$$A_\phi^+(\phi) = A_y^+((\phi-\phi_i)\rho_0)\exp\left(i\chi(\phi-\phi_i)^2/2 + i\chi^2(\phi-\phi_i)^4/8\xi_i^2\right)$$

with $\chi = \xi_i\rho_0/L_\nabla$ and $\xi_i = (\rho_i - \rho_0)/L_\nabla$. One can see an additional poloidal phase modulation over ϕ due to not conserving beam structure over y' in the WKB region. For example, for the Gaussian incident beam with a local poloidal width a_ϕ and a flat wave front in a vicinity of the surface $\rho = \rho_i$ we retain the quadratic phase modulation resulted from the different curvature of the wave front and the cut-off surfaces. Note, that in tokamak conditions $\chi >> 1$, so the curvature may be essential even for narrow beams with a small local width compared to the curvature itself. Strong phase-modulation in the poloidal direction results in strong degradation of the O-X coupling efficiency for wave beams with a flat wave front. For example, in case of a new ECRH launching system at the FTU tokamak [7] the maximum coupling efficiency is about 40% for a standard wave beam without phase-front tailoring [6].

3. Effects of a finite electron temperature

In a next generation fusion experiment the electron temperature may achieve a value at which thermal electron motion results in non-neglidgible contribution to the O-X mode-coupling. This is possible when relativistic factor of thermal electrons, $\beta_e^2 = T_e/m_e c^2$, becomes comparable to the small parameter of the geometric-optics, $1/k_0 L_n$. The thermal effects may be taken into account by retaining only the first-order terms over β_e^2 in the plasma dielectric tensor [8], and assuming condition $N_\perp << N_\parallel$ usual for the O-X mode-coupling. In this approximation, the dielectric tensor remains diagonal in the Stix representation with the following diagonal components:

$$\varepsilon_\pm = 1 - \frac{X}{1 \pm Y}\left(1 + \frac{N_\parallel^2\beta_e^2}{(1\pm Y)^2}\right), \quad \varepsilon_\parallel = 1 - X\ (1 + 3N_\parallel^2\beta_e^2),$$

where $X = \omega_{pe}^2/\omega^2$ and $Y = \omega_{ce}/\omega$. One can see that the small perturbation of the diagonal components results only to shifts of the cut-off surfaces $\varepsilon_\parallel = 0$ and $\varepsilon_+ = N_\parallel^2$.

Thermal shift of the cut-off layers results in modification of the optimal launching conditions for the wave beams. For example in the slab geometry with $Y = Y_0 = \text{const}$ and $X = 1 + x/L_n$, the optimal propagation direction with taken into account the finite temperature shift may be defined as

$$N_{\parallel}^2 = \frac{Y_0}{1+Y_0}\left(1 + \frac{3(1+Y_0)^2 - 1}{(1+Y_0)^2}\beta_e^2\right).$$

One can see that thermal corrections may be easily compensated by tuning $N_{\parallel}$-spectrum, but can lead to essential degradation of the O-X mode-coupling efficiency if this tuning is omitted. Level of this degradation may be estimated as a coupling coefficient calculated with thermal effects being taken into account but for the propagation direction optimal for the cold plasma:

$$T = \exp(-\delta),\ \delta = AL_n k_0 \beta_T^4 \quad ,\ A = \pi\sqrt{2Y_0}[3(1+Y_0)^2 - 1]^2(1+Y_0)^{-6}.$$

Here coefficient δ is calculated for the case of the slab geometry, however in a more complex tokamak geometry it differs only by a multiplier of the order of unity. For the case of ITER $\beta_e^2 = 1/50$, $\lambda = 1.7\text{mm}$, $L_n = 200 - 1000$ mm (depending on a scenario), what corresponds to $T = 10^{-1} - 10^{-4}$, i.e. not-compensated thermal degradation of the coupling efficiency may be very essential.

4. Conclusion

In the present paper two effects are considered that may result in essentialdegradation of the O-X coupling efficiency. These are the influence of the magnetic flux surface on the curvature of the phase fronts in optimal beams and the shift of the cut-off layers due to spatial dispersion induced by thermal electron motion. It is shown that each of these effects must be taken into account while planning future O-X-B heating experiments with overdense plasma, from the other side these effects may be responsible (along with plasma fluctuations [9] and parametric decay instabilities [10]) for low O-X-B heating efficiency in modern experiment.

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References

[1] H. P. Laqua, Plasma Phys. Control. Fusion 49, R1 (2007)
[2] E. Mjølhus J. Plasma Phys. 31, 7 (1984)
[3] T. H. Stix *The Theory of Plasma Waves* (New York: McGraw-Hill, 1962)
[4] E. D. Gospodchikov, A. G. Shalashov, E. V. Suvorov, Plasma Phys. Control. Fusion 48, 869 (2006)
[5] A. Yu. Popov, Plasma Phys. Control. Fusion 49, 1599 (2007)
[6] T. A. Khusainov, E. D. Gospodchikov, A. G. Shalashov, Plasma Phys. Rep. 38 (2), 83 (2012)
[7] A. Bruschi, W. Bin, S. Cirant, G. Granucci, et al. Fusion Science and Technology 55 (1) 94 (2009)
[8] V. V. Zheleznyakov *Radiation in Astrophysical Plasmas* (Dordrecht: Kluwer, 1996)
[9] H. P. Laqua, V. Erckmann, H. J. Hartfuß, et al. Phys. Rev. Lett. **78** (18), 3467 (1997)
[10] A. Surkov, G. Cunningham, A. Gurchenko, et al. Proc. 32nd EPS Conference on Plasma Phys. (Tarragona, 27 June - 1 July 2005) ECA Vol. 29C, P-5.103 (2005)