Quasilinear Response to Resonant Magnetic Field Perturbations in a Tokamak*

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Introduction

Resonant magnetic field perturbations (RMPs) from external coils are a useful tool for mitigation of Edge Localized Modes (ELMs) in tokamaks. Linear theory predicts strong shielding of the perturbations by the plasma. At the same time, quasilinear MHD analysis suggests that RMPs may modify background plasma parameters and fully penetrate. In this report the problem of RMP interaction with the plasma is treated in quasilinear approximation within kinetic theory for cylindrical tokamak geometry. The linear problem is solved by the KiLCA code (Kinetic Linear Cylindrical Approximation) and the quasilinear problem - by a 1-D balance code. For this, we corrected our linear and quasilinear models by introducing a particle and energy conserving collision operator. Unlike the linear model, the quasilinear model is very sensitive to the details of the collision operator and it must be fully consistent with the conductivity model inside the Maxwell equation solver. It is shown that the new collision operator ensures Onsager symmetry of the quasilinear transport coefficient matrix and avoids artifacts such as fake heat convection which may appear in simple collision models.

Basic equations

Both, the linear plasma conductivity and the quasilinear transport coefficients are determined by the solution of the kinetic equation, $\hat{L}_V f = \hat{L}_{\text{cp}} f$ where $\hat{L}_V$ is the Vlasov operator, $\hat{L}_{\text{cp}} = \hat{L}_{\text{c}} + \hat{L}_{\text{cI}}$ is an energy preserving collision operator,

$$\hat{L}_{\text{c}} = \frac{\partial}{\partial u_\parallel} D \left(\frac{\partial}{\partial u_\parallel} + \frac{u_\parallel - V_\parallel}{v_T^2(r)} \right) \tag{1}$$

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is the Ornstein-Uhlenbeck operator and $\hat{L}_{\mathrm{cI}}$ is an energy conserving term given by

$$\hat{L}_{\mathrm{cI}}\tilde{f}(v_\perp,v_\parallel) = -\frac{\nu}{\sqrt{2\pi}v_T}\exp\left(-\frac{v_\parallel^2}{2v_T^2}\right)\left(\frac{v_\parallel^2}{v_T^2}-1\right)\int\limits_{-\infty}^{\infty}\mathrm{d}v'_\parallel\left(\frac{{v'_\parallel}^2}{v_T^2}-1\right)\tilde{f}(v_\perp,v'_\parallel). \tag{2}$$

This type of kinetic equation can be solved in cylindrical geometry up to the end in terms of a Green's function. The gyroaverage of the perturbed distribution function needed in the quasilinear problem has the form

$$f_{\mathbf{m}}(v_\parallel) = -\int\limits_{-\infty}^{\infty}\mathrm{d}v'_\parallel\, G_{\mathbf{m}\mathrm{p}}(v_\parallel,v'_\parallel)\left(A_1+\frac{v'^2_\parallel+v_\perp^2}{2v_T^2}A_2\right)f_0(v_\perp,v'_\parallel)\,v^r_{\mathbf{m}}(v_\perp,v'_\parallel) \tag{3}$$

where $f_{\mathbf{m}}$ and $v^r_{\mathbf{m}}$ are the amplitudes of the Fourier series over toroidal and poloidal angles of the perturbed distribution function and of radial guiding center velocity, respectively. $G_{\mathbf{m}\mathrm{p}}$ is the Green's function, and the thermodynamic potentials are

$$A_1 = \frac{1}{n}\frac{\partial n}{\partial r}+\frac{e}{T}\frac{\partial \Phi}{\partial r}-\frac{3}{2T}\frac{\partial T}{\partial r}, \qquad A_2 = \frac{1}{T}\frac{\partial T}{\partial r}. \tag{4}$$

These potentials determine the particle and energy fluxes,

$$\Gamma^{(\mathrm{EM})}_{(\mathrm{e,i})} = -n_{\mathrm{e,i}}\left(D_{11}A_1+D_{12}A_2\right), \qquad Q^{(\mathrm{EM})}_{(\mathrm{e,i})} = -n_{\mathrm{e,i}}T_{\mathrm{e,i}}\left(D_{21}A_1+D_{22}A_2\right), \tag{5}$$

through quasilinear diffusion coefficients. Retaining in $v^r_{\mathbf{m}}$ only parallel motion along the perturbed magnetic field and the $\mathbf{E}\times\mathbf{B}$-drift (these are the dominating processes for electrons), these coefficients are

$$\begin{aligned} D_{kl} &= \frac{1}{\sqrt{8\pi}v_{\mathrm{T}}B_0^2}\mathrm{Re}\sum_m\int_{\mathbb{R}}\mathrm{d}v_\parallel\int_{\mathbb{R}}\mathrm{d}v'_\parallel\, G_{\mathbf{m}p}(v_\parallel,v'_\parallel)\exp\left(-\frac{v'^2_\parallel}{2v_T^2}\right) \\ &\quad\times(v_\parallel B^r_{\mathbf{m}}+cE_{\mathbf{m}\perp})^*(v'_\parallel B^r_{\mathbf{m}}+cE_{\mathbf{m}\perp})a_{kl}(v_\parallel,v'_\parallel), \end{aligned} \tag{6}$$

$$a_{11}=1, \quad a_{12}=1+\frac{{v'_\parallel}^2}{2v_T^2}, \quad a_{21}=1+\frac{v_\parallel^2}{2v_T^2}=a_{12}, \quad a_{22}=2+\frac{v_\parallel^2+{v'_\parallel}^2}{2v_T^2}+\frac{v_\parallel^2{v'_\parallel}^2}{4v_T^4}. \tag{7}$$

Also, the Fourier amplitude of parallel equilibrium current density which is responsible for shielding the RMPs can be expressed in terms of the radial component of the magnetic perturbation field $B^r_{\mathbf{m}}$ and the the component of the electrostatic field $cE_{\mathbf{m}\perp}$ tangential to the unperturbed flux surface and perpendicular to the magnetic field,

$$j_{\mathbf{m}\parallel} = -\frac{nev_T}{\nu B_0}\left(\left((A_1+A_2)I^{10}_{\mathrm{p}}+\frac{1}{2}A_2I^{21}_{\mathrm{p}}\right)cE_{\mathbf{m}\perp}+\left((A_1+A_2)I^{11}_{\mathrm{p}}+\frac{1}{2}A_2I^{31}_{\mathrm{p}}\right)v_TB^r_{\mathbf{m}}\right). \tag{8}$$

The mismatch between the perturbed magnetic flux surfaces and the perturbed equipotential surfaces is the reason for quasilinear transport in this approximation. The lowest order Larmor radius approximation used in 6) and in (8) is sufficient for the electrons. These quantities are finally expressed via velocity moments of Green's function

$$I^{mn}_{(p)} = \frac{\nu}{\sqrt{2\pi} v_T^{m+n+1}} \int\limits_{-\infty}^{\infty} dv_\parallel \int\limits_{-\infty}^{\infty} dv'_\parallel \, G_{\mathbf{m}(\mathrm{p})}(v_\parallel, v'_\parallel) \exp\left(-\frac{{v'_\parallel}^2}{2v_T^2}\right) v_\parallel^m {v'_\parallel}^n \tag{9}$$

which are determined by similar moments of Green's function $\tilde{G}$ defined in Ref. [1] as follows

$$I^{mn}_{\mathrm{p}} = I^{mn} + \frac{\left(I^{m0} - I^{m2}\right)\left(I^{n0} - I^{n2}\right)}{1 - I^{00} + 2I^{20} - I^{22}}, \tag{10}$$

Due to the property $I^{21}_{\mathrm{p}} = I^{30}_{\mathrm{p}}$ the diffusion tensor satisfies Onsager symmetry.

Using (6), balance equations for plasma density n_e, toroidal ion rotation velocity V_i^{φ}, and electron and ion temperatures $T_{e,i}$ presented in Ref. [2] were solved for JET like parameters in experiments with ELM mitigation by C-coil. Only the 3/1 mode of the coil spectrum has been retained. Modelled are 4 variants of starting equilibria obtained by scaling the toroidal rotation velocity V^{ϕ} by factors 0.8 and 1, see Fig. 1, and by changing the anomalous diffusion coefficient by a factor 2. The results show that quasilinear effects do not lead to a significant increase in field penetration and may also lead to even stronger shielding despite that the parallel electron current in the resonant zone is reduced, see Fig. 2. In contrast to earlier MHD theories, the main quantity responsible for quasilinear relaxation is the electron temperature. The sensitivity of this quantity has been noticed earlier in Ref. [3]. In all cases, the perpendicular electron fluid velocity is evolving to zero in the resonant zone. In MHD theory, this would lead to field penetration. In kinetic theory, the point of field penetration is not the same, see Fig. 1.

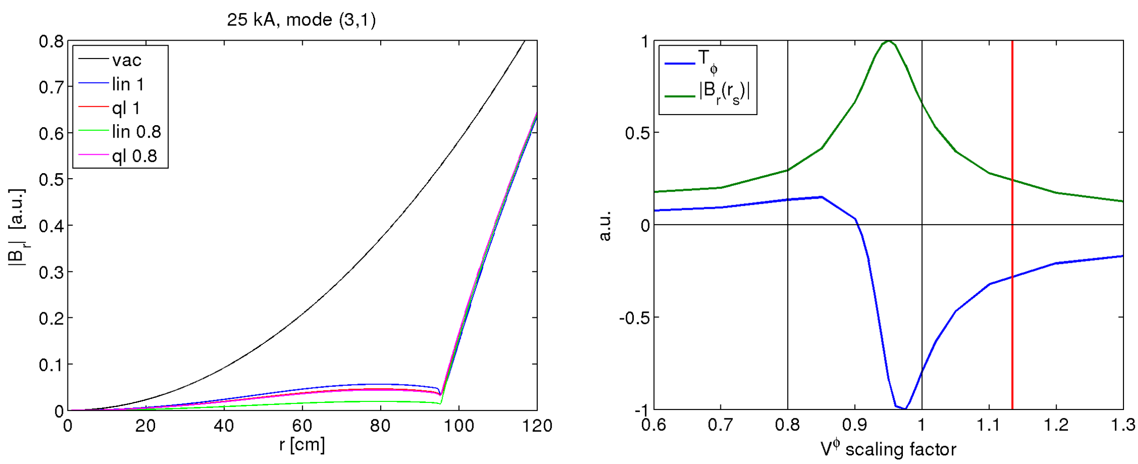

Fig. 1. Left: Radial profiles of $|B^r_{\mathbf{m}}|$ before and after quasilinear relaxation. Right: Toroidal torque and $|B^r_{\mathbf{m}}|$ at the resonant surface as functions of toroidal velocity scaling parameter. Parameter values used for computations and corresponding to zero electron fluid velocity at the resonant surface are indicated by black lines and red lines respectively.

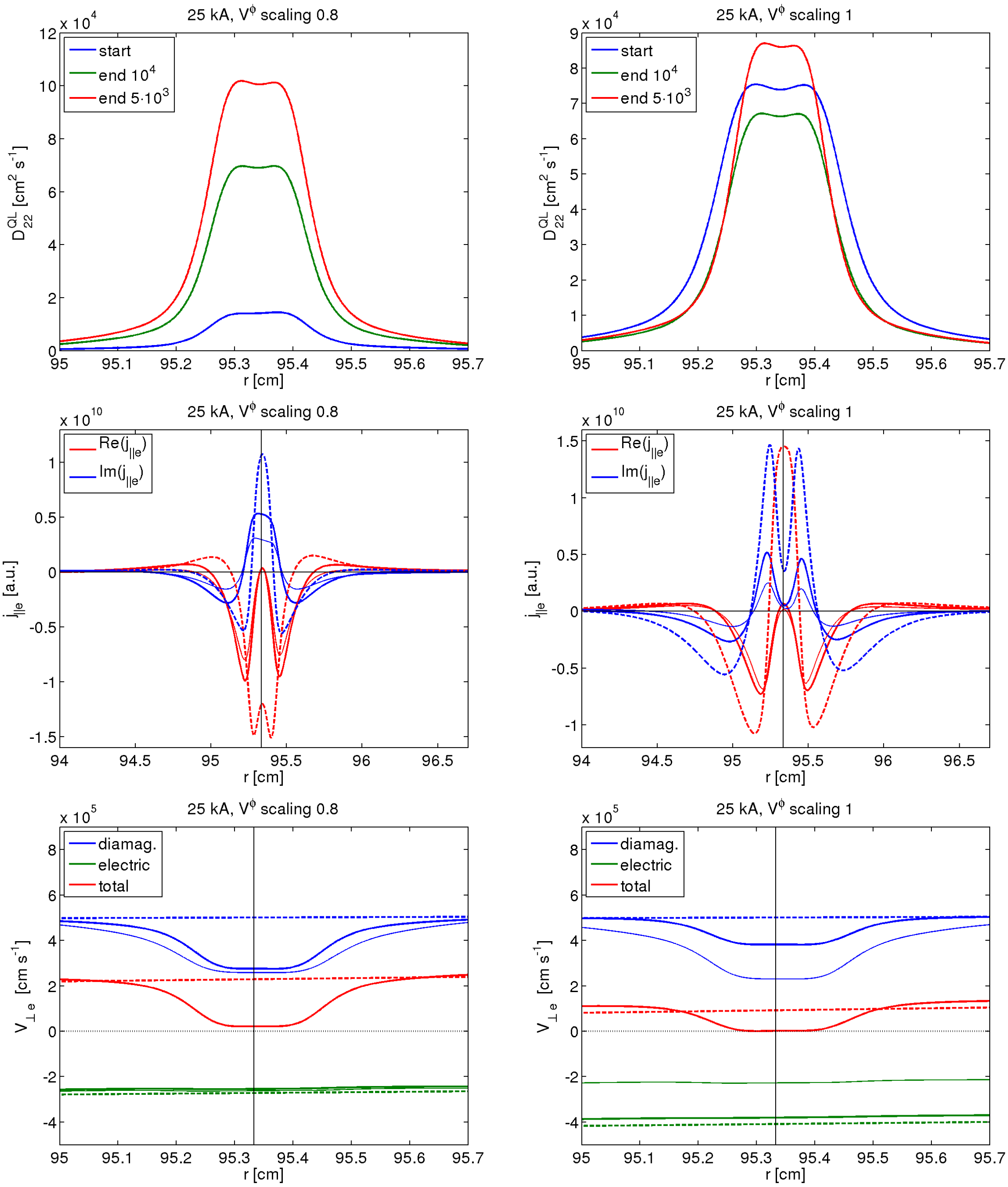

Fig. 2. Quasilinear heat conductivity coefficient (top), parallel electron current (middle), and perpendicular components of electron fluid velocity (bottom). Dashed lines for the currents and rotation velocity components show initial values. Thin lines correspond to evolution with reduced anomalous coefficients.

References

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