Modelling local time evolution of strong heat pulses in magnetically confined plasmas

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1. Introduction

Cold heat pulse propagation has remained for two decades a quasi-paradoxical feature of energy transport in magnetic confinement fusion (MCF) plasmas, both tokamaks\textsuperscript{[1-5]} and stellarators\textsuperscript{[6-9]}. In these experiments\textsuperscript{[1-9]}, see for example Fig.1, the electron temperature $T_e$ at the plasma edge is rapidly reduced by a transient local increase in radiation, typically induced by injection of a pellet which produces radiating impurity ions and cold electrons. The magnitude of the local gradient of electron temperature $\nabla T_e$ at the edge is correspondingly increased. In response to rapid edge cooling, the excursion of electron temperature from its equilibrium value in the core plasma may, over time, have either sign. As in any physical system, understanding the response of MCF plasmas to nonlinear heat pulses is central to understanding the equilibrium and non-equilibrium transport processes. However these experimental observations continue to challenge the interpretive capacity of theoretical plasma physics. The development of a generic model for the temporal evolution, at a given location, of energy flux and temperature during cold pulse experiments is reported here. It is motivated by recent high resolution measurements of cold pulse propagation experiments in the Large Helical Device\textsuperscript{[7-9]}. The experiments yield plots\textsuperscript{[9]} of the time evolution of the deviation of local heat flux and electron temperature gradients from their equilibrium values determined by self-consistent turbulent transport: respectively $\delta q_e(\rho, t)/n_0$ and $\delta \nabla T_e(\rho, t)$, where $\rho = r/a$, $a$ denotes the local minor radius of LHD (on average $a = 0.6$m) and $r$ denotes distance from the axis of symmetry. Violation of simple diffusive transport is clearly demonstrated, see Fig.1. Corresponding measurements of the time evolving local excursion (which may have either sign) of electron temperature from its equilibrium value were also obtained. These observations\textsuperscript{[9]} motivate the construction, from first principles, of a generic three-wave system of coupled nonlinear differential equations which generate key features of the observed phenomenology of local cold pulse time evolution.

2. Model outline

In our model, the perturbed heat flux and electron temperature gradient undergo a variant of coupled critically damped oscillation, and are in turn coupled to the local electron temperature; this oscillates as a nonlinear pendulum, driven by the perturbed heat pulse and electron temperature gradient, and damped by the equilibrium turbulent
plasma transport processes. The model results map well to the measurements of local cold pulse time evolution in LHD. In particular, the model matches the electron cyclotron emission measurements encapsulated in the published Lissajous diagrams, Figs.1(h,i) of Ref.[9], for $\delta q_e(\rho, t)/n_e$ and $\delta \nabla T_e(\rho, t)$ evaluated in the core plasma at radius $\rho = 0.19$. These embody measurements of LHD plasmas where, respectively, the edge cold pulse initiates either a substantial rise in central $T_e$, or a decline and recovery in central $T_e$. In both cases the Lissajous figure is closed, and the sense of circulation over time is the same: anticlockwise from the start, using the system of axes chosen in Ref.[9]. In the first case, the initial perturbation at $\rho = 0.19$ is a sharp increase in the magnitude of $\delta q_e/n_e$, which rapidly becomes increasingly negative, while $\delta \nabla T_e$ does not initially change from zero. At the end of this phase, the Lissajous circulation heads in the direction of increasingly negative $\delta \nabla T_e$, while the magnitude of the negative $\delta q_e/n_e$ declines. In the second case, the initial perturbation at $\rho = 0.19$ is a sharp increase in the magnitude of $\delta q_e/n_e$, which rapidly becomes increasingly positive, while $\delta \nabla T_e$ does not initially change from zero. At the end of this phase, the Lissajous circulation heads in the direction of increasingly positive $\delta \nabla T_e$, while the magnitude of the positive $\delta q_e/n_e$ declines[9].

The single-circuit closed Lissajous figures motivate our first conjecture, that the pulse propagation depicted in Figs.1(b,c,h,i) of Ref.[9] incorporates aspects of damped sine and cosine wave physics. The over-relaxation of central temperature $T_e$ (cooling followed by heating followed by cooling, with respect to its initial value $T_{e0}$) shown in Fig.1(b) of Ref.[9], for example, motivates our second conjecture, that the local
temperature incorporates aspects of damped nonlinear pendulum physics. Our third conjecture is that the key aspects of the observed local phenomenology can be captured by a coupled nonlinear three-wave system for the variables $\delta q_e(\rho, t)/n_e$, $\delta \nabla T_e(\rho, t)$ and $T_e - T_{e0}$.

Physically, it appears reasonable to assume that if the temperature gradient exceeds its equilibrium magnitude, so that $-\delta \nabla T_e$ is positive, this drives an increase in the outward turbulent flux above its equilibrium value, such that $\delta q_e/n_e$ tends to become increasingly positive. In our model, therefore, the local temporal rate of change of the mismatch between the turbulent outward heat flux and its equilibrium value, given the temperature gradient at that instant, is driven by the excess of the magnitude of the temperature gradient over its equilibrium value. Any excess heat flux will act so as to reduce the excess temperature gradient, driving relaxation towards the local equilibrium temperature gradient and its corresponding local equilibrium turbulent flux. In our model, the temporal rate of change of the local excess of electron temperature gradient over its equilibrium value is therefore driven by $\delta q_e/n_e$. In our third model equation, the time evolution of $T_e - T_{e0}$ is driven by the other perturbations and damped by the steady-state transport. Our model equations depend explicitly on time, but not on space, although the latter enters implicitly through the values of key local equilibrium plasma properties.

3. Model results

Figures 2(a,b) show time evolving traces derived from our model using experimentally derived transport coefficients in the underlying nonlinear coupled equations. We emphasise that the model traces are not fitted: they represent the time evolving solutions of the coupled nonlinear system of equations, given the initial conditions of the pulse on arrival and given ambient plasma conditions.

![Figure 2(a) Comparison (top panel) between measured and modelled time evolution of local electron temperature rate of change in the LHD core plasma at $\rho = 0.19$, for a case where the perturbing heat pulse generates a local temperature rise (second panel). The lower two panels show measured evolution of perturbed heat flux and temperature gradient.](image)
Figure 2(b) Comparison (top panel) between measured and modelled time evolution of local electron temperature rate of change in the LHD core plasma at $\rho = 0.19$, for a case where the perturbing heat pulse generates a local temperature decline (second panel). The lower two panels show measured evolution of perturbed heat flux and temperature gradient.

The success of the model suggests that its key physics elements, outlined above, correspond to those of the real system: the local response of the LHD core plasma to the incident strongly nonlinear heat pulse. It may therefore be generically applicable to similar experiments in other MCF plasmas.

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