Effects of second order magnetic barriers on the diffusion of magnetic field in stochastic magnetic fields in the DIII-D tokamak

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Magnetic field lines in fusion plasmas are trajectories of Hamiltonian systems. The problem of magnetic field line transport can be treated in the framework of the theory of Hamiltonian systems. In this context, the concepts of symmetry, nonlinear resonance and chaos are essential in understanding the dynamical evolution of Hamiltonian systems.

A magnetic field line moves through three-dimensional Cartesian space (x,y,z). But the Cartesian coordinates are not canonical coordinates for magnetic field lines. To obtain canonical coordinates, the magnetic field is expressed in a symplectic form [1] in terms of canonical coordinates $(\psi_t,\vartheta,\varphi)$ .The equations for the evolution of magnetic field lines have the canonical form $d\vartheta/d\varphi = \partial\psi_p(\psi_t,\vartheta,\varphi)/\partial\varphi$, $d\psi_t/d\varphi = -\partial\psi_p(\psi_t,\vartheta,\varphi)/\partial\vartheta$. A toroidal coordinate (φ) then plays the role of time, a toroidal flux (ψ_t) and a poloidal angle (θ) are the canonical coordinates. The poloidal flux, $\psi_p(\psi_t,\theta,\varphi)$ is the Hamiltonian. In an axisymmetric field, such as an equilibrium field in divertor tokamaks, one can always transform these coordinates to action-angle coordinates (ψ,θ) with $\chi = \psi_p(\psi)$the Hamiltonian. The safety factor q characterizing the winding of the magnetic field lines is given by $q(\psi)=1/\iota(\psi)=d\psi/d\chi$. The rotational transform is $\iota(\psi)=1/\ q(\psi)$. $q(\psi)$ is called the safety factor. The magnetic perturbations, in general, are not uniform along the toroidal, φ, and poloidal, θ, axes; and break the symmetry of the equilibrium field along the toroidal axis. In the presence of these non-axisymmetric magnetic perturbations, the total Hamiltonian χ can be written as $\chi(\psi,\theta,\varphi) = \chi_0(\psi) + \chi_1(\psi,\theta,\varphi)$. Here $\chi_0 = \int\iota(\psi)d\psi$ is the equilibrium Hamiltonian. The perturbed part of the Hamiltonian χ_1 is a doubly 2π- periodic function of θ and φ and it can always be written as a Fourier series, $\chi_1(\psi,\theta,\varphi) = \sum_{m,n} [\chi_{mn}(\psi) \cos(m\theta - n\varphi + \xi_{mn})]$ with m and n are the poloidal and toroidal mode numbers and ξ_{mn} the phases of the Fourier modes with mode numbers (m,n).

Any arbitrary small perturbation leads to the formation of magnetic islands near resonance surfaces. Resonance occurs when $q(\psi=\psi_{mn})=m/n$. These surfaces are called rational surfaces; and they are topologically unstable. When the islands grow in size and overlap, the magnetic surface structure is lost. Regions with chaotic field line trajectories arise. For sufficiently small perturbation, the KAM theory [2] states that these regions are limited and invariant tori (KAM surfaces) remain. In the absence of particle collisions and magnetic drifts, these KAM surfaces act as transport barriers inhibiting radial transport of heat and particles which predominantly follow magnetic field lines. This leads to non-uniform chaotic zones consisting of interwoven chaotic, islands and regular regions, which have complicated influence on the chaotic dynamics, and present a challenge to control the chaos. Chaos control is actively studied in many fields including plasma and accelerator physics. Recently, a control method based on a *localized* control of chaos in Hamiltonian systems has been proposed and extensively described in [3] and applied to problems in fusion plasmas [4-7]. The aim of this method is to reduce transport along chaotic field lines by adding a small, but well chosen, field perturbation to create KAM barriers in chaotic regions. An important and valuable feature of this method is that the perturbation that is required to control chaos is small compared to the size of the perturbation that creates the chaos in the first place.

In this paper, we employ the control and our mapping method to study the effects of second order magnetic barriers on the stochastic fields in the DIII-D tokamak. We use a symplectic map to calculate the trajectories of field lines. This map is derived from an analytic equilibrium generating function (EGF) constructed in action-angle coordinates (ψ,θ) from the experimental data of the Grad–Shafranov equilibrium solver EFIT [8,9,10] for the DIII-D tokamak. This map preserves the symplectic topological invariance of the Hamiltonian system. It has also been used to demonstrate the sensitivity of stochastic broadening from a set of qualitatively different magnetic perturbations [10,11]. We call this map the DIII-D map. We analyze the DIII-D map with and without second order control terms to study the formation of the stochastic layer created by two locked tearing modes, and the resulting local diffusion of field lines. The mean square radial deviation of field lines in the predominantly chaotic region is calculated as the strength of the magnetic perturbation is varied. For magnetic perturbation, we chose two tearing modes with mode numbers $(m, n) = \{(3, 1), (4, 1)\}$, $\chi_1(\theta, \varphi)=\delta[\cos(3\theta-\varphi)+\cos(4\theta-\varphi)]$. The (3,1) mode is resonant at $\psi_{31}\approx 1.3768$, and the (4,1) mode is resonant at $\psi_{41}\approx 1.6622$. δ is the amplitude of magnetic perturbation. Following [4,6], we add a second order magnetic perturbation given by $\chi_C^{(2)}(\theta,\varphi)=-\frac{1}{2}\delta^2 A\left[B\cos(3\theta-\varphi)+C\cos(4\theta-\varphi)\right]^2$ to the poloidal flux $\chi=\chi_0+\chi_1$. Here χ_0 is the equilibrium poloidal flux, ψ_b denotes the location of the magnetic barrier, $\omega_b = \iota(\psi_b)=1/q(\psi_b)$, $A=[d/d\psi(\iota(\psi))]_{\psi=\psi_b}$, $B=3/(3\omega_b-1)$, and $C=4/(4\omega_b-1)$. Fig. 1 shows the phase portrait of field lines trajectories computed without [Figs. 1a and 1b] and with the control term [Fig. 1c]. Fig. 1c clearly shows that the phase space is more regular, less chaotic than the corresponding uncontrolled one (Fig. 1b) for the same physical parameters.

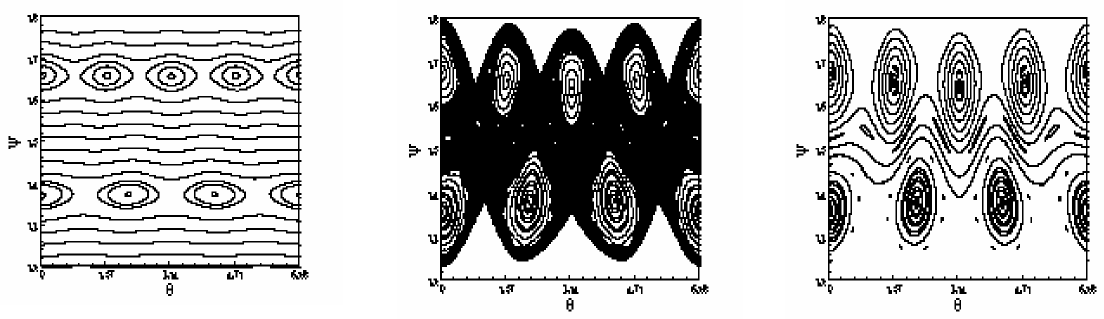

Fig. 1. (a) Phase portrait of magnetic field lines in DIII-D when the amplitude of resonant magnetic perturbations is (a) $\delta=10^{-4}$, and (b) $\delta=1.26\times10^{-3}$; and (c) same as (b) but with control term at $\psi_b=1.53981485409455$ added to the poloidal flux. In these plots, 21 lines are integrated. Each line is iterated for 10000 toroidal circuits of DIII-D. Initial values of ψ are chosen so that they are spaced almost uniformly in the interval (1.2,1.8).

Fig. 1b shows a sizable non-uniform chaotic layer between the resonance surfaces. The presence of surviving islands, surrounding the stable periodic points, leads to the deviations from the Gaussian diffusive law. The reason for this is the stickiness property of the KAM surfaces. In Fig. 2, we show the forward image obtained for the same physical parameters as in Fig. 1b. To study the effect of the second order control on the diffusion of the chaotic field lines in DIII-D, we calculate the radial excursions of the field lines by means of the mean square displacement (MSD). We start with 1000 field lines at a fixed initial value of $\psi = \psi_0$ in the chaotic region, (1.5,1.53) with N=1000 randomly chosen initial values of θ in the interval $(0,2\pi)$. The amplitude of the perturbation is fixed at $\delta=1.26\times10^{-3}$. Each field line is advanced for 10000 toroidal circuits of DIII-D. The MSD is calculated as $\sigma_n^2 = \left\langle \left(\delta\psi_n\right)^2 \right\rangle = \frac{1}{N}\sum_{i=1}^{N}\left(\psi_{ni}-\psi_0\right)^2$. Fig. 3 shows σ_n^2 with and without the control term.

Figs. 3a and b show the mean square displacements with and without the control term. The results clearly show a significant decrease in radial excursions of the field line in the chaotic region between the two primary resonances when the control term is added. Fig. 3a shows that initially a super-diffusive regime appears; and after about 10 iterations, the radial field line excursions are limited and a persistent sub-diffusive regime follows. Comparison of the results obtained from DIII-D and previous results on the ASDEX UG indicates that stronger barrier can be built in the DIII-D than in the ASDEX UG. High magnetic shear near the separatrix in the DIII-D is inferred as the possible cause of this. This work is supported by the US DOE grants DE-FG02-01ER54624 and DE-FG02-04ER54793. This research used resources of the NERSC, supported by the Office of Science, US DOE, under contract DE-AC02-05CH11231.

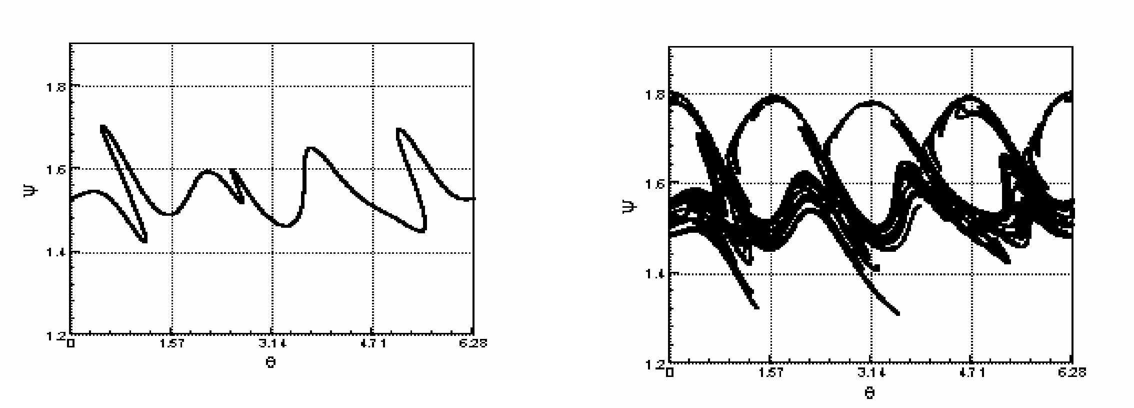

Fig. 2. Forward images of a ensemble of initial conditions within the chaotic region for the same parameters as in Fig. 2b. (a) after 5 iterations, (b) after 25 iterations.

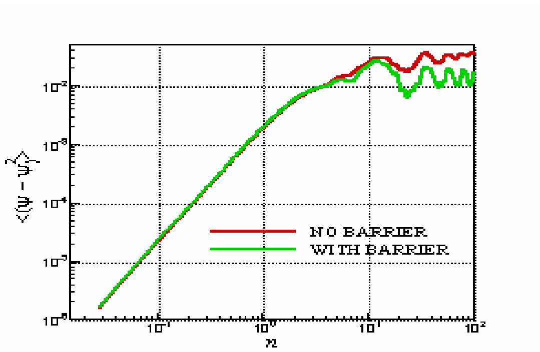

Fig. 3a. MSD for a single magnetic surface for 10000 circuits.

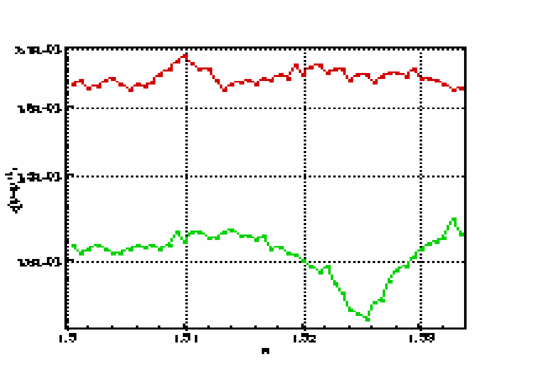

Fig. 3b. MSD for 50 magnetic surfaces in the neighborhood of the magnetic barrier.

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