Hydromagnetic solitary and shock waves in a magnetoplasma

P. K. Shukla[1], B. Eliasson[2], L. Stenflo[3]

[1] *International Centre for Advanced Studies in Physical Sciences and Institute for Theoretical Physics, Faculty of Physics & Astronomy, Ruhr University Bochum, D-44780 Bochum, Germany, and Department of Mechanical and Aerospace Engineering & Center for Energy Research, University of California San Diego, La Jolla, CA 92093, U. S. A.*

[2] *International Centre for Advanced Studies in Physical Sciences and Institute for Theoretical Physics, Faculty of Physics & Astronomy, Ruhr University Bochum, D-44780 Bochum, Germany*

[3] *Department of Physics, Linköping University, SE-581 83 Linköping, Sweden*

We present a theory for large amplitude compressional dispersive Alfvénic (CDA) shocks and solitary waves [1] that are propagating in a warm collisional plasma across a uniform magnetic field ($\hat{\mathbf{z}}B_0$), where $\hat{\mathbf{z}}$ is the unit vector along the z-axis in a Cartesian coordinate system, and B_0 is the strength of the magnetic field. The cold and collisionless limit was studied by Adlam and Allen [2] more than fifty years ago, while Nairn *et al.* [3] derived a Kortweg-de Vries (KdV) equation that governs the dynamics of small amplitude nonlinear compressional dispersive Alfvénic (CDA) waves in a cold magnetoplasma. The restoring forces comes from the wave magnetic pressure, whereas the ion mass provides the inertia to sustain the CDA waves. The CDA wave electric field is $\mathbf{E}_\perp = \hat{\mathbf{x}}E_x + \hat{\mathbf{y}}E_y$, where $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ are the unit vectors along the x and y-axes, respectively, whereas the CDA wave magnetic field is aligned along the z-axis. In a quasi-neutral plasma with $n_e = n_i \equiv n$, where n_e and n_i are the electron and ion number densities, respectively, the x components of the electron and ion fluid velocities are equal (viz. $u_{ex} = u_{ix} \equiv u$), whereas the x and y components of the electron fluid velocities differ owing to the electron polarization drift. The electrons carry currents only along the y-direction. The CDA waves compress the magnetic field-lines without bending them, and are accompanied by density perturbations.

The nonlinear propagation of one-dimensional CDA waves along the x-axis in our quasi-neutral collisional magnetoplasma is thus governed by the ion continuity equation [1]

$$\frac{Dn}{Dt} + n\frac{\partial u}{\partial x} = 0, \tag{1}$$

the momentum equation

$$\frac{Du}{Dt} + \frac{1}{2n}\frac{\partial B^2}{\partial x} + \frac{\beta}{n}\frac{\partial n^3}{\partial x} = 0, \tag{2}$$

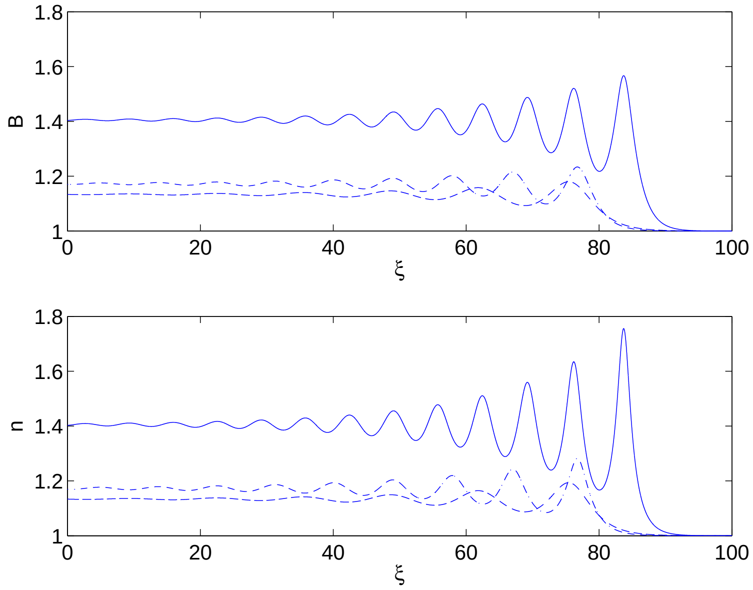

Figure 1: Profiles of the magnetic field and plasma density for $\alpha = 0.1$, using $M = 1.3$ and $\beta = 0$ (solid curves), $M = 1.1$ and $\beta = 0$ (dashed curves), and $M = 1.3$ and $\beta = 0.1$ (dash-dotted curves). After Ref. [1].

and the induction equation derived from Faraday's law,

$$\frac{\partial B}{\partial t} + \frac{\partial (uB)}{\partial x} - \frac{\partial}{\partial x}\left[\frac{D}{Dt}\left(\frac{1}{n}\frac{\partial B}{\partial x}\right)\right] - \alpha\frac{\partial}{\partial x}\left(\frac{1}{n}\frac{\partial B}{\partial x}\right) = 0, \quad (3)$$

where n has been normalized by n_0, B by B_0, u by C_A, t by $1/\omega_{LH}$ and x by $\lambda_e = c/\omega_{pe}$. Here $D/Dt = (\partial/\partial t) + u\partial/\partial x$, $\beta = 4\pi n_0 k_B T/B_0^2$, $T = T_e + T_i$, $\alpha = \nu_{ei}/\omega_{LH}$, B is the sum of the ambient and wave magnetic fields, ν_{ei} the electron-ion collision frequency, $C_s = [k_B T/m_i]^{1/2}$ the effective ion-acoustic speed, $C_A = B_0/\sqrt{4\pi n_0 m_i}$ the Alfvén speed, $\omega_{LH} = \sqrt{\omega_{ce}\omega_{ci}}$ the lower-hybrid resonance frequency, $\omega_{ce} = eB_0/m_e c$ ($\omega_{ci} = eB_0/m_i c$) the electron (ion) gyrofrequency, $\omega_{pe} = (4\pi n_0 e^2/m_e)^{1/2}$ the electron plasma frequency, m_e (m_i) the electron (ion) mass, e the magnitude of the electron charge, k_B Boltzmann's constant, T_e (T_i) the electron (ion) temperature, and n_0 the equilibrium plasma number density. In Eq. (2), we have adopted the adiabatic pressure laws [viz. $p_j = p_{j0}(n/n_0)^3$, where $p_{j0} = n_0 k_B T_j$ and where j equals e for electrons and i for ions] for the electron and ion fluids.

Assuming that n, u and B are functions of $\xi = x - Mt$, where $M = U/C_A$ is the Mach number and U the constant propagation speed, we have from Eqs. (1)–(3), respectively, $u = M(1 - 1/n)$,

$$M^2\left(\frac{1}{n} - 1\right) + \frac{1}{2}(B^2 - 1) + \beta(n^3 - 1) = 0, \quad (4)$$

and

$$\frac{\partial}{\partial \xi}\left(\frac{1}{n}\frac{\partial B}{\partial \xi}\right) - \frac{\alpha}{M}\frac{\partial B}{\partial \xi} - B + n = 0, \quad (5)$$

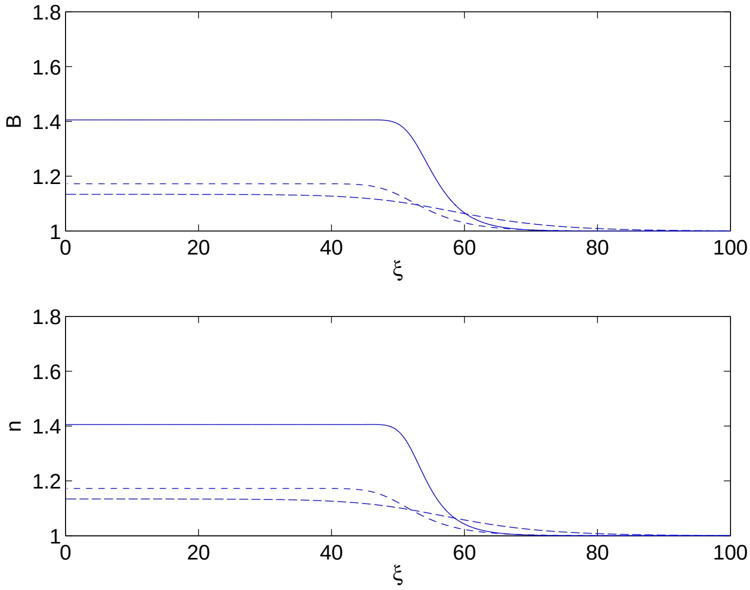

Figure 2: Profiles of the magnetic field and plasma density for $\alpha = 1.5$, using $M = 1.3$ and $\beta = 0$ (solid curves), $M = 1.1$ and $\beta = 0$ (dashed curves), and $M = 1.3$ and $\beta = 0.1$ (dash-dotted curves). After Ref. [1].

where we used the boundary conditions $u = 0$, $n = 1$, $B = 1$ and $\partial B/\partial\xi = 0$ at $\xi \to +\infty$. In the unperturbed state at $\xi \to -\infty$, we have $\partial/\partial\xi = \partial^2/\partial\xi^2 = 0$ in Eq. (5) so that $n = B$. Inserting the latter into Eq. (4) we obtain

$$M^2\left(\frac{1}{B} - 1\right) + \frac{1}{2}(B^2 - 1) + \beta(B^3 - 1) = 0, \tag{6}$$

which exhibits the existent diagram for M versus B for given values of β. Eliminating a common factor $(B - 1)$, Eq. (6) can be expressed as

$$\frac{M^2}{B} - \frac{1}{2}(B + 1) - \beta(B^2 + B + 1) = 0, \tag{7}$$

which gives the amplitude of B at $\xi = -\infty$. In the cold limit $\beta = 0$, we have from (7) $M^2 = B(B+1)/2$, which reveals that the nonlinear structures have super Alfvénic speed for $B > 1$.

Numerical solutions of Eqs. (4) and (5) for different values of M, β and α are displayed in Figs. 1–3. In Fig. 1, we used $\alpha = 0.1$, which leads to oscillatory shock structures. The amplitudes of the structures increase with increasing values of M, while they decrease significantly even for small values of β. On the other hand, for a larger value of $\alpha = 1.5$, we see in Fig. 2 that the shock structures are more or less monotonic, while their amplitudes depend on M and β in the same manner as in Fig. 1. For vanishing α, the shock structure will dissolve into a train of solitary CDA waves, as seen in Fig. 3. The amplitudes of the solitary waves increase with increasing M, while they decrease with increasing values of β.

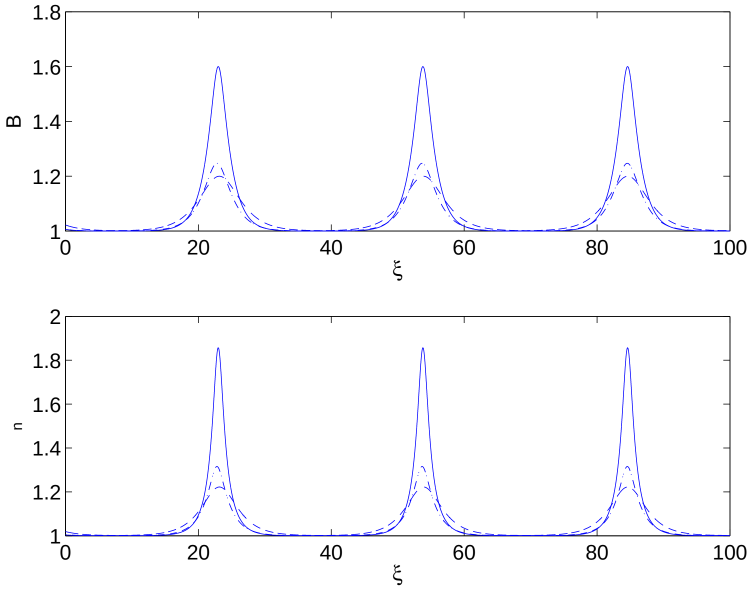

Figure 3: Profiles of the magnetic field and plasma density for $\alpha = 0$, using $M = 1.3$ and $\beta = 0$ (solid curves), $M = 1.1$ and $\beta = 0$ (dashed curves), and $M = 1.3$ and $\beta = 0.1$ (dash-dotted curves). After Ref. [1].

In summary, we have derived the governing nonlinear equations for large amplitude nonlinear CDA waves in a warm collisional magnetoplasma. A numerical analysis of the nonlinear equations in a stationary moving frame reveals that the nonlinear CDA waves appear in the form of either monotonic or oscillatory shock waves, or as a train of solitary waves. We stress that the CDA shock wave pressure might be responsible for the cross-field proton acceleration/energization over length scales that are either comparable or far exceed the electron skin depth in a magnetized electron-ion plasma.

Acknowledgments This research was partially supported by the Deutsche Forschungsgemeinschaft through the project SH21/3-1 of the Research Unit 1048.

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