Strong electromagnetic waves and pulsar termination shocks

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Rapidly spinning, magnetized neutron stars lose their rotational energy in a relativistic outflow that can be regarded as a large-amplitude wave. Close to the star, where the pair plasma in the magnetosphere is dense, the wave can be described by the MHD equations. The wave/outflow terminates at the point where its ram pressure matches that of the surrounding medium. Downstream of this "shock", observations of nebular synchrotron radiation imply that a significant fraction of the energy flux is deposited in relativistic particles. But the energetics is strongly dominated by the electromagnetic fields, and in an ideal radial MHD flow there is no plausible way of transferring energy from Poynting flux to particles. We discuss a solution to this problem, in which the MHD wave converts into a strong electromagnetic mode that forms a precursor to the shock. This mode efficiently accelerates particles to relativistic energies at the expense of the field energy. When a strong EM wave propagates radially, its velocity decreases and its ram pressure asymptotes to a constant value, a property we use to match it to the surrounding medium. Once the wave has decelerated, instabilities set in that thermalize its energy and complete the formation of the shock front.

MHD wave and the σ-paradox

Pulsars are rapidly rotating neutron stars, endowed with a magnetic field that is frequently assumed to be a dipole orientated obliquely to the rotational axis. In vacuum, such objects generate large-amplitude waves beyond the light cylinder surface $r_{\rm L} = c/\omega$, where the corotation velocity equals c [3]. The presence of an electron-positron plasma thought to be generated close to the star, opens up the field lines that cross this surface. The plasma flows outwards along them, exerting a torque on the spinning star and extracting its rotational energy. Close to the light cylinder the outflow can be described as an MHD wave that carries the entire pulsar spindown power L, characterized by the dimensionless parameter $a_{\rm L} = \left(e^2 L/m^2 c^5\right)^{1/2}$. The magnetization parameter σ, which is the ratio of the energy flux carried by the fields to that carried by the particles, is large at launch, and the modest particle flux is quantified by the large dimensionless parameter μ, which is the Lorentz factor each particle would have if the entire luminosity were carried by the particles only.

In the MHD model, a termination shock forms roughly where the ram pressure of the pulsar wind matches that of the surrounding medium. Observations of pulsar wind nebulae, downstream of this shock, indicate that a significant fraction of the wind energy is transferred to the radiating particles. This is possible only when $\sigma \ll 1$ upstream, in contrast to its large value at launch. However, in an ideal, radial MHD flow σ does not decrease with the distance. This is known as the σ-problem, since the transition from highly-magnetized magnetospheric plasma to the particle dominated shocked nebular flow is not understood. A possible solution is that the MHD model loses its validity and the outflow converts into a large-amplitude EM wave before reaching the shock [8, 4], but the conversion mechanism is so far unknown. Here, we propose that conversion is driven by the boundary conditions imposed by the external medium, which the MHD mode cannot satisfy. Thus, the EM mode can be thought of as a shock precursor that is causally connected to the surroundings and matches their pressure. Since the wave is strong, it efficiently accelerates particles at the expense of the field energy.

Superluminal strong waves in pulsar winds

Since EM waves do not propagate in the overdense plasma close to the pulsar, mode conversion can happen only beyond a critical radius $r > r_{\rm c}$, where, because of the spherically expanding flow, the plasma density drops below a critical value. Waves launched far outside this cut-off distance resemble vacuum waves, whereas those created close to the cut-off have properties strongly influenced by the presence of plasma. In spherical geometry, the latter have larger amplitudes and drive particles to almost the maximal possible Lorentz factor $\gamma \sim \mu$ in less than half a period.

Propagation of strong plane waves in a plasma can be described by the cold two-fluid equations and Maxwell equations. Under pulsar conditions the plasma is positronic so that EM waves are purely transverse. The $e^{\pm}$ fluids move with equal momentum $p_{\|}$ parallel to the propagation direction, but have oppositely directed, equal amplitude oscillations in transverse momentum $p_{\perp}$. This generates a conduction current, that, in the frame in which the wave has zero group speed, exactly balances the displacement current. In the lab. frame these waves have superluminal phase velocity, but subluminal group speed $c\beta_*$ and, in general, the wave group speed does not coincide with the parallel component of the fluid 3-velocity, so that particles stream through the wave. In the following we consider, for simplicity, a monochromatic, circularly polarized solution.

Radial propagation can be treated by perturbation analysis, using as a small parameter $\varepsilon = c/\omega r \ll 1$, the ratio of the wavelength to the radial distance r at which the wave is considered. Expanding the relevant equations, one obtains to lowest order the plane-wave solution, dependent only on the wave phase. The first order equations determine the slow radial evolution of the phase-averaged, plane-wave quantities, which is governed by (1) the continuity equation, (2) energy conservation, and (3) the evolution of the radial momentum flux. In contrast to the

MHD wave, the radial momentum flux is not conserved in spherically expanding EM modes. This follows from the fact that a radially co-moving volume element expands in the perpendicular direction, causing the finite perpendicular fluid momentum to do work. Because the total energy flux is conserved, this changes its division between the perpendicular and parallel degrees of freedom. However, in addition to the conserved particle flux and energy flux, it can be shown [9] that the phase-averaged Lorentz factor of the particles, measured in the laboratory frame $\langle \gamma_{\rm lab} \rangle$ is an integral of motion for both circularly and linearly polarized modes.

To find the initial properties of the EM mode at the conversion radius, one has to solve jump conditions between the MHD and the EM wave, to ensure that they carry the same particle, energy and radial momentum fluxes [4, 1]. Fig. 1 shows the Lorentz factor of a strong EM wave $\gamma_* = \left(1-\beta_*^2\right)^{-1/2}$, obtained from the jump conditions (red curves), and its radial evolution (blue curves) for different launching radii. There are two solutions of the jump conditions that describe two possible EM modes: a free-escape mode (higher branch) and a confined mode (lower branch). Their behaviour is very different: at large distances the free-escape wave accelerates whereas the confined one decelerates. Keeping in mind that the wind solution should be matched to the slowly expanding nebula, we concentrate only on the confined mode.

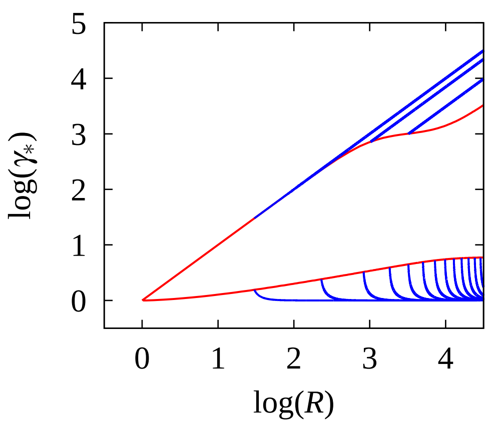

Figure 1: Red: The Lorentz factor at of a circularly polarized EM wave, corresponding to an MHD wind with $\mu = 10^4$, $\sigma = 100$, $a_{\rm L} = 3.4 \times 10^{10}$; against conversion radius. Blue: the radial evolution of an EM wave launched at a point on the red curve. Radius is normalized $R = r\mu\omega/(ca_{\rm L})$

The evolution equations imply that the ram pressure of the confined mode tends to a constant value at large radius. In a generic solution this value should be equal to the external pressure. Since only the wave launched at the correct radius has the correct asymptotic pressure, we are able to find an unique solution that matches asymptotically a given pressure $p_{\rm ext}$ of the surrounding medium. To uniquely determine a wave at launch/conversion requires four quantities to be specified: the conversion radius R_0, initial group speed β_{*0}, and initial particle momenta $p_{\parallel 0}$, $p_{\perp 0}$. The jump conditions define three of them, leading to the red curves in Fig. 1, and $p_{\rm ext}$ can be used to determine the fourth one: R_0. This is possible analytically, since the conservation of $\langle \gamma_{\rm lab} \rangle$ allows us to connect the asymptotic wave pressure with the initial wave parameters unambiguously. Therefore, a unique, stationary precursor-solution is determined by the MHD wave parameters σ, μ, a_L, and the external pressure $p_{\rm ext}$, unless $p_{\rm ext}$ is larger than the ram pressure at $r = r_{\rm c}$, in which case no such precursor can be formed. This situation is unlikely to occur for isolated pulsars, but might be relevant in binary systems.

Damping and shock formation

Self-consistent EM waves are damped by nonlinear inverse Compton scattering (i.e. radiation reaction on the fluids accelerated in the wave fields) [2] as well as inverse Compton scattering of ambient low energy photons. Even though energy is removed from the system overall, the evolution equations show that particles gain more energy from the fields than they lose in radiation, and that both the wave and the particle streaming slow down. Thus, this effect mimics that caused by radial expansion. When, by either of these mechanisms, the streaming vanishes, strong waves become very unstable to small density perturbations in the direction of motion [7, 5, 10]. Parametric instabilities then quickly set in and destroy the wave. This point is the location of the termination shock, beyond which the flow energy is thermalized.

Summary

In a self-consistent picture the conversion of an MHD pulsar wind into an EM wave that forms a shock precursor is determined not only by the wind parameters, but also by the external medium. There are two regimes of interest: (1) when the external pressure is so high that an EM wave cannot propagate; (2) when the external pressure is lower. In the latter case, the EM wave slows down either by radiation damping or by the effects of spherical expansion. Eventually it becomes unstable and thermalizes at a “shock front”.

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