Modulated longitudinal wave packets in three dimensional bcc configuration lattice

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Since 1994, when "plasma crystals" were observed for the first time in experiments [1–3], there have been a number of studies of crystal structure and phase transitions; see, e.g., [4]. Most of the experiments were performed in radiofrequency (rf) discharges, where the micron sized particles are levitated in the electric field of the lower electrode sheath in ordered structures with one or a few horizontal particle layers. The waves in one-dimensional (1D) and two-dimensional (2D) lattice consisting of charged particles have been studied extensively [5,6]. Three-dimensional (3D) crystals were also observed in some experiments, where the particles order themselves in fcc, bcc, and hcp lattices [7]. Wang *et.al.*[8] has studied the linear and nonlinear waves in (3D) simple cubic configuration. The state of an infinite system of particles with Yukawa pair interaction is determined by the coupling strength between the particles (parameter Γ is measured in units of the potential energy of interaction between neighboring particles normalized by their mean kinetic energy) and the lattice parameter κ. For $\Gamma \sim 10^3$, $\kappa \sim 1-2.5$, the state of particles can crystallize into bcc configuration.

In this paper we will derive a nonlinear equation implying the evolution of longitudinal dust lattice wave in special directions, by considering a potential energy of the type of Deby-Huckle and using continuum approximation.

Consider a three-dimensional cubic crystal with lattice spacing a consisting of negative charge dust grains q and mass M, modeled as point charges. In order to find the equation of motion for the (n,m,l)'th particle, we only consider the forces exerted on that particle by 8 particle of first neighbors (at the distance of $\sqrt{3}/2a$) and 6 of the second neighbours (at the distance of a) which are expressed as follow at equilibrium position; $(n+n_i, m+m_i, l+l_i)$. At equilibrium let the (n,m,l)'th particle in the origin, then the positions of the eight nearest neighbour particles are $(n_i, m_i, l_i) \equiv (\pm a/2, \pm a/2, \pm a/2,)$, $i = 1-8$ and the second neighbours are at $(n_i, m_i, l_i) \equiv (\pm a, 0, 0)$, $(0, \pm a, 0)$, $(0, 0, \pm a)$, $i = 9-14$. Particle displacement of i'th particle from equilibrium position can be written as (u_i, v_i, w_i), so distance between particle in origin $(0,0,0)$ and i'th particle obtain as the form of $(n_i + u_i - u,\ m_i + v_i - v,\ l_i + w_i - w)$, here (u, v, w) refers to displacement of the particle in origin, from its equilibrium position.

We assume that there is elastic force between two arbitrary particles. In order to reach that mean we expand the Debye–Huckel interaction potential energy, namely $U(r) = Q^2 \exp[-r/\lambda_D]/4\pi\varepsilon_0 r$ around equilibrium at $r = r_0$.

$$U(r) \cong \frac{1}{2} G_{1_i} (r - r_{0_i})^2 + \frac{1}{3} G_{2_i} (r - r_{0_i})^3 + \frac{1}{4} G_{3_i} (r - r_{0_i})^4 + \cdots \tag{1}$$

upon defining $G_{1_i} = \frac{\partial^2 U}{\partial r^2}\big|_{r=r_{0i}}$, $G_{2_i} = \frac{1}{2}\frac{\partial^3 U}{\partial r^3}\big|_{r=r_{0i}}$, $G_{3_i} = \frac{1}{6}\frac{\partial^4 U}{\partial r^4}\big|_{r=r_{0i}}$ and r_{0_i} is equal to a if $1 \le i \le 8$ (first neighbours) and it is equal to $\sqrt{3}/2\,a$ when $9 \le i \le 14$ (for second neighbours) and the distance between particle in origin and the i'th particle is $r_i = \sqrt{(x_i)^2 + (y_i)^2 + (z_i)^2}$ where

$$x_i = (u_i + n_i) - u_{nml} = \Delta u_i + n_i \tag{2-a}$$

$$y_i = (v_i + m_i) - v_{nml} = \Delta v_i + m_i \tag{2-b}$$

$$z_i = (w_i + l_i) - w_{nml} = \Delta w_i + l_i \tag{2-c}$$

In the case of propagating the longitudinal wave in (1,0,0)-direction ($\Delta v = \Delta w = 0$), Continuum approximation can be used when the typical length scale of the wave is greater than the inter-particle spacing. In this way we expand u_i around u ,

$$u_i = u + n_i \frac{\partial u}{\partial x} + \frac{n_i^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{n_i^3}{6}\frac{\partial^3 u}{\partial x^3} + \frac{n_i^4}{24}\frac{\partial^4 u}{\partial x^4} \tag{3}$$

so the equation of motion for (nml)th particle in the lattice is

$$\frac{\partial^2 u}{\partial t^2} = \alpha_1 \frac{\partial^2 u}{\partial x^2} + \beta_1 \frac{\partial^4 u}{\partial x^4} + \gamma_1 \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2} + \delta_1 \left(\frac{\partial u}{\partial x}\right)^2 \frac{\partial^2 u}{\partial x^2}. \tag{4}$$

in which this parameters are defined as $\alpha_1 = \left(a^2/3G_1 + a^2 G_1'\right)/M$, $\beta_1 = \left(a^4/144\,G_1 + a^4/12\,G_1'\right)/M$, $\gamma_1 = \left(2a^2/3G_1 + \sqrt{3}a^3/9\,G_2 + 2a^3 G_2'\right)/M$ and $\delta_1 = \left(-2a^2/9\,G_1 + 2\sqrt{3}a^3/9\,G_2 + a^4/12\,G_3 + 3a^4 G_3'\right)$.

By the same calculations for the longitudinal wave in (1,1,0)-direction ($\Delta w = 0$, $\Delta u = \Delta v$), and (1, 1, 1)-direction($\Delta u = \Delta v = \Delta w$), one can obtain the continuous equation of motions for both directions, respectively.

for propagation along (1, 1, 0)-direction equation of motion is in the form of

$$\frac{\partial^2 u}{\partial t^2} = \alpha_2 \frac{\partial^2 u}{\partial x^2} + \beta_2 \frac{\partial^4 u}{\partial x^4} + \gamma_2 \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2} + \delta_2 \left(\frac{\partial u}{\partial x}\right)^2 \frac{\partial^2 u}{\partial x^2}, \tag{5}$$

by introducing parameters $\alpha_2 = \left(4a^2/3G_1 + a^2 G_1'\right)/M$, $\beta_2 = \left(a^4/9\,G_1 + a^4/12\,G_1'\right)/M$, $\gamma_2 = \left(8a^2/3G_1 + 16\sqrt{3}a^3/9\,G_2 + 3a^2 G_1' + 2a^3 G_2'\right)/M$ and

$$\delta_2 = \left(-56a^2/9\,G_1 + 32\sqrt{3}a^3/9\,G_2 + 16a^4/3G_3 - 9a^2/2\,G_1' + 6a^3 G_2' + 3a^4 G_3'\right)/M \ .$$

when propagation direction is along (1,1,1)-direction, we obtain,

$$\frac{\partial^2 u}{\partial t^2} = \alpha_3 \frac{\partial^2 u}{\partial x^2} + \beta_3 \frac{\partial^4 u}{\partial x^4} + \gamma_3 \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2} + \delta_3 \left(\frac{\partial u}{\partial x}\right)^2 \frac{\partial^2 u}{\partial x^2}, \tag{6}$$

by defining $\alpha_3 = \left(7a^2/3G_1 + a^2 G_1'\right)/M$, $\beta_3 = \left(61a^4/144\,G_1 + a^4/12\,G_1'\right)/M$, $\gamma_3 = \left(2a^2/3G_1 + 4\sqrt{3}a^3 G_2 + 3a^2 G_1' + a^3 G_2'\right)/M$ and

$$\delta_3 = \left(4a^2/9\,G_1 + 2\sqrt{3}a^3/9\,G_2 + 547a^4/12\,G_3 - 3a^2 G_1' + 9a^3 G_2' + 3a^4 G_3'\right)/M \ .$$

We now proceed by considering small-amplitude oscillations of the form

$$u = \varepsilon u_1 + \varepsilon^2 u_2 + \varepsilon^3 u_3 + \cdots \ , \qquad (7)$$

$$u_n = \sum_j u_{nj} e^{j\theta} + C.C \ , \qquad (8)$$

$$\theta = kX_0 - \omega T_0 \,. \qquad (9)$$

at each lattice site. Introducing multiple scales in time and space, i.e. $X_n = \varepsilon^n x$, $T_n = \varepsilon^n t$ $(n = 0,1,2,\cdots)$, we develop the derivatives in Eq. (41) in powers of the smallness parameter ε and then collect terms arising in successive orders. The equation thus obtained in each order can be solved and substituted to the subsequent order, and so forth. This reductive perturbation technique is a standard procedure for the study of the nonlinear wave propagation often used in the description of localized pulse propagation, prediction of instabilities, etc. This procedure leads to a dispersion relation of the form in first order of ε,

$$\omega^2 = \alpha k^2 - \beta k^4 \ , \qquad (10)$$

so, the group velocity is

$$v_g = \frac{\alpha k - 2\beta k^3}{\omega} \,, \qquad (11)$$

and we obtain in the third order, the nonlinear Schrodinger equation,

$$i\frac{\partial u_{11}}{\partial \tau} + P\frac{\partial^2 u_{11}}{\partial \xi^2} + Qu_{11}|u_{11}|^2 = 0 \ , \qquad (12)$$

where the slow variables (τ , ξ) are (T_2 , $X_1 - v_g T_1$),The dispersion coefficient (P) and the nonlinearity coefficient (Q) are $P = \left(\alpha - v_g^2 - 6\beta k^2\right)/2\omega$, $Q = -k^2\gamma^2/6\beta\omega - \delta k^4/2\omega - \gamma^2 k^4/\left(2\omega(v_g^2 - \alpha)\right)$ respectively.

In Fig. 1 the dispersion coefficient of longitudinal waves has been showed. it is obvious that the dispersion coefficient depends on the propagation direction, this is because of bcc configuration is an anisotropic lattice. In Fig. 2 nonlinearity coefficient of longitudinal waves has been showed. As it is shown in the figure, because of the anisotropy the bcc lattice, nonlinearity coefficient depends on the propagation direction of the wave, too.

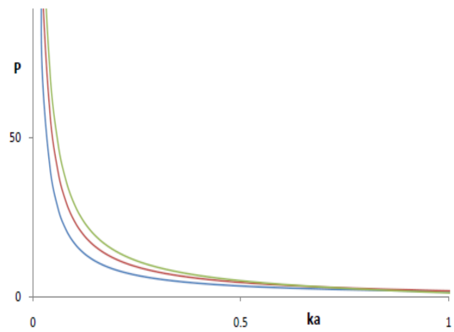

Fig.1 The dispersion coefficient of the longitudinal dust-lattice wave as a function of the normalized wave-number, for wave propagating along (1,0,0)-blue line, (1,1,0)-red line, (1,1,1)-green line directions.

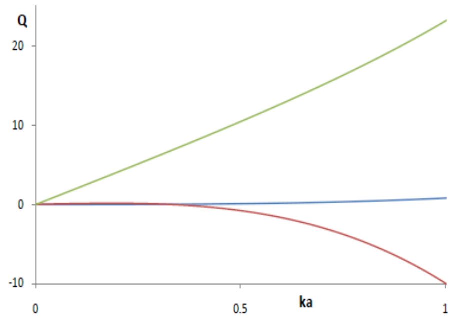

Fig. 2 The nonlinearity coefficient of the longitudinal dust-lattice wave as a function of the normalized wave-number, for wave propagating along (1,0,0)-blue line, (1,1,0)-red line, (1,1,1)-green line directions.

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