Parametric growth of seed pulses due to Brillouin scattering in relativistic laser-plasma interaction

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Parametric plasma processes received renewed interest in the context of generating ultraintense and ultrashort laser pulses up to the exawatt-zetawatt regime [1,2]. Stimulated backward Raman- and Brillouin scattering can be used to transfer energy from a long pump pulse to a short seed pulse. By these mechanisms, pulse amplification beyond the damage threshold of optical gratings seems to be possible. First experiments report 30-fold amplification of a 400 fs seed pulse by a 3.5 ps pump pulse via Brillouin backscattering [3].
For Brillouin scattering and seed pulse amplification at high intensities, the strong coupling (sc) regime is of special interest [4]. Here, we investigate how sc modes can be used in the relativistic regime.

We start with the dispersion relation, which covers high frequency electron waves, as well as lower frequency ion acoustic modes. We make the assumption, that the variation n_{e1} of the electron density due to the interaction of pump and scattered electromagnetic wave can be written as a superposition $n_{e1} = n_{e1,h} + n_{e1,l}$. Here $n_{e1,h}$ and $n_{e1,l}$ are the high frequency and the low frequency component, respectively. This decomposition holds in the regime $\gamma_i m_i >> \gamma_e m_e$, where $(\omega_{pe}/\gamma_e)^2 + 3\, v_{the}^2/\gamma_e k^2 >> (c_s\, k)^2$. The relativistic Lorentz factors of electrons and ions are γ_e and γ_i, respectively. The thermal electron velocity is denoted by v_{the} , the ion acoustic velocity by c_s.
High frequency perturbations $n_{e1,h}$ of the electron density are due to Raman scattering. The dispersion relation for relativistic Raman scattering is obtained from a fluid-Maxwell model, taking into account the relativistic mass increase of the electrons [5]. For the low frequency contribution $n_{e1,l}$, which is due to the acoustic modes of the ions, we assume an isothermal equation of state for the electron pressure. We treat the ion motion non-relativistic, but retain the mass increase of the electrons. In this way, we obtain a dispersion relation for Brillouin scattering in the presence of relativistic electrons.
Combining both, high and low frequency modes, we obtain the dispersion relation

$$D^+D^- = \frac{e^2 a_0^2}{\gamma_{e0}^2 m_e^2 c^4}\left[c^2k^2\left(\frac{\Omega_{pe0}^2}{D_e^h}+\frac{\Omega_{pi0}^2}{D_e^l}\right)+\Omega_{pe0}^2\right](D^+ + D^-)\,, \tag{1}$$

$$D^{\pm} = \left(\sqrt{\omega_0^2-\Omega_{pe0}^2}\pm ck\right)^2 - (\omega_0\pm\omega)^2+\Omega_{pe0}^2 \tag{2}$$

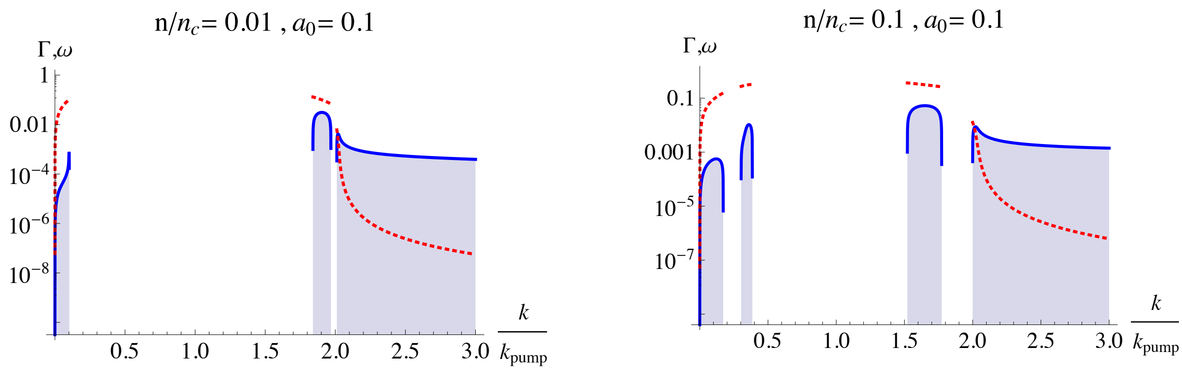
$$D_e^l \equiv \frac{k^2 T_e}{m_i\gamma_{i0}} - \omega^2 = c_s^2k^2-\omega^2\,, \tag{3}$$

$$D_e^h \equiv \Omega_{pe0}^2 - \omega^2 + \frac{3v_{the}^2}{\gamma_{e0}}k^2\,, \tag{4}$$

$$\Omega_{pe} = \frac{\omega_{pe}}{\gamma_{e0}}\,,\quad \Omega_{pi} = \frac{\omega_{pi}}{\gamma_{i0}}\,,\quad \gamma_{e,i} = \sqrt{1+\left(\frac{m_{e,i}}{m_e}\frac{ea_0}{mc^2}\right)^2}\,. \tag{5}$$

The dispersion relation (1) is a sixth-order polynomial in ω and its solutions give insight into the interaction of circularly polarized relativistic radiation with Langmuir and ion acoustic waves. Since the dispersion relation has been derived under rather strict assumptions, the solutions have been compared to numerical solutions of the according linearized two-fluid Maxwell model. The results from Eq. (1) are in very good agreement with the numerical results for all scenarios discussed here.

Figure 1 shows results for the exponential growth rate Γ and the frequency ω of unstable modes obtained from Eq. (1) for three different plasma densities at a fixed pump intensity. The plasma density is normalized to the critical density n_c, while the vector potential a_0 of the pump is de-dimensionalized by e/mc^2.



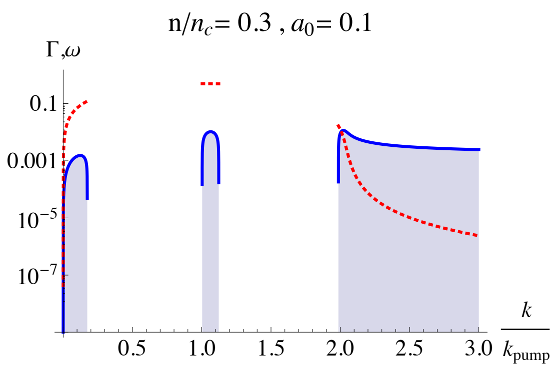


Fig.1: Growth rates Γ (blue line) and frequencies ω (red line) of unstable modes obtained from Eq.(1), normalized to the pump frequency. The pump amplitude is $a_0 = 0.1$, the plasma densities are $0.01n_c$, $0.1n_c$ and $0.3n_c$, respectively.

In all three subfigures of Fig. 1, the right most branch corresponds to Brillouin scattering. We observe, that for $k = 2k_{pump}$ we have $\Gamma \approx \omega >> c_s\, k$. This is referred to as strongly coupled (sc) Brillouin scattering [4]. For large pump intensities (about I ≥ 10^{16} W/cm^2) we enter the regime of sc Brillouin where the ion acoustic wave becomes a nonlinear driven wave.

The amplification of a seed pulse by a pump pulse via Raman or Brillouin scattering requires the frequencies and wave-numbers of the pump, the seed and the plasma wave to fulfill the matching conditions, $\omega_{seed} = \omega_{pump} \pm \omega$ and $k_{seed} = k_{pump} \pm k$. The frequency ω determines the minimal seed pulse duration for amplification. For standard Brillouin scattering this is of the order of ps, for Raman scattering about 5 fs and for sc-Brillouin scattering about 50 fs. Thus, sc-Brillouin allows for the amplification of short laser pulses. In fact, since Raman amplification operates at low plasma densities ($n \sim 0.01 n_c$), and Brillouin amplification at much higher plasma densities ($n \sim 0.1 n_c$), the frequencies for Raman and Brillouin amplification can become comparable, see Fig.1.

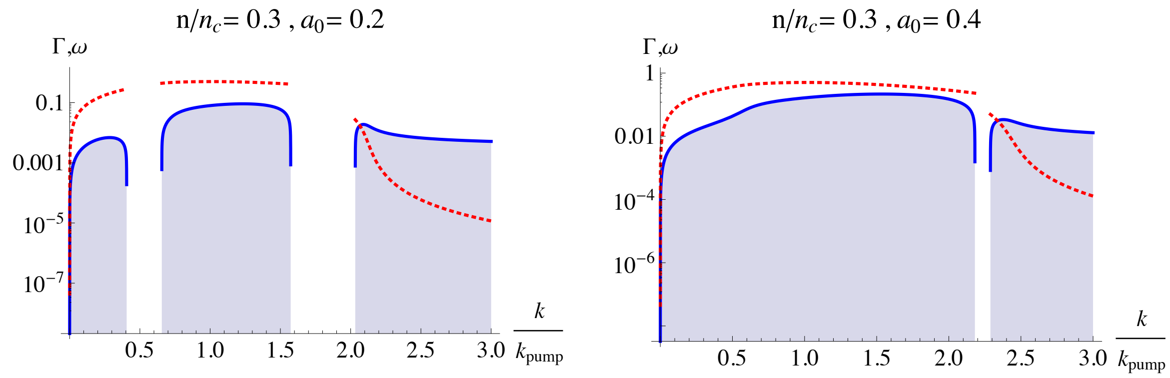


Fig.2: Growth rates Γ (blue line) and frequencies ω (red line) of unstable modes obtained from Eq.(1), normalized to the pump frequency. The pump amplitude is $a_0 = 0.2$ (left) and $a_0 = 0.4$ (right), the plasma density $n = 0.3\, n_c$.

Figure 2 shows how the regions of Raman and Brillouin scattering will change once we increase the pump intensity. At larger intensities, we observe a broadening of the Raman backward branch (middle peak in Fig. 2, left). Eventually this branch will merge with the already merged Raman forward/modulational instability branches on the left. At the same time, we observe a shift in the spectrum of the Brillouin modes towards larger wave-numbers. Brillouin amplification schemes which are based on pump pulses with relativistic amplitudes have to take this shift into account. The short seed pulses will at the same time have a broad spectrum. Hence, in the relativistic regime, they might not only experience Brillouin scattering, but partially Raman scattering as well.

We carried out Vlasov simulations demonstrating Brillouin amplification, results of which are exemplified in Fig. 3.

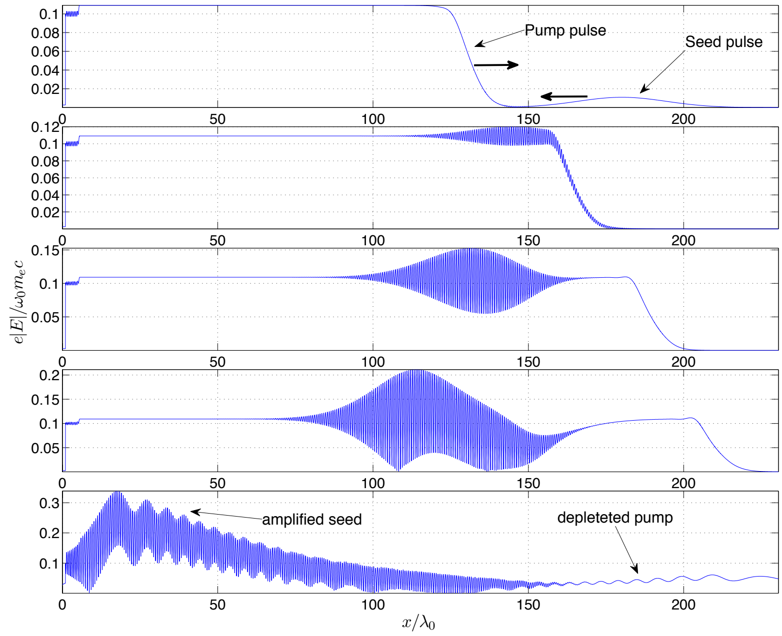


Fig.3: Amplification of a seed pulse of amplitude a_0=0.01, FWHM 40 λ_0 by a pump pulse of amplitude a_0=0.1. Shown is the total electric field. The plasma has a density of 0.3n_c and begins at 5 λ_0 and ends at 250 λ_0. Seed and pump pulse are circularly polarized and have the same wavelength. The plots show the field intensity at times (normalized to $2\pi/\omega_0$) t=0, 40, 70, 95, 195.

The amplification is a two stage process. As long as the seed pulse amplitude is much smaller than the pump pulse amplitude, we observe exponential growth. The measured growth rate agrees with the predictions from the dispersion relation (1). Once the seed has the same amplitude as the pump, we enter the regime of pump depletion. Now the seed may also loose energy again to the pump, which leads to oscillations at the tail of the seed. These oscillations are similar to the π-pulses, known from the theory of Raman amplification. In the pump depletion regime of Brillouin scattering, we observe a growth of the seed amplitude with $t^{3/4}$. This is in agreement with predictions from a reduced three-wave interaction model [4].

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