Interaction of ultrarelativistic electron and proton bunches with dense plasmas

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Introduction

High-energy bunch electrons generate the wake plasma wave (wake) in such a way that the energy from a bunch of electrons is transferred to the plasma wave through Cherenkov resonance radiation producing high electric fields (wakefields). The idea to accelerate the charged particles in a plasma medium using collective plasma fields belongs to G. I. Budker, V. I. Veksler and Ia. B. Fainberg (Proceed. of the CERN Sympos., 1956).

Recently, the possibility of generation of high power wakefields (proton-bunch-driven plasma-wakefield acceleration) of terra-watt amplitude using the ultrarelativistic proton bunches was introduced [4].

In the work [5] this idea along with the employment of ultrarelativistic electron bunch was discussed at the qualitative level. Namely, we make an estimation of plasma parameters, maximum amplitude of the generated wakefield when the ultrarelativistic electron and proton bunches are employed and plasma, bunch lengths at which the maximum amplitude of the wakefield can be gained. Here, we would like to briefly present the main idea of the work.

Ultrarelativistic electron bunch

Let us start our analysis with the ultrarelativistic monoenergetic electron bunch of density n_b and velocity $\vec{u}$, noting that $\gamma = 1/\sqrt{1-u^2/c^2} >> 1$.

Such a bunch interacts with the cold (no thermal motion) isotropic plasma being in a rest of density $n_p >> n_b$ and generates the plane wave $E = E_0 \exp(-i\omega t + i\vec{k}\cdot\vec{r})$, here ω is the frequency and $\vec{k}$ - the wave vector. We choose an axis Z directed along the velocity of the bunch $\vec{u}$, and put the external field as absent [1]. Dispersion relation in the laboratory frame of reference (plasma in a rest) describing the amplification of a plasma wave (longitudinal-transverse) with the help of a bunch can be written as following [1] :

$$(k^2c^2 - \omega^2 + \omega_p^2 + \omega_b^2\gamma^{-1})\left(1 - \frac{\omega_p^2}{\omega^2} - \frac{\omega_b^2\gamma^{-3}}{(\omega - k_z u)^2}\right) - \frac{k_\perp^2 u^2}{\omega^2}\frac{\omega_p^2\omega_b^2\gamma^{-1}}{(\omega - k_z u)^2} = 0, \quad (1)$$

here $\omega_p = \sqrt{4\pi e^2 n_p/m}$, $\omega_b = \sqrt{4\pi e^2 n_b/m}$ are Langmuir plasma electron and bunch frequencies respectively (in GSU units) with n_p, n_b being the plasma and bunch densities, k_z and $k_\perp$ are the longitudinal (directed along the velocity of the bunch $\vec{u}$) and transverse components of the wave vector $\vec{k}$. We put no boundary conditions on the plasma-bunch system. This relation represents the plane wave solution of Maxwell's equations for a plasma-bunch system making use of Lorentz transformations for the electric field $\vec{E}$ and dielectric permittivity $\varepsilon(\omega,\vec{k})$. The plasma ion terms were neglected in the derivation, i.e. only the interaction of the electron bunch with the high-frequency plasma oscillations are considered. The plasma wave emerging from an initial fluctuation amplifies with the time and the system becomes

[1]In ultrarelativistic bunches bunch divergence can be neglected because when the electron bunch is ejected into the dense plasma, within the time $t \sim 1/\omega_p$ the neutralization of the bunch charge occurs prohibitting the bunch divergence [2]

unstable. The bunch terms are the most significant in the frequency range of Cherenkov resonance, i.e. when the bunch speed u coincides with the phase velocity of the plasma oscillations ω_p/k.

Hence the general solution of the equation (1) $\omega(\vec{k})$ at $\omega \approx k_z u \approx \omega_p$ with the positive imaginary part ($\Im m\omega > 0$) will be the following:

$$\omega = k_z u(1+\delta) = \omega_p(1+\delta) \quad \delta = \frac{-1+i\sqrt{3}}{2}(\frac{n_b}{2n_p}\frac{1}{\gamma})^{1/3}(1-\frac{u^2}{c^2}\frac{\omega_p^2}{\omega_p^2+k_\perp^2 u^2})^{1/3}. \tag{2}$$

Equation (2) shows that the oscillations with the frequency $\omega \approx k_z u$ are unstable with ($\Im m\omega = \Im m\delta > 0$). From Eq. (2) follows that at $\Re e\delta < 0$ - $u > \omega/k_z$ meaning that the bunch electrons overtake the wave and transfer part of their kinetic energy to the wave potential energy producing the high wakefields.

For a start we would like to be far away from the concrete magnitudes of the concrete system parameters and consider two opposite limiting cases:

A . In a case of dense plasma when

$$\omega_p^2 >> k_\perp^2 u^2 \sim u^2/R_0^2 \tag{3}$$

Then, the value δ is given by the expression

$$\delta = \frac{-1+i\sqrt{3}}{2}\left(\frac{n_b}{2n_p}\right)^{1/3}\frac{1}{\gamma}. \tag{4}$$

B . In a case of rare plasma when the inverse to (3) inequality is satisfied then the value δ becomes

$$\delta = \frac{-1+i\sqrt{3}}{2}\left(\frac{n_b}{2n_p}\frac{1}{\gamma}\right)^{1/3}. \tag{5}$$

Let us outline again that while deriving the equations (2-5) we took into account that value δ is negligibly small what is sustained by the inequalities $n_b << n_p$ and $\gamma^2 >> 1$.

It is obvious that the saturation of instability can occur when the kinetic energy of bunch electrons, in the wake frame of reference, will become less than the saturation amplitude of the potential of the plasma wake, generated by this bunch in dense plasma, which is measured in the same frame of reference. In this case the bunch electrons get trapped by the wake, i.e. there will be no relative motion between the bunch electrons and the wake, thus, no energy exchange between the bunch and wake occurs, the bunch and the wake become stationary. Hence, the stationary saturation amplitude of the plasma wake potential can be obtained from the following equation:

$$\frac{e\Phi_0}{mc^2} = \frac{1}{\gamma}\left\{\frac{1}{\sqrt{1-\frac{u^2\delta^2\gamma^4}{c^2(1-2\delta(\gamma^2-1))^2}}}-1\right\}, \tag{6}$$

here $\Phi_0\gamma$ is measured in the wake frame of reference. In the relativistic limit when ($\delta\gamma^2 << 1$) we will get the result already obtained in [6] for dense plasma Eq. (4), whereas in the ultrarelativistic limit when ($\delta\gamma^2 >> 1$) we obtain

$$\frac{e\Phi_0}{mc^2} \approx \frac{0.154}{\gamma}, \tag{7}$$

here Φ_0 is measured in the laboratory frame of reference. Eq. (7) leads to the following conclusion: the amplitude of the potential of the plasma wake generated by the electron bunch is dependent on the relativity factor $\Phi_0 = 0.154mc^2/(e\gamma)$. This is demonstrated in the Figure . We can observe a maximum $\Phi_{0max} \simeq 770$ V for dense plasma and $\Phi_{0max} \simeq 15.5$ k V for rare plasma, see Fig. 1.

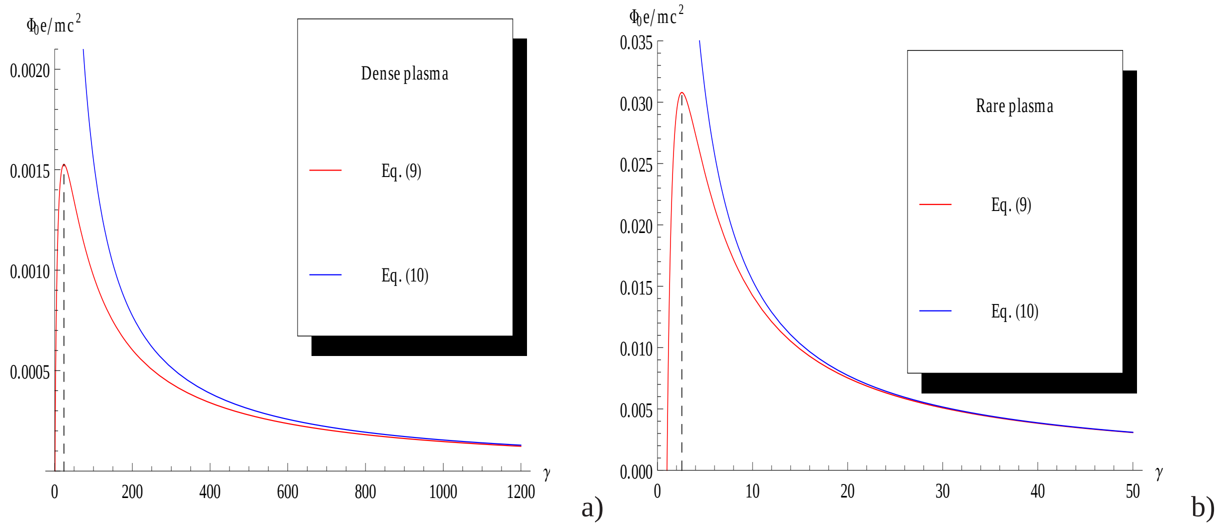

Figure 1: a) The relative stationary saturation amplitude of the plasma wake potential generated by the ultrarelativistic electron bunch Eq. (6) for a) dense (4) at $n_b = 2 \cdot 10^{12} cm^{-3}$, $n_p = 10^{16} cm^{-3}$, $\gamma_0 = 24$ and b) rare (5) plasmas at $n_b = 2 \cdot 10^{12} cm^{-3}$, $n_p = 6 \cdot 10^{12} cm^{-3}$, $\gamma_0 = 2.6$. In a) and b) the relative stationary saturation amplitude in the ultrarelativistic limit (7) is presented for comparison [5].

Ultrarelativistic proton bunch

The idea is that to slow the protons down using the plasma wakefield is much harder than the electrons and as a result the significantly higher wakefield amplitude can be gained in comparison with that produced by the electron bunch.

The dispersion relation (1) can be easily generalized for the case of plasma-proton-bunch interaction. Correspondingly, we need to make the substitution $n_b \to n_{bi}$ (n_b - electron bunch density), i.e. substitute the n_b by the proton bunch density n_{bi} multiplied by the ratio of electron mass m to the ion mass M, m/M.

Having solved Eq. (1), the solution generalizing Eq. (2) for the proton bunch wiil have the similar to (2) form where proton δ_1 differs from δ, from Eq. (3), in $n_b \to n_{bi}\frac{m}{M}$. In the dense and rare plasmas when the constraint (3) and inverse one to it are correspondingly satisfied the value δ_1 takes the corresponding similar to (4), (5) forms.

Similarly, we can estimate the maximum amplitude of the generated by the proton bunch wakefield, so called the amplitude of saturated instability. We would like to note that ultrarelativity of the proton bunch can be reached at much higher proton energies.

By analogy with Eq. (6), the stationary saturation amplitude of the plasma wake potential, produced by the proton bunch in dense plasma, can be obtained from the following equation:

$$\frac{e\Phi_0}{Mc^2} = \frac{1}{\gamma}\left\{ \frac{1}{\sqrt{1 - \frac{u^2\delta_1^2\gamma^4}{c^2(1-2\delta_1(\gamma^2-1))^2}}} - 1 \right\}, \qquad (8)$$

here $\Phi_0\gamma$ is measured in the wake frame of reference. Correspondingly, in the ultrarelativistic limit when $\delta_1\gamma^2 >> 1$ we get an estimate similar to the Eq. (7) with $m \to M$. We have estimated it for dense plasma leading to $\Phi_{0max} \simeq 0.1$ M V and rare plasma leading to $\Phi_{0max} \simeq 5.74$ M V, see Fig. 2.

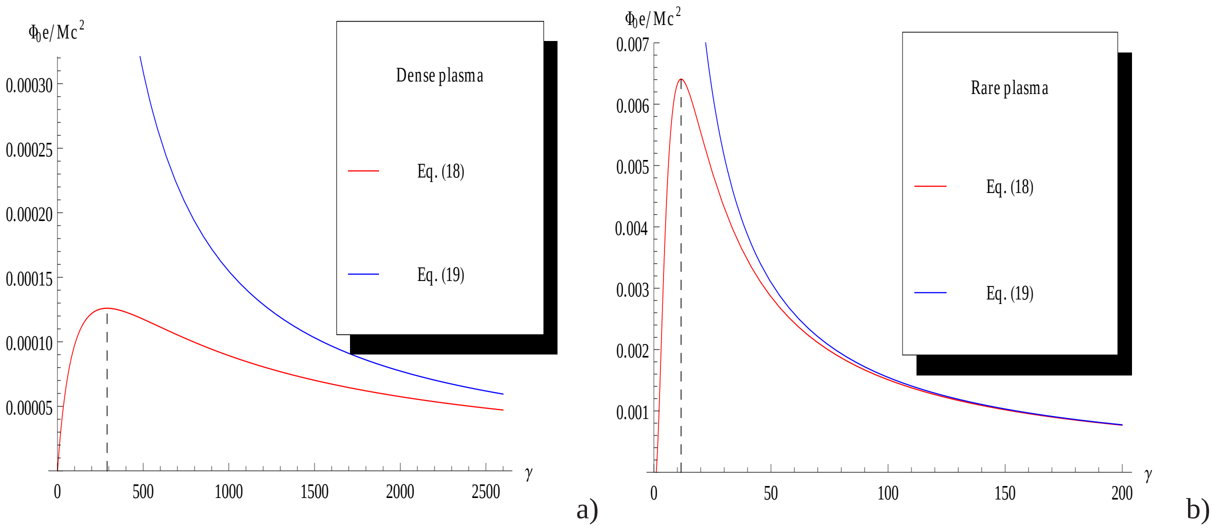

Figure 2: a)The relative stationary saturation amplitude of the plasma wake potential generated by the ultrarelativistic proton bunch Eq. (8) for a) dense at $n_b = 2 \cdot 10^{12} cm^{-3}$, $n_p = 10^{16} cm^{-3}$, $\gamma_0 = 289$ and b) rare plasmas at $n_b = 2 \cdot 10^{12} cm^{-3}$, $n_p = 6 \cdot 10^{12} cm^{-3}$, $\gamma_0 = 11.6$. In a) and b) the relative stationary saturation amplitude in the ultrarelativistic limit is presented for comparison [5].

Results and Discussions

In the present work for the first time the analytical problem of interaction of ultrarelativistic electron and proton bunches with dense plasmas ($n_p >> n_b$) has been solved. These bunches remain relativistic in the frame of reference of wakes generated by these bunches compared to those considered earlier which were nonrelativistic. On the basis of the conducted analysis of the resonance Cherenkov interaction of the ultrarelativistic monoenergetic electron as well as proton bunches with plasma, we can make the following conclusion: the wake amplitude growth produced by the bunches gets saturated with an increase of bunch energy at a not quite high level. The saturation amplitude of the electric wakefield possesses a maximum in dependence on the relativity factor γ and should be tuned in accordance with the constraints for Cherenkov resonance in either dense or rare plasmas, plasma and bunch densities. This amplitude can be increased by increasing the plasma and bunch densities. The highest amplitude of the electric wakefield produced by the electron bunch can be generated in dense plasma at $n_p = 10^{16} cm^{-3}$ and is of order 14.5 M V/m, whereas that produced by the proton bunch is the highest in the rare plasma at $n_p = 6 \cdot 10^{12} cm^{-3}$ and is of order 63 M V/m. These magnitudes are less than those gained with the help of contemporary quite powerful pulse lasers (10^{15} W/cm^2). From our calculations follows that the resonance Cherenkov instabilities arise for electron as well as for proton bunches when the plasma wake amplitudes are much less than the breakdown threshholds [5].

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