A multidimensional description of pulse solitons in superthermal plasmas

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Ion-acoustic shock excitations can be generated during the nonlinear evolution of a plasma fluid. The inspiration for our work here comes from previous theoretical research [1-3] and is also motivated by experimental observations, in which electrostatic solitary structures were detected in laser-plasma experiments [4-5].

The laser-plasmas which we are investigating are not thermalised, and therefore it cannot be assumed that the electrons obey a Maxwellian distribution. Here we model the behaviour of non-Maxwellian electrons via the Cairns- Tsallis distribution [6]. The reductive perturbation technique is used [7] to derive the Zakharov-Kutznetsov (ZK) equation [2]. Different types of shock solutions can be obtained using the hyperbolic tangent (tanh) method [8], depending on the relationship between the system parameters.

The Fluid Equation Model. We are modelling ion-acoustic waves propagating in a magnetised electron-ion plasma. The magnetic field $\mathbf{B_0}$ is uniform and we choose for it to lie along the z-axis of our Cartesian coordinate system. The plasma consists of cold ions and Cairns-Tsallis distributed electrons [6]. The system of reduced fluid equations used is as follows:

$$\frac{\partial n_i}{\partial t} + \nabla n_i \mathbf{u_i} = 0 \quad (1) \qquad \frac{\partial \mathbf{u_i}}{\partial t} + \mathbf{u_i} \nabla \mathbf{u_i} = -\nabla \phi + \mathbf{u_i} \times \Omega \quad (2)$$

where n_i and $\mathbf{u_i}$ are ion density and ion velocity, ϕ is the electric potential, and $\Omega = \omega_{ci}/\omega_{pi}$ is the ratio of ion cyclotron frequency ($\omega_{ci} = \frac{ZeB_0}{m_i}$) to ion plasma frequency ($\omega_{pi} = \left(\frac{Zn_0e^2}{\varepsilon_0 m_i}\right)^{1/2}$). We assume that at equilibrium the electron and ion densities are approximately equal, i.e. $n_{e0} \simeq n_{io} \simeq n_0$. The system is closed by Poisson's Equation, in which the electron distribution function is non-Maxwellian, but instead described by a Cairns-Tsallis distribution [6], which in reduced form is given by:

$$n_e = (1+(q-1)\phi)^{(1/(q-1))+1/2}\left(1 - \left(\frac{16q\alpha}{3-14q+15q^2+12\alpha}\right)\phi + \left(\frac{16(2q-1)q\alpha}{3-14q+15q^2+12\alpha}\right)\phi^2\right),$$

where q is the strength of nonextensivity and α is a parameter determining the number of nonthermal electrons present in the plasma model. In order to make this analytically tractable, we expand using a Taylor series, truncated at second order, and so Poisson's equation can be expressed as $\nabla^2\phi \simeq 1 + C_1\phi + C_2\phi^2 - n_i$, where

$$C_1 = \frac{q+1}{2} - \frac{16q\alpha}{3-14q+15q^2+12\alpha}, \qquad C_2 = \frac{3+2q-q^2}{8} + \frac{24q\alpha(q-1)}{3-14q+15q^2+12\alpha}.$$

We have employed the following normalisations: we have normalised lengths by the modified

Debye length $\lambda_{De} = \left(\frac{\varepsilon_0 k_B T_e}{Z n_0 e^2}\right)^{\frac{1}{2}}$, time by the inverse plasma frequency $\omega_{pi} = \left(\frac{Z n_0 e^2}{\varepsilon_0 m_i}\right)^{\frac{1}{2}}$, number density by the total ion density Zn_0, electrostatic potential by $\frac{k_B T_e}{e}$, and velocities by the ion-acoustic sound speed $c_{i,s} = \left(\frac{k_B T_e}{m_i}\right)^{\frac{1}{2}}$.

The system is subjected to following boundary conditions, which guarantee structures localised near the origin: $\phi \to 0, \quad \nabla\phi \to 0, \quad \nabla^2\phi \to 0, \quad n \to n_0, \quad \mathbf{u} \to 0 \quad$ as $|x| \to \infty$.

Reductive Perturbation Theory. To investigate the behaviour of the small ion acoustic waves, we use reductive perturbation theory, in which the independent variables are stretched as follows: $X = \varepsilon^{\frac{1}{2}}\bar{x}$; $\quad Y = \varepsilon^{\frac{1}{2}}\bar{y}$; $\quad Z = \varepsilon^{\frac{1}{2}}(\bar{z} - v_{ph}\bar{t})$; $\quad T = \varepsilon^{\frac{3}{2}}\bar{t}$.

We have assumed that the ion streaming velocity is along the z-axis, and expand in the wave amplitude while keeping one order higher than in linear theory. Due to the anisotropy introduced into the system by the magnetic field, the coordinates of velocity perpendicular to the magnetic field, u_x and u_y appear at higher order in ε than the parallel component u_z. This means that the gyromotion is treated as a higher order effect in this model: $\bar{n} = 1 + \varepsilon n_1 + \varepsilon^2 n_2 + ...,\quad \bar{\phi} = \varepsilon\phi_1 + \varepsilon^2\phi_2 + ...,$

$\bar{u}_x = \varepsilon^{\frac{3}{2}} u_{x1} + \varepsilon^2 u_{x2} + ..., \quad \bar{u}_y = \varepsilon^{\frac{3}{2}} u_{y1} + \varepsilon^2 u_{y2} + ..., \quad \bar{u}_z = \varepsilon u_{z1} + \varepsilon^2 u_{z2} + ...$

Lowest Order Terms. We substitute the stretched variables above into the reduced fluid equations, and extract the lowest order terms to produce a series of compatibility conditions, which allows us to express the phase speed v_{ph} as $v_{ph} = c_1^{-1/2}$. The phase velocity is thus dependent on the nonextensivity and nonthermality parameters q and α, as shown in the figure below.

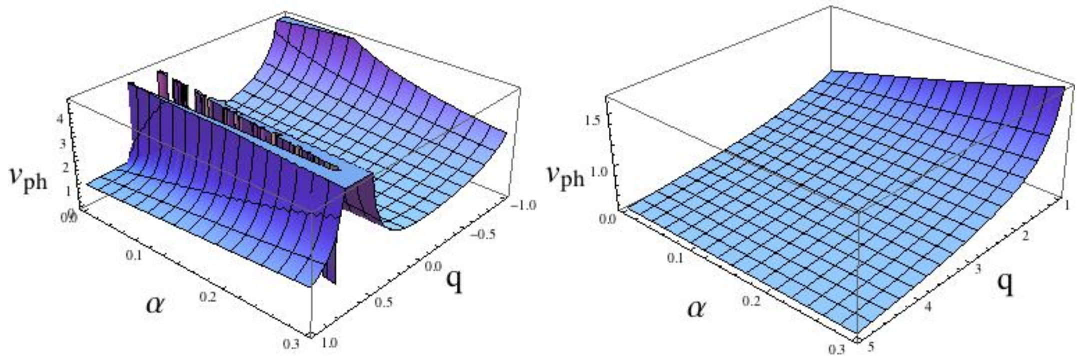

Figure 1: Plots of the phase velocity v_{ph} against q and α, where $v_{ph} = c_1^{-1/2}$. The left-hand plot shows $-1 \le q \le 1$, and the right-hand plot $q > 1$.

2nd Order. The second order terms yield compatibility conditions, which when combined, lead to the derivation of the ZK equation :

$$\frac{\partial\phi_1}{\partial T} + A\phi_1\frac{\partial\phi_1}{\partial Z} + B\frac{\partial^3\phi_1}{\partial Z^3} + C\frac{\partial}{\partial Z}\left(\frac{\partial^2\phi_1}{\partial X^2} + \frac{\partial^2\phi_1}{\partial Y^2}\right) = 0, \tag{3}$$

where the nonlinearity coefficient A, and dispersion coefficients B and C are defined as

$$A = \frac{3}{2}\left(\frac{q+1}{2} - \frac{16q\alpha}{3-14q+15q^2+12\alpha}\right)^{\frac{1}{2}} - \frac{\frac{3+2q-q^2}{8} + \frac{24q\alpha(q-1)}{3-14q+15q^2+12\alpha}}{\left(\frac{q+1}{2} - \frac{16q\alpha}{3-14q+15q^2+12\alpha}\right)^{\frac{3}{2}}},$$
$$B = \frac{1}{2\left(\frac{q+1}{2} - \frac{16q\alpha}{3-14q+15q^2+12\alpha}\right)^{\frac{3}{2}}}, \qquad C = \frac{1}{2\left(\frac{q+1}{2} - \frac{16q\alpha}{3-14q+15q^2+12\alpha}\right)^{\frac{3}{2}}}\left(1+\frac{1}{\Omega^2}\right). \tag{4}$$

ZK Solution - Hyperbolic Tangent (tanh) Method. The general solution of equation (3) can be found using the hyperbolic tangent (tanh) method [8], and is given by:

$$\psi_{sol} = \psi_0 \,\mathrm{sech}^2\left(\frac{[lX+mY+nZ-UT]}{W}\right), \tag{5}$$

where $\psi_0 = \frac{3U}{A_0}$ is the amplitude of the excitation, $W = \alpha^{-1} = 2\sqrt{\frac{B_0}{U}}$ is the soliton width, $A_0 = An$, $B_0 = Bn^3 + Cn\left(l^2+m^2\right)$, and A, B are as defined above.

Polarity Change of Soliton. The polarity of the soliton depends on whether the coefficient A_0 is positive or negative. The contour plots below show variation of A_0 with q and α.

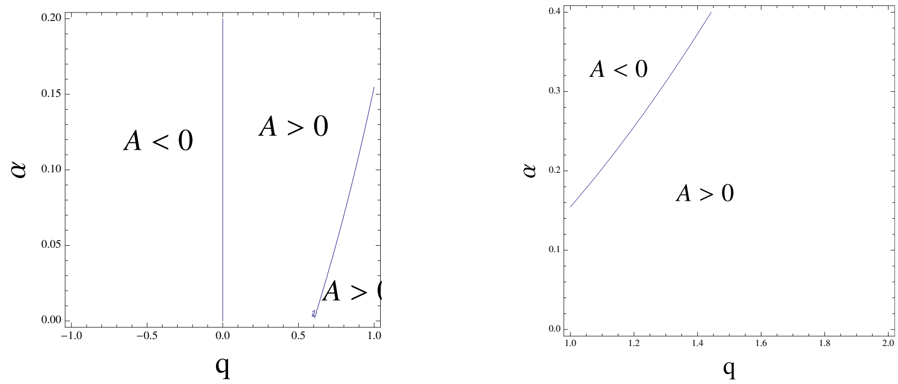

Figure 2: Contour Plot of $A_0 = 0$. Negative values marked in dark blue. Left hand plot ($-1 \leq q \leq 1$), and right-hand plot ($q \geq 1$). We have taken $n = 1$.

Effect of Nonthermality on Soliton Structure. We now look at how soliton shape changes with α, which represents the degree of nonthermality of the plasma. Figure 3 shows soliton structure variation with alpha, with negligible nonextensivity ($q = 1$).

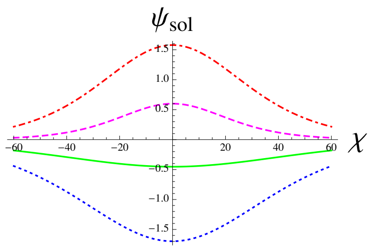

Figure 3: ψ vs χ for $\alpha = 0.1$ (red dotdashed line), $\alpha = 0.2$ (blue dotted line), $\alpha = 0.3$ (green continuous line) and the Maxwellian distribution (purple dashed line), taking $\Omega = 0.1, U = 0.1, q = 1.2$. Based on equation (5).

Effect of Nonextensivity on Soliton Structure. We now look more closely at how soliton shape changes with the nonextensive parameter q, when nonthermality is negligible (that is, $\alpha = 0$).

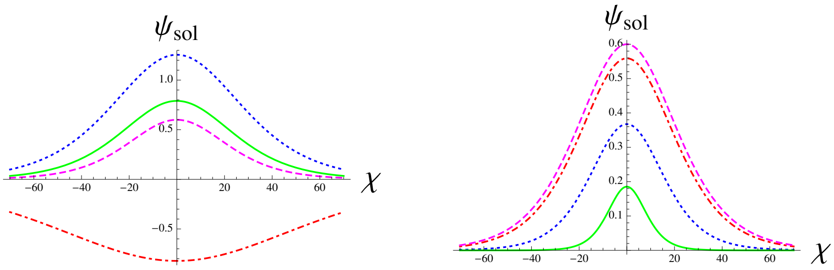

Figure 4: ψ vs χ for $q = -0.4$ (red dotdashed line), $q = 0.1$ (blue dotted line), $q = 0.8$ (green continuous line) and the Maxwellian distribution (purple dashed line) in left plot; and $q = 1.1$ (red dotdashed line), $q = 2.1$ (blue dotted line), $q = 6.1$ (green continuous line) and the Maxwellian distribution in right plot, taking $\Omega = 0.1$, $U = 0.1$, $\alpha = 0$. Based on equation (5).

Conclusion. Through reductive perturbation analysis, we have derived a Zakharov - Kutznetsov equation, which shows how an ion acoustic wave in a magnetised plasma evolves in time, and is influenced by the extent of the medium's nonlinearity and dispersion. Solving this using the hyperbolic tangent method, we have found a soliton excitation, whose shape and polarity varies with changes in nonextensivity and nonthermality.

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