Radiation Reaction in Ultra-relativistic Laser Electron Interactions

K. Seto[1], H. Nagatomo[1], J. Koga[2] and K. Mima[1]

[1]Institute of Laser Engineering, Osaka Univ., Osaka, Japan

[2]Advanced Beam Technology Division, Japan Atomic Energy Agency, Kyoto, Japan

I. Introduction

With the progress of the ultra-short pulse laser technologies, intensities will reach 10^{22}W/cm^2 [1]. We can distinguish physical regimes by the laser intensity in laser-electron interactions. With the intensity of 10^{18}W/cm^2, the electron becomes relativistic since the laser drives it to the speed of light. This is the region where we need to consider the ponderomotive force effect. As the intensity goes up to 10^{22}W/cm^2, it is predicted that the 'radiation reaction' effect appears [2], which is our main interest in this work. Radiation reaction is called 'radiation damping', as well. This effect is related to the bremsstrahlung from the electron. We can consider the bremsstrahlung which is the kinetic energy loss of the electron. If this energyloss is significant, the electron loses most of its inertia and is accelerated by the external force easily. This mechanism of the inertia change due to the bremsstrahlung is called radiation reaction or radiation damping. In order to formulate the phenomena, we need to treat it as a certain force in the equation of motion.

The original research of radiation reaction was related to the classical model of the electron by H. A. Lorentz in 1916 [3]. He considered the electron without quantum theory. The summary of his model is as follows: The charge of the electron is distributed on the surface of the sphere with the classical electron radius r_{classic}=O(10^{-15}). One of the small charged elements interacts with other elements through the Liénard-Wiechert electromagnetic field. When the electron moves, the self-interaction force (not the external force) works on the electron due to the directivity of the Liénard-Wiechert field. This self-interaction is the radiation reaction. The equation of motion with radiation reaction in the nonrelativistic regime by Lorentz is called the Lorentz-Abraham (L-A) equation,

$$m\dot{\mathbf{v}} = \mathbf{F}_{\mathrm{ex}} + m_0\tau_0\ddot{\mathbf{v}}\,. \tag{1}$$

Here, $\tau_0 = e^2/6\pi\varepsilon_0 m_0 c^3$ and c is the speed of light. P. A. M. Dirac suggested the relativistic equation of L-A's with Lorentz metric g, the signature of this is $(+---)$ [4]

$$m_0\frac{dw^\mu}{d\tau} = QF_{\mathrm{Laser}}{}^{\mu\nu}w_\nu + \left[m_0\tau_0\frac{d^2w^\mu}{d\tau^2} + \frac{m_0\tau_0}{c^2}\frac{dw^\nu}{d\tau}\frac{dw_\nu}{d\tau}w^\mu\right] \tag{2}$$

where, τ is the proper time, w is the 4-velocity defined as $w = \gamma(c, \mathbf{v})$ and m_0 is the experiential rest mass of the electron. This Lorentz-Abraham-Dirac (LAD) equation and L-A equation have a significant defect in the energy exponential divergence called the run-away.

There are many methods to solve this (the approximation of the radiation reaction force [5], new assumptions of the model [6]). In this work, a new equation of motion with the radiation reaction is derived. In the next chapter, we will show the outline of the derivation in the single electron system with the "off-shell" models.

II. Radiation Reaction with the "Off-Shell"-renormalized mass model

The LAD equation is satisfied with the relativistic relation of $w_\mu \, dw^\mu / d\tau = 0$. In another expression, we often use the relation,

$$\left(m_0 w_\mu\right)\left(m_0 w^\mu\right) = m_0{}^2 c^2 , \tag{3}$$

which is called the "on-shell" state. This Eq.(3) leads to concrete definitions of the 4-velocity and the4-force, etc. But, the radiation reaction is the self-interaction in QED [7]. In this theory, it can be transformed from particles to fields, or from fields to particles. Our rest mass in the classical theory is the renormalized mass in QED. This mass includes the electromagnetic mass which is converted from the Liénard-Wiechert (Coulomb) field. And these fields imply the vacuum polarization around an electron. We need to consider the "off-shell" model with a variable-renormalized mass. Now, we consider the equation of motion with the radiation reaction by proceeding stages.

step 1) Newton's equation is, $m_0 \, dw^\mu / d\tau = F_{\mathrm{ex}}{}^\mu$. However, we need to take into account the electromagnetic mass (dressed mass) m_{EM} in the self-interaction of electron. Therefore, the equation of motion becomes, $\left(m_0 + m_{\mathrm{EM}}\right) dw^\mu / d\tau = F_{\mathrm{ex}}{}^\mu$. Now, it is noted m_{EM} is constant. But, this motion is different from the original Newton's equation. Here, the external force is defined as

$$F'_{\mathrm{ex}}{}^\mu = \frac{m_0 + m_{\mathrm{EM}}}{m_0} F_{\mathrm{ex}}{}^\mu , \tag{4}$$

then, the equation of motion is

$$\left(m_0 + m_{\mathrm{EM}}\right) \frac{dw^\mu}{d\tau} = F'_{\mathrm{ex}}{}^\mu . \tag{5}$$

These two equations reproduce the original Newton's equation.

step 2) The energy loss of the bremsstrahlung is expressed by $m_0 \tau_0 \left(dw^\nu / d\tau\right)\left(dw_\nu / d\tau\right)$ in the covariant form [5,6]. From the theory of Einstein, the mass is equivalent to the energy (the relation, $E = mc^2$). Therefore, if we treat the bremsstrahlung as the mass changing,

$$\frac{dm}{d\tau} = -\frac{m_0 \tau_0}{c^2} \frac{dw^\nu}{d\tau} \frac{dw_\nu}{d\tau}. \tag{6}$$

The mass of the electron can't be represented by a constant any longer.

step 3) The mass before the renormalization is defined as

$$m(\tau) = m_0 + m_{\mathrm{EM}}(\tau) = m_0 \times f(\tau). \tag{7}$$

Here, we consider that m_0 is the renormalized mass. Now, the 4 momentum is defined as $p^\mu = m w^\mu$, the equation of motion becomes

$$\frac{dp^\mu}{d\tau} = F'^{\ \mu}_{\mathrm{ex}}. \tag{8}$$

Equation (4) and Eq.(7) are substituted for Eq.(8),

$$\frac{d}{d\tau}\left[m(\tau) w^\mu \right] = \frac{m(\tau)}{m_0} F_{\mathrm{ex}}^{\ \mu}, \tag{9}$$

$$m_0 \frac{dw^\mu}{d\tau} + m_0 \left[\frac{d}{d\tau} \ln \frac{m(\tau)}{m_0} \right] w^\mu = F_{\mathrm{ex}}^{\ \mu}. \tag{9'}$$

Here, we replace $m_0 \mapsto m(\tau)$, in the bremsstrahlung energy loss of Eq.(6),

$$\frac{d}{d\tau}\left[\ln m(\tau) \right] = \frac{1}{m(\tau)} \frac{dm(\tau)}{d\tau} = -\frac{\tau_0}{c^2} \frac{dw^\nu}{d\tau} \frac{dw_\nu}{d\tau}. \tag{10}$$

Solving this equation,

$$m(\tau) = m_0 \times \exp\left[-\int_{-\infty}^{\tau} d\tau' \left(\frac{\tau_0}{c^2} \frac{dw^\nu}{d\tau} \frac{dw_\nu}{d\tau} \right)(\tau') \right]. \tag{11}$$

In the classical theory, the electromagnetic mass always has an infinite value [3,5]. It is introduced that the infinite parameter $\Lambda_0 \to \infty$ with $\Lambda(\tau)$, (11) is replaced,

$$m(\tau) = m_0 \times \exp\left[-\int_{-\infty}^{\tau} d\tau' \left(\frac{\tau_0}{c^2} \frac{dw^\nu}{d\tau} \frac{dw_\nu}{d\tau} \right)(\tau') + \Lambda_0 + \Lambda(\tau') \right]. \tag{11'}$$

Here, we assume $\Lambda(\tau) << \Lambda_0$ and recover the original mass state. By another expression, the mass before the renormalization without the radiation reaction is

$$m(\tau) = m_0 \times \exp \Lambda_0. \tag{12}$$

This recovery process must be treated in QED or any higher-theory, not classically. The two equations, Eq.(9)' and Eq.(11) become

$$m_0 \frac{dw^\mu}{d\tau} = F_{\mathrm{ex}}{}^\mu + m_0 \left(\frac{\tau_0}{c^2} \frac{dw^\nu}{d\tau} \frac{dw_\nu}{d\tau} - \frac{d\Lambda}{d\tau} \right) w^\mu . \tag{13}$$

The mass relation of Eq.(11)' has, of course, time dependence. Since, Λ_0 is too large, we cannot observe the recovering process in Eq.(11)', the component of $\Lambda(\tau)$ can be negligible. It can be noted $\Lambda(\tau)$ is taken into Λ_0. In a rough estimation, Eq.(13) without the mass recovery process is,

$$m_0 \frac{dw^\mu}{d\tau} = F_{\mathrm{ex}}{}^\mu + \frac{m_0 \tau_0}{c^2} \frac{dw^\nu}{d\tau} \frac{dw_\nu}{d\tau} w^\mu , \tag{14}$$

This equation is the approximation of the LAD equation, without the Schott term. Instead of the on-shell relation of $w^\mu w_\mu = c^2$, Eq.(13) with the Eq.(11)' and $F_{\mathrm{ex}}{}^\mu = 0$ becomes

$$m(\tau) w^\mu w_\mu = m_0 c^2 \times \exp[\Lambda_0] . \tag{15}$$

This is the off-shell relation which is corrected to the on-shell condition.

III. Summary

In this research, we investigated radiation reaction which is a new generation physics, but an old problem of the classical theory. Our final result is Eq.(13) or Eq.(9) with Eq.(11)'. The new point of view is by using the mass before renormalization (see Eq.(8)). Based on previous reviews, the motion follows Eq.(14) in the ultra-intense laser electron interaction [2,6]. Therefore, $d\Lambda/d\tau$ should be smaller than other terms, the mass recovers more slowly than the radiation duration. The detail of this mechanism will be discovered by QED, or other theories. This research is supported by a research granted from The Murata Science Foundation.

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