The ion-ion hybrid Alfvén resonator in a fusion environment

W. A. Farmer, G. J. Morales, S. T. Vincena, and J. E. Maggs

Physics and Astronomy Department

University of California, Los Angeles

Los Angeles, CA 90095 USA

Abstract

In a tokamak, the periodic variation in the strength of the magnetic field along a field-line results in two conjugate ion-ion hybrid points that give rise to an inherent shear Alfvén wave resonator. Modes trapped within such a resonator could have consequences for plasma heating, proposed alpha channeling schemes, and instabilities. In addition, the modes could provide useful diagnostic signatures. Recent experiments in the linear device LAPD at UCLA have demonstrated the existence of such a resonator in a magnetic mirror configuration. Motivated by these results, and also by related magnetospheric studies, the properties of a similar resonator in a fusion environment are explored. Theoretical results relevant to the modeling of the resonator in ITER-like plasmas are presented.

Introduction

In a single species, magnetized plasma, shear Alfvén waves propagate at frequencies, ω, below the ion-cyclotron frequency and are evanescent at higher frequencies. In the presence of two ion species, however, it has long been established [1] that the properties of Alfvén waves are significantly different [2]. The cold-plasma dielectric for two ion-species is

$$\varepsilon_\perp = \left(\frac{c}{v_A}\right)^2 \frac{1-\frac{\omega^2}{\omega_{ii}^2}}{\left(1-\frac{\omega^2}{\Omega_1^2}\right)\left(1-\frac{\omega^2}{\Omega_2^2}\right)}, \tag{1}$$

where the frequencies Ω_1 and Ω_2 are the cyclotron frequencies of the respective ions, c is the speed of light, v_A is the Alfvén speed, and ω_{ii} is the "ion-ion hybrid frequency" at which the perpendicular dielectric vanishes [3]. The Alfvén speed and the ion-ion hybrid are related to the ion-cyclotron and plasma frequencies as follows,

$$v_A = \frac{\Omega_1\Omega_2}{\sqrt{\omega_{p1}^2\Omega_2^2+\omega_{p2}^2\Omega_1^2}}c, \quad \omega_{ii}^2 = \frac{\omega_{p1}^2\Omega_2^2+\omega_{p2}^2\Omega_1^2}{\omega_{p1}^2+\omega_{p2}^2}. \tag{2}$$

For large perpendicular wave number, $k_\perp$, (i.e., $\lambda_\perp \ll c/\omega_{pi}$) the dispersion relation for the shear Alfvén wave can be approximated by

$$k_{||}^2 = k_0^2\varepsilon_\perp\left(1-\frac{k_\perp^2}{k_0^2\varepsilon_{||}}\right), \tag{3}$$

with $k_0 = \omega/c$. In this parameter regime, the compressional mode is evanescent. The group velocity of the waves is predominantly along the magnetic field, and the ion-ion hybrid frequency acts as a cutoff for parallel propagation. Shear waves propagate for frequencies satisfying $0 < \omega < \Omega_1$ (the lower propagation band) and $\omega_{ii} < \omega < \Omega_2$ (the upper propagation band) where species 2 has been assumed to have the larger charge-to-mass ratio. Due to the dependence of the ion-ion hybrid frequency on magnetic field strength, if the wave propagates along a field line of increasing strength, the wave will be reflected at the location, $\omega = \omega_{ii}(\mathbf{r})$. If a magnetic well is present, then it is possible to trap the resulting waves between two conjugate reflection points resulting in a resonator configuration. This concept was introduced to explain wave features in the Earth's magnetosphere [4] and has recently been verified in the laboratory [5].

In a tokamak environment, the toroidal magnetic field, B_t, varies as $1/(R + r\cos\theta)$, R being the major radius, r is the radius of the flux surface from the magnetic axis, and θ is the poloidal angle. As a shear wave propagates along a field line from the outboard side to the inboard side (from $\theta = 0$ towards $\theta = \pi$), it will encounter regions of increasing magnetic field. Further, in a fusion environment, it is necessary to operate with two ion species, deuterium and tritium. Thus, a resonator configuration is naturally present inside a tokamak device operating under fusion conditions.

Model

To illustrate what a resonator might look like in ITER, the ion-cyclotron and ion-ion hybrid frequencies are shown in Fig. 1 as a function of distance along a magnetic field line for the expected parameters of the device. The cyan region corresponds to the lower propagation band and is the frequency region where Toroidal Alfvén Eigenmodes (TAE's) can exist (at low frequency). The yellow region corresponds to the upper propagation band. In order to have a resonator configuration, it is important that the mode not interact with the deuterium cyclotron frequency. This confines the candidate resonator modes to the region indicated by the red cross-hatching.

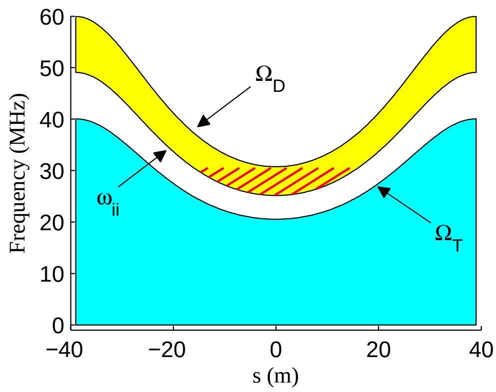

Figure 1: Frequencies along a field line in ITER. Plasma is composed of equal concentrations of D-T at an electron density of $n_e = 1\times10^{14}$ cm^{-3}. Safety factor, q = 2; major radius, R = 6.21 m; minor radius, a = 2 m; field at magnetic axis, B_0 = 53 kG.

Neglecting the corrections due to field line curvature and magnetic shear and using the cold-plasma dielectric tensor, the wave equation takes the form

$$\nabla \times (\nabla \times \mathbf{E}) - k_0^2 \overleftrightarrow{\varepsilon}(\mathbf{r}, \omega) \cdot \mathbf{E} = 0. \tag{4}$$

Decomposing **E** into its eigenvectors, the eigenvector associated with the shear root is

$$\tilde{\mathbf{E}}_S = \left(\left(k_\perp^2 - k_0^2 \varepsilon_{||}\right) \vec{e}_1 + i\varepsilon_{xy} \frac{k_\perp^2 - k_0^2 \varepsilon_{||}}{k_0^2 \varepsilon_\perp - k_\perp^2 - k_S^2} \vec{e}_2 + k_\perp k_S \vec{e}_3 \right) \tilde{E}_S, \tag{5}$$

$$k_S^2(\omega, s) = k_0^2 \varepsilon_\perp + \frac{-k_\perp^2 (\varepsilon_{||} + \varepsilon_\perp) \pm D}{2\varepsilon_{||}}, \quad D = \sqrt{k_\perp^4 \left(\varepsilon_{||} - \varepsilon_\perp\right)^2 - 4k_0^2 \varepsilon_{xy}^2 \varepsilon_{||} \left(k_\perp^2 - k_0^2 \varepsilon_{||}\right)}, \tag{6}$$

with the background magnetic field defined as $\mathbf{B}_0 = B_0 \vec{e}_3$ and the wave vector as $\mathbf{k} = k_\perp \vec{e}_1 + k_S \vec{e}_3$. Here, k_S is the root of the dispersion relation associated with the shear mode. With these definitions, Eq. (4) can be cast as

$$\frac{d^2 \tilde{E}_S}{ds^2} + [k_S(s)]^2 \tilde{E}_S = 0, \tag{7}$$

with s parameterizing the distance along the field line.

In order to solve Eq. (7), a WKB method is employed. This leads to the quantization condition

$$\int_{-s_0(\omega)}^{s_0(\omega)} k_S(\omega, s) ds = \left(n + \frac{1}{2}\right) \pi, \tag{8}$$

where n is indexed starting at zero. The parameter, s_0, is the turning point of the effective potential and corresponds to the position at which k_S vanishes. Since s_0 also depends on the frequency, Eq. (8) is an implicit equation that must be solved numerically. The eigenfunctions can similarly be calculated, though their functional form is not shown for brevity.

Results

Figure 2 illustrates a WKB solution in the top panel and the corresponding phase velocity of the wave as a function of position in the bottom panel. The parameters are the same as in Fig. 1 with the perpendicular wavelength, $\lambda_\perp = 10$ cm. The waves are assumed to propagate along a magnetic flux surface at $r = 2$ m. For reference, the thermal velocities of electrons and deuterium ions at 10 keV are displayed as red and orange dashed lines, respectively. Additionally, the velocity of a 3.5 MeV alpha particle is shown as a blue dashed line. The wave solution corresponds to $n = 27$ with a frequency of 25.44 MHz. Because of the nature of the solutions, a wide variation in phase velocities of the eigenmodes exists. An $n = 3$ mode (25.06 MHz) has a minimum phase velocity roughly equivalent to the electron thermal speed, and an $n = 1000$ mode (30.59 MHz) has a minimum phase velocity roughly equivalent to the deuterium thermal velocity.

On this flux surface, the band of trapped frequencies ranges from 25.1 to 30.7 MHz with the initial frequency spacing of 16 kHz. At flux surfaces closer to the magnetic axis, the well is longer in position space, and the frequency range shifts to higher values. At the flux surface, $r = 60$ cm, the frequencies of trapped modes range from 30.1 to 36.5 MHz with an initial frequency spacing of 11.6 kHz. At $r \approx 60$ cm, ω_{ii} on the inboard side is equal to Ω_D on the outboard side. At flux surfaces approaching the magnetic axis, no trapped modes exist.

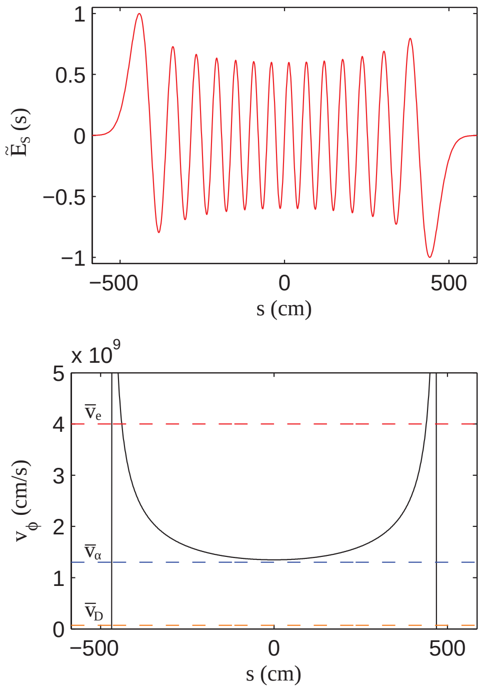

Figure 2: Top panel illustrates the n=27 WKB wave solution, and bottom panel shows the phase velocity. Dashed lines correspond to the thermal velocities of the electrons (red), the deuterium ions (orange), and the velocity of a 3.5 MeV alpha particle (blue).

Conclusions

From this analysis, it is apparent that the ion-ion hybrid resonator could plausibly exist in a magnetically confined, fusion environment. It is also clear that the resonator can be overmoded with many possible frequencies and phase velocities existing within the well. Further, these modes will be confined to the outboard side of the device, and will be more significant towards the exterior of the plasma than at the core. The wide range of phase velocities gives rise to possible wave-particle resonant interactions that could have a variety of effects. The ion-ion hybrid resonator warrants further theoretical and experimental studies to explore the possible consequences for burning plasmas.

References

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