Dynamical Modelling of Neoclassical Tearing Mode Suppression by ECCD

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Introduction

The stabilization of neoclassical tearing modes (NTMs) by electron cyclotron current drive (ECCD) has been studied extensively in the framework of the generalized Rutherford equation (GRE) [1], which describes the evolution of the full width w of the associated magnetic island. Including only the most relevant terms in the context of the present paper the GRE is written as [1]:

$$0.82\frac{\tau_r}{r_s}\frac{dw}{dt} = r_s\Delta_0'(w) + r_s\Delta_{bs}' + r_s\Delta_{CD}', \qquad (1)$$

where $\tau_r = \mu_0 r_s^2/\eta$ for the plasma resistivity η at the resonant surface r_s of the island. $\Delta_0'(w)$ is the classical stability index. The driving mechanism of NTMs, which is the perturbation of the bootstrap current, is represented by the second term. The last term refers to the stabilizing effect of ECCD. Conventionally, Δ_{CD}' is obtained by averaging the ECCD over an island rotation period τ_{rot}. This requires that τ_{rot} is much shorter than the collisional time scale τ_{coll} on which the EC driven current is generated/decays. When this assumption is not valid, the EC driven current δJ_{CD} becomes time dependent, oscillating and moving through the island with the rotation period. As a result Δ_{CD}' will oscillate with the rotation period as well. In this contribution we study these oscillations in Δ_{CD}' and their consequences for the suppression of NTMs by ECCD.

Model equations

We will be presenting the results in terms of a normalized GRE. Time is normalized to τ_{NTM} which is defined as the inverse growth rate coming from the maximum of the bootstrap term, i.e. $\tau_{NTM} \equiv \frac{0.82\tau_r}{r_s\Delta_{bs,max}'}$. Spatial scales like island sizes and deposition widths are normalized by r_s. The classical stability index Δ_0' is expressed in terms of the saturated island size, w_{sat} at which the NTM growth saturates in the absence of other stabilizing effects, i.e. $\Delta_0' = -\Delta_{bs}'(w = w_{sat})$. Finally, the small island limit of the bootstrap term is supposed to be determined by the effects of incomplete pressure flattening from the competition between parallel and perpendicular transport [2]. This way the GRE is obtained in the following normalized form,

$$\frac{d\bar{w}}{d\bar{t}} = -\frac{2\bar{w}_{sat}\bar{w}_{marg}}{\bar{w}_{sat}^2 + \bar{w}_{marg}^2} + \frac{2\bar{w}\bar{w}_{marg}}{\bar{w}^2 + \bar{w}_{marg}^2} + \bar{\Delta}_{CD}', \qquad (2)$$

where $\bar{t} \equiv t/\tau_{NTM}$, $\bar{w} \equiv w/r_s$ and $\bar{\Delta}'_{CD} = \Delta'_{CD}/\Delta'_{bs,max}$. The contribution of ECCD is written as [3, 4]

$$r_s\Delta'_{CD} = -\frac{16\mu_0 L_q r_s}{B_p \pi w^2}\left[\int_{-\infty}^{\infty} dx \oint d\xi J_{CD}\cos\xi\right]. \tag{3}$$

To evaluate this term as a function of time a proper equation for the evolution of the driven current density is needed taking into account the effects of rotation and a finite collision time.

The power deposition profile is written as the product of the total power P_{tot} with a normalized profile function, $\tilde{p}_{CW}(x,\xi)$ which is defined as Gaussian in the radial direction localized at r_{dep} and with a full width w_{dep}, and as a top hat function with an angular width $\Delta\xi$ moving through the island with the rotation period τ_{rot} in the helical direction. We will only consider CW application of the ECCD power, with a radial deposition that is perfectly aligned with the resonant radius of the NTM, i.e. $r_{dep} = r_s$. Parallel transport is assumed to be instantaneous, such that the driven current density is a flux function. We will be using the normalized flux label $\Omega = 8x^2/w^2 - \cos\xi$, with $\Omega = -1$ at the O-point of the island and $\Omega = 1$ at the X-point. Here, $x = r - r_s$ is the displacement from the resonant surface. Outside the island the label $\sigma = sgn(x)$ distinguishes the two different surfaces with identical Ω on opposite sides of r_s.

The ECCD efficiency is assumed to be a simple constant, $\eta_{CD} \equiv I_{CD}/P_{tot}$. The driven current density then depends only on the flux surface averaged power density $P_{EC}(\Omega,t)$ as a function of time. Because the island evolution is incompressible, the evolution of the island will not affect the current density $J_{CD}(S_\Omega)$ expressed as a function of the total area enclosed by the flux surface: $J_{CD}(S_\Omega)$ decays on a collisional time scale τ_{coll} and is generated by the instantaneous power deposited on that surface, i.e.

$$\frac{\partial J_{CD}(S_\Omega,\sigma,t)}{\partial t} = -\frac{J_{CD}(S_\Omega,\sigma,t)}{\tau_{coll}} + 4\pi^2 R r_s \frac{\eta_{CD}}{\tau_{coll}} P_{EC}(S_\Omega,\sigma,t). \tag{4}$$

Only on those surfaces that are being reconnected due to island growth the current density must be averaged over the two contributing surfaces, i.e. for $S_{sep}(t^-) < S_\Omega \leq S_{sep}(t^+)$:

$$J_{CD}\left(S_\Omega,t^+\right) = \frac{1}{2}\left(J_{CD}\left(S_\Omega,\sigma=1,t^-\right) + J_{CD}\left(S_\Omega,\sigma=-1,t^-\right)\right), \tag{5}$$

while on surfaces that get disconnected due to island shrinkage the current density is conserved, i.e. for $S_{sep}(t^+) < S_\Omega \leq S_{sep}(t^-)$:

$$J_{CD}\left(S_\Omega,\sigma=\pm 1,t^+\right) = J_{CD}\left(S_\Omega,t^-\right). \tag{6}$$

Here, S_{sep} represents the enclosed surface area within the separatrix and t^- and t^+ refer to the time steps just before and just after the surfaces are reconnected/disconnected, respectively. Equations (4), (5) and (6) describe the time dependent evolution of the driven current in the case of an evolving island.

Numerical results

We first solve equations (4) and (3) for constant island width w. The results are normalized to $\Delta'_{REF} \equiv \Delta'_{CD}(\Delta\xi = 2\pi, \tau_{coll} = 0)$, which corresponds to Δ'_{CD} as obtained conventionally by averaging the power deposition and current drive profiles over a rotation period [3, 4]. For finite τ_{coll} the build up of the driven current takes time reaching a quasi-steady state for $t \gg \tau_{col}$ [5]. Consequently the same holds for Δ'_{CD}. A helically localized power results in oscillations of the driven current and of Δ'_{CD} at the plasma rotation frequency on top of this quasi-stationary state. As equations (4) and (3) are linear, the time average over a full rotation period of the CD term, $< \Delta'_{CD} >_{\tau_{rot}}$, is identical to the reference case: $\lim_{t\to\infty} \langle \Delta'_{CD} \rangle_{\tau_{rot}} = \Delta'_{REF}$. The Δ'_{CD} oscillations in the quasi-steady state are shown in Fig. 1 for $\tau_{coll}/\tau_{rot} = 0$, 0.3, 1.0, and ∞. The case of $\tau_{coll}/\tau_{rot} = 0$ can also be seen as representing the Δ'_{CD} efficiency as a function of island phase in case of a locked mode. As τ_{coll}/τ_{rot} increases, the amplitude of the oscillations decreases, and the phase of the Δ'_{CD} oscillation is shifted relative to the island rotation.

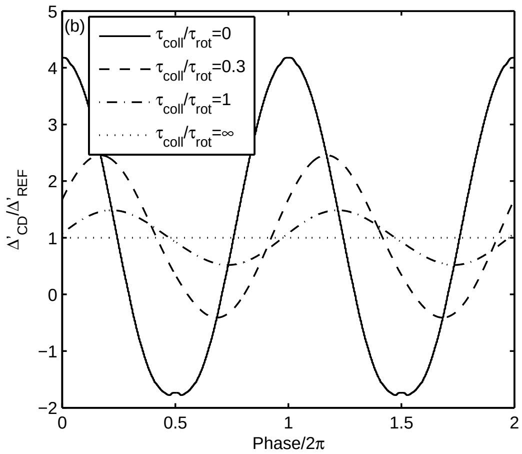

Figure 1: Δ'_{CD} oscillations in quasi-steady state for $\Delta\xi = 0.04\pi$ and $w = w_{dep}$.

The extrema of Δ'_{CD} simulations at quasi-steady state as a function of w/w_{dep} for three different τ_{coll}/τ_{rot} are shown in Fig. 2. The results are shown in terms of extrema and not simply the oscillation amplitudes, because the periodic Δ'_{CD} oscillation is not a simple sinusoidal oscillation around its reference value. When looking at the maximum and the minimum as a function of τ_{coll}/τ_{rot} for a fixed island width, the extrema in Δ'_{CD} are seen to saturate at a maximum value for $\tau_{coll}/\tau_{rot} < 10^{-1}$. As τ_{coll}/τ_{rot} is increased above 10^{-1}, the extrema approach the reference value, leaving only modest oscillations for $\tau_{coll}/\tau_{rot} > 1$. The amplitude of the Δ'_{CD} oscillations remain almost unchanged when the helical power deposition width is increased up to $\Delta\xi = 0.4\pi$.

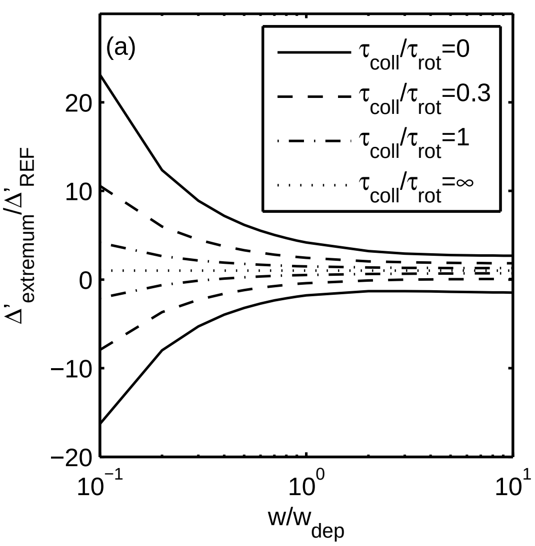

Figure 2: Dependence of extrema of Δ'_{CD} on w/w_{dep}.

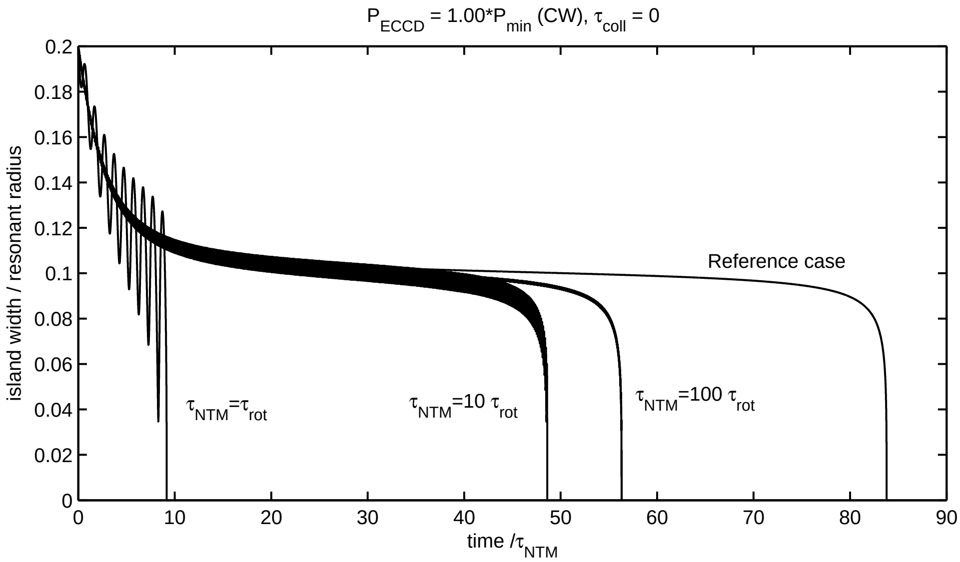

Figure 3: Island width evolution for a power marginally above $P_{min}(CW)$ in the cases of reference, $\tau_{NTM}/\tau_{rot} = 1, 10$ and 100, with $(\Delta\xi = 0.04\pi, \tau_{coll} = 0)$.

We now investigate how the oscillations in Δ'_{CD} influence the stabilization of NTMs. The parameters are chosen as representative of a 2/1 NTM in ITER: $\bar{w}_{sat} = 0.2$ and $\bar{w}_{marg} = 0.0125$, with a narrow EC radial deposition $\bar{w}_{dep} = 0.01$ and helical width $\Delta\xi = 0.04\pi$. A minimum required power P_{min} for NTM stabilization is determined for the reference case. Fig. 3 shows numerical results with an ECCD power marginally above P_{min} for $\tau_{coll}/\tau_{rot} = 0$ and various values of $\tau_{NTM}/\tau_{rot} = 1, 10$ and 100. An oscillation in the island width is found, whose amplitude increases proportional to the rotation period. This oscillation results in a net increase of the stabilizing effect of ECCD and a reduction in the time required for full suppression. Also the minimum power required for full suppression is reduced below the value of P_{min} in the reference case. In case of $\tau_{NTM}/\tau_{rot} = 1$ and $\tau_{coll}/\tau_{rot} = 0$, the minimum required power for full NTM stabilization is reduced to 95% of P_{min}.

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