Efficiency of theoretical approaches, applied in plasma polarimetry

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The paper analyses efficiency of seven theoretical approaches used in plasma polarimetry: quasi-isotropic approximation (QIA), Stokes vector formalism (SVF), complex polarization angle (CPA), complex amplitude ratio (CAR) and three angular variables (AVT) techniques: "azimuth angle ψ - ellipticity angle χ", "amplitude ratio α - phase difference δ" and "azimuth angle ψ - phase difference δ". All the methods stem from the unique theoretical basis – electrodynamics of weakly anisotropic plasma, presented by quasi-isotropic approximation. In principle they are equivalent to each other, though they have drastically different mathematical form: single ODE of the first order for CPA and CAR, system of two ODE for AVT and QIA and system of three ODE for SVF. It is shown that in conditions characteristic for modern tokamaks all analyzed methods are comparable in calculation accuracy and rate.

1. Introduction

Plasma polarimetry deals with the theoretical methods describing evolution of electromagnetic wave polarization in magnetized plasma. The equations of all existing methods could be derived from equations of quasi-isotropic approximation (QIA) of geometrical optics method [1-3]. For the magnetized plasma with negligible dissipation, in cold plasma approximation, the change of polarization state along the ray is described by the QIA equations

$$\begin{cases} \dot{\Gamma}_1 = -\frac{1}{2} i\left(2\Omega_0 - \Omega_\perp - \Omega_1\right)\Gamma_1 + \frac{1}{2} i\left(\Omega_2 + i\Omega_3\right)\Gamma_2 \\ \dot{\Gamma}_2 = \frac{1}{2} i\left(\Omega_2 + i\Omega_3\right)\Gamma_1 - \frac{1}{2} i\left(2\Omega_0 - \Omega_\perp + \Omega_1\right)\Gamma_2 \end{cases} \tag{1}$$

where parameters $\Omega_{1,2,3}$ are the components of vector $\mathbf{\Omega}$

$$\mathbf{\Omega} = \begin{pmatrix} \Omega_1 \\ \Omega_2 \\ \Omega_3 \end{pmatrix} = \frac{\omega_p^2}{2c\omega^3} \begin{pmatrix} \omega_c^2 \sin^2 \alpha_\parallel \cos 2\alpha_\perp \\ \omega_c^2 \sin^2 \alpha_\parallel \sin 2\alpha_\perp \\ 2\omega\omega_c \cos \alpha_\parallel \end{pmatrix} = \frac{\omega_p^2}{2c\omega^3} \begin{pmatrix} \frac{e^2}{m^2c^2}\left(B_x^2 - B_y^2\right) \\ \frac{e^2}{m^2c^2} 2B_x B_y \\ 2\omega \frac{e}{mc} B_z \end{pmatrix} \tag{2}$$

widely used in plasma polarimetry [4], and Ω_0 with $\Omega_\perp = \sqrt{\Omega_1^2 + \Omega_2^2} = \Omega_0 \sin^2 \alpha_\parallel$ are auxiliary parameters. $\omega_p = 4\pi e^2 N_e / m$ is the plasma frequency , $\omega_c = eB_0/mc$ is the electron cyclotron frequency and $\alpha_\parallel$ with $\alpha_\perp$ are the angles between the ray and the components of magnetic field vector $\mathbf{B} = (B_x, B_y B_z) = (B\sin\alpha_\parallel \cos\alpha_\perp, B\sin\alpha_\parallel \sin\alpha_\perp, B\cos\alpha_\parallel)$.

Equations of polarization state evolution described in the frame of other methods has been presented and discussed in subsequent publications: [4,5] - Stokes vector formalism (SVF)

$$\dot{\mathbf{s}} = \mathbf{\Omega} \times \mathbf{s} \tag{3}$$

[1,6] - complex polarization angle (CPA),

$$\dot{\theta} = \tfrac{1}{2}\Omega_3 + \tfrac{1}{2} i(\Omega_1 \sin 2\theta - \Omega_2 \cos 2\theta) \tag{4}$$

[7] - complex polarization ratio (CAR)

$$\dot{\zeta} = \frac{1}{2}\left[2i\Omega_1\zeta - i\Omega_2\left(1-\zeta^2\right) + \Omega_3\left(1+\zeta^2\right)\right] \tag{5}$$

[6] - "azimuth angle ψ - ellipticity angle χ"

$$\begin{cases} \dot{\psi} = \tfrac{1}{2}\Omega_3 - \tfrac{1}{2}(\Omega_1 \cos 2\psi + \Omega_2 \sin 2\psi)\tan 2\chi \\ \dot{\chi} = \tfrac{1}{2}(\Omega_1 \sin 2\psi - \Omega_2 \cos 2\psi) \end{cases} \tag{6}$$

[8] -"amplitude ratio α - phase difference δ"

$$\begin{cases} \dot{\alpha} = \tfrac{1}{2}\left(-\Omega_2 \sin\delta + \Omega_3 \cos\delta\right) \\ \dot{\delta} = \Omega_1 - \left(\Omega_2 \cos\delta + \Omega_3 \sin\delta\right)\cot 2\alpha \end{cases} \tag{7}$$

[9] - "azimuth angle ψ - phase difference δ"

$$\begin{cases} \dot{\psi} = \tfrac{1}{2}\Omega_3 - \tfrac{1}{2}(\Omega_1 \cos 2\psi + \Omega_2 \sin 2\psi)\ \tan\delta \sin 2\psi \\ \dot{\delta} = \Omega_1 - (\Omega_2 \cos\delta + \Omega_3 \sin\delta)\cos\delta / \tan 2\psi \end{cases} \tag{8}$$

Thus, we have seven systems of equations describing evolution of electromagnetic wave polarization state in magnetized plasma, which never were undergone to comparative analysis until now.

2. Numerical simulations

In order to compare all models the specific plasma configuration has been chosen: plasma with circular cross-section of the magnetic flux surfaces and parabolic plasma density profile $N_e = N_0\left(1-\rho^2\right)$, where $\rho = r/a$ is the normalized radius of the flux surface in the plasma with

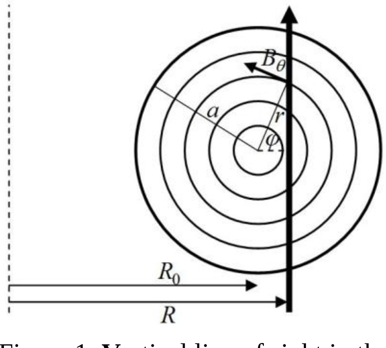

Figure 1. **V**ertical line of sight in the plasma with circular flux surfaces

minor radius a (Fig.1). In the case of a large aspect-ratio circular plasma with a current density distribution $j = j_0\left(1-\rho^2\right)^\nu$, providing the total current I_0, the poloidal magnetic field is $B_\theta = \mu_0 I_0 \left(1-\left(1-\rho^2\right)^{\nu+1}\right)/2\pi a\rho$ and the toroidal component of magnetic field is $B_R = B_0\, R_0/R$ [10]. The plasma parameters to numerical simulations have been chosen similar to these of the large thermonuclear plasma devices, like JET: $B_0 = 4.8\,\mathrm{T}$, $R_0 = 3\,\mathrm{m}$, $a = 1.2\,\mathrm{m}$, $N_0 = 1\cdot 10^{20}\,\mathrm{m}^{-3}$, $I_0 = 3\,\mathrm{MA}$. The laser beam, with wavelength $\lambda = 195\,\mu\mathrm{m}$ (DCN laser) and initial polarization $(\psi_0, \chi_0) = (\pi/4, 0)$, propagates along a vertical chords with $R = -1.2 \div 1.2\,\mathrm{m}$ (taken as a z-axis) , in a poloidal plane of a tokamak. For such a cords $\mathbf{B} = (B_R, B_\theta \sin\varphi, B_\theta \cos\varphi)$.

Numerical calculations of the beam polarization state after crossing the plasma have been done using standard Matlab procedures, with “ode45” differential equations solver and relative tolerance $\mathrm{RelTol} = 1\cdot 10^{-6}$. For all models the code was the same, except subroutines with differential equations. Results of calculations for all seven methods being under consideration, with the aim of comparison recalculated to variables (ψ, χ), are presented in Figure 2. As differences between individual lines on Figure 2 are unnoticeable, Figure 3 presents their deviation from the averaged value, calculated over all methods: $(\psi - \overline{\psi}, \chi - \overline{\chi})$. Finally the mean computation time for single vertical chord and for each method is presented on Figure 4.

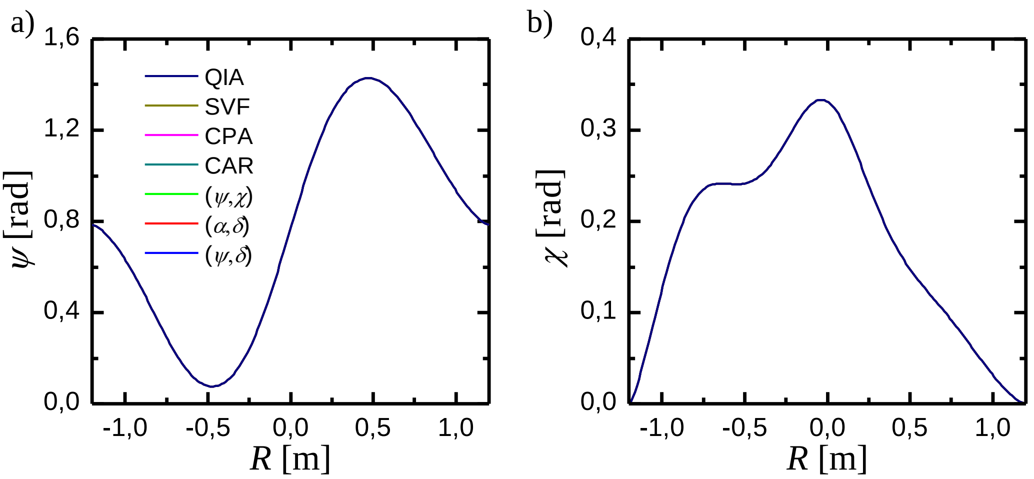

Figure 2. The phase shift (a) and ellipticity angle (b) calculated from all analyzed methods

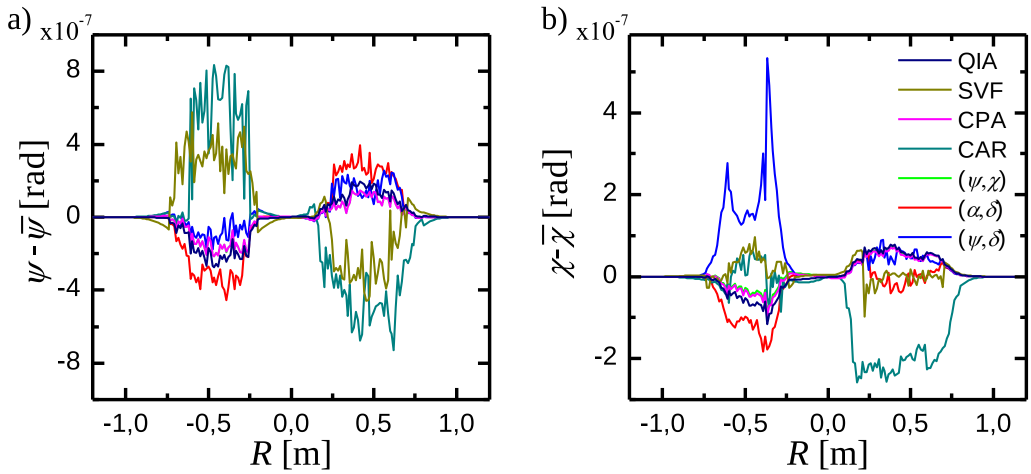

Figure 3. The phase shift (a) and ellipticity angle (b) deviation from medium value

3. Conclusions

All seven methods under consideration operate with system of differential equations: single equation for complex variables in case of CPA and CAR, two equations for complex variables in case of QIA, two equations for real variables in case of AVT methods ((ψ,χ), (α,δ) and (ψ,δ)) and three equations for three variables in frame of SVF. Nevertheless different mathematical structure, there are not significant differences between individual methods in polarization state evolution calculations, both in computation accuracy (~10^{-7}rad) and in mean computation time (~40ms).

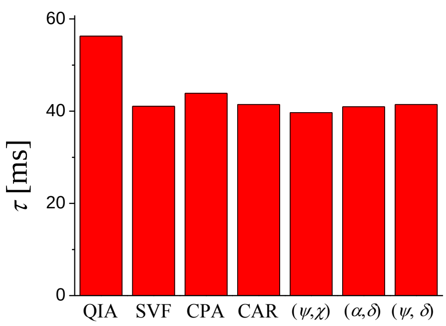

Figure 4. Mean computation time

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