On the relevance of Lorentzian pulses and exponential spectra for the understanding of fusion edge plasma turbulence

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Over the last three decades, the dynamics of turbulent transport in magnetically confined toroidal plasmas such as those in tokamaks and stellarators has been probed experimentally by examining time series of edge fluctuation data measured with Langmuir probes. One of the conclusions of these analysis was that edge fluctuation data exhibit self-similar properties, at least in certain regimes, among others leading to power spectra characterized by power law sections extending over a meaningful range of frequencies [1–5].

Recently, Maggs et al. [6] have proposed that this type of results should be reinterpreted in terms of an uncorrelated superposition of Lorentzian pulses with a narrow distribution of pulse durations. The resulting power spectra would be predominantly exponential, which would fit many of the spectra reported in literature and, in particular, their own experimental data obtained in the LAPD-U linear plasma device [7]. According to these authors, a consequence of this proposal is that the underlying dynamics would be low-dimensional quasi-deterministic chaos, contrasting with the hypothesis of self-organized critical dynamics proposed in the past [8, 9].

Any experimental signal $s(t)$ can be approximated by a sum of pulses:

$$s'(t) = \sum_{i=1}^{N} \frac{a_i}{\sqrt{\tau_i}} L\left(\frac{t-t_i}{\tau_i}\right). \quad (1)$$

Here, $L(t)$ is the basic pulse shape, and t_i, $i = 1,\ldots,N$ is a set of times at which pulses of amplitude a_i and duration τ_i occur. For discretely sampled data, the absolute difference between the signal s and the sum s' can be made arbitrarily small, provided $L(t)$ satisfies some minimal requirements.

We note that it is useful to choose $L(t)$ as similar as possible to the typical shape of actual observed pulses in the data series – which can in principle be determined using techniques such as conditional averaging. Fig. 1 shows an example of such a mean pulse shape obtained from fusion edge data.

In Fig. 1, we also present fits to 3 simple analytic functions, namely the Lorentzian $L(t) = 1/\pi(1+t^2)$, the Gaussian, and the function $G(t) = 1/(1+e^{|t|})$. The fit quality can be assessed using the R^2 statistic, and we find: Lorentzian: $R^2 = 0.9733$, Gaussian: $R^2 = 0.9203$, exponential function $G(t)$: $R^2 = 0.9718$, showing that $L(t)$ and $G(t)$ provide equally good fits.

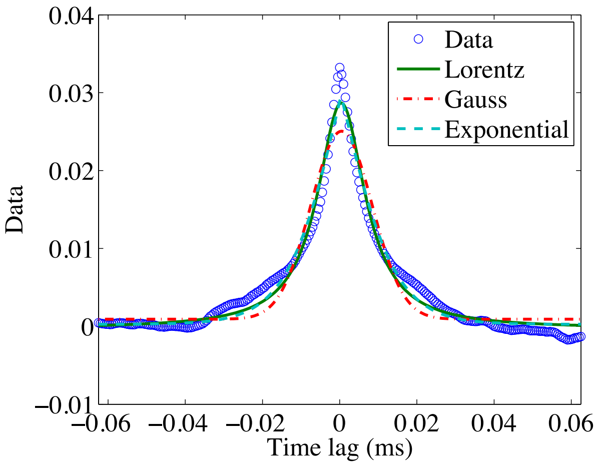

Figure 1: Conditionally averaged ion saturation current fluctuation data (W7-AS): 967 events. Fits to a Lorentzian $L(t)$, a Gaussian and the exponential function $G(t)$ show that $L(t)$ and $G(t)$ are equally good fits.

The spectrum of a Lorentzian is exponential ($\hat{L}(\omega) \propto e^{-|\omega|}$), whereas the spectrum of the alternative function $G(t)$ is power-law ($\lim_{\omega\to\infty} \hat{G}(\omega) \propto \omega^{-2}$). Since the Lorentzian $L(t)$ and the function $G(t)$ provide equally good fits to the data, while the corresponding spectra are in a totally different class, this seems to suggest that the pulse shape does not, in fact, predetermine the shape of the experimental power spectrum.

The relation between the pulse shape and the power spectrum can be analyzed as follows. For a fixed value of $\tau_i = \overline{\tau}$ (as assumed in [6]), Eq. (1) can be rewritten as

$$s'(t) = \int_{-\infty}^{\infty} \left[\sum_{i=1}^{N} \frac{a_i \delta(t'-t_i)}{\sqrt{\overline{\tau}}} \right] L\left(\frac{t-t'}{\overline{\tau}}\right) dt' = \int_{-\infty}^{\infty} f(a_i, t_i, \overline{\tau}; t') L\left(\frac{t-t'}{\overline{\tau}}\right) dt', \qquad (2)$$

i.e., $s'(t)$ is a convolution of a pulse distribution function f specifying the number, amplitude, and time of the pulses, and the pulse shape function L. Thus, its Fourier transform can be written as the product of the Fourier transforms of the pulse distribution function f, and L:

$$P_{s'}(\omega) = P_f(\omega) P_L(\omega), \qquad (3)$$

where P is the power spectrum, i.e., $P_{s'} = |\hat{s}'|^2$. Assuming, in addition, that the spectrum of $f(...;t)$ is essentially flat (in the frequency range of interest), the spectrum of s' will therefore have the same shape as the pulse shape spectrum (i.e., exponential when $L(t)$ is a Lorentzian). Note that this requires satisfying *two* conditions:

- the signal s may successfully be approximated by s' using only a single pulse duration $\overline{\tau}$ (or a rather narrow distribution of scales), and
- the distribution of pulse amplitudes f is such that its spectrum is flat (i.e., the pulses are random, uncorrelated).

We conclude that it is not sufficient to require that the signal consists of Lorentzian pulses with a fixed duration (as in [6]) in order to produce an exponential spectrum. This can easily be demonstrated by generating a signal based on the following simple algorithm:

1. Choose a number N of pulse amplitudes a_i, $i = 1, ..., N$.

2. Choose $N-1$ time intervals Δt_i.
3. Construct the pulse time array from $t_i = t_{i-1} + \Delta t_{i-1}$.
4. Compute the signal $s(t)$ from Eq. (1), using $\tau_i = \overline{\tau}$. The resulting signal is a sum of fixed-duration Lorentzian pulses.

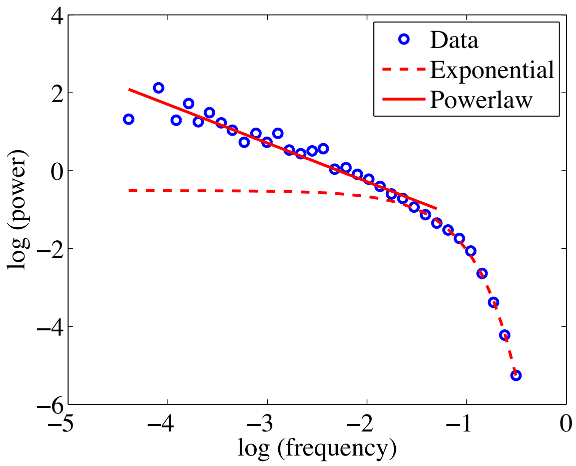

Figure 2: Spectrum for artificial data according to Eq. (1), constructed using the algorithm described in the text, showing that even a signal built from fixed-duration Lorentzian pulses is not always completely exponential.

By way of example, we set $N = 5000$ and $\overline{\tau} = 3$. For Δt_i we take a uniformly distributed random variable in the range $[0, 10)$, and for a_i a fractional Gaussian noise (fGn) with a certain degree of persistence (Hurst parameter $H = 0.8$) [10]. Computing the spectrum from this artificial signal, we obtain the Fourier spectrum shown in Fig. 2. The spectrum is clearly of the power-law type for low frequencies (extending over nearly 3 orders of magnitude of f), and exponential for high frequencies (as expected). While we do not pretend this simulation to be a model of any physical system, it certainly shows that a signal consisting of a sum of fixed-duration Lorentzian pulses does not possess an exponential spectrum *unless the pulses are uncorrelated in time.*

Conclusions. In this work, we have examined the claim put forward in [6] that pulse shapes and spectra of fluctuations in the edge of fusion plasmas might be explained by a sequence of Lorentzian pulses, associated with exponential spectra.

It was established that the proposed exponential spectral shapes are only obtained from Lorentzian pulses if *two* conditions are met: (i) the data are described well by a pulses with a narrow pulse duration distribution *and* (ii) the temporal distribution of pulse amplitudes has a flat spectrum (implying no temporal correlations). Earlier work shows that fusion edge plasma data are often characterized by broad distributions of pulse amplitudes and durations, and by strong temporal correlations [1, 11–13], which invalidates both these assumptions.

Thus, we conclude that the dynamics of turbulence in the edge of magnetically confined toroidal plasmas are often quite different from and richer than in linear plasmas. These differences might be related to the fact that parallel losses in the latter may limit the range of times over which temporal correlations can be established, whereas in toroidal plasmas the existence of closed magnetic surfaces allows persistence for times much longer than typical local turbulent scales, especially inside the last closed magnetic surface. We also note that the interaction between profiles and turbulence (plasma self-organization) may be a source of long-range temporal correlations [14], which may require conditions of good confinement and strong gradients. Although the analysis reported here seems to describe the typical situation for tokamaks and stellarators, it cannot be excluded that there are specific regimes and/or regions in toroidal plasmas that might exhibit dynamics more similar to the results reported by Maggs et al. at LAPD-U. For instance, the decorrelation between pulses should be faster in a region of strong poloidal sheared flows or in the far scrape-off layer, where magnetic field lines are connected to the first wall.

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