# Modelling of Sheath Effects on Radio-Frequency Antennas

K. Crombé[1, 2], V. Kyrytsya[2] and D. Van Eester[2]
[1]*Department of Applied Physics, Ghent University, Belgium*
[2]*Laboratory for Plasma Physics, Association EURATOM-BELGIAN STATE, Royal Military Academy, Brussels, Belgium*

## Introduction

The large voltages on radio frequency (RF) antennas that are used for heating of fusion plasmas, can create a thin sheath layer with largely negative potential and thus strong electric near-fields that attract and accelerate positively charged ions. The possible damage to antenna and in-vessel components due to local overheating and sputtering, is one of the main concerns for high power antennas in future fusion reactors. Good predictive simulation tools that take these sheath effects into account are still lacking. A practical implementation for modelling codes was proposed in [1], where sheath properties are introduced by means of a non-linear sheath boundary condition (SBC) on antenna surfaces. The sheath is represented by a scalar dielectric medium with relative permittivity $\varepsilon_{sh} = 1 + i\nu_{sh}$, i.e. a lossy vacuum layer. It is assumed that the electrons are inertia-free and therefore accelerated immediately into the metal surface, and that the power lost in the sheath is purely coming from ions accelerated in the rectified sheath potential. The sheath width ($\Delta_{sh}$) is determined by the Child-Langmuir law, and the sheath potential depends on the electric field component normal to the surface. Continuity of the normal component of the displacement vector at the sheath plasma interface leads to the general description of the sheath as boundary condition $E_t = \nabla_t \left((\Delta_{sh}/\varepsilon_{sh})\, \mathbf{n}\cdot\boldsymbol{\varepsilon}_{\mathbf{pl}}\cdot\mathbf{E}\right) = \nabla_t \left((\Delta_{sh}/\varepsilon_{sh})\, D_n\right)$, where $E_t$ is the tangential component of electric field and $D_n$ the normal component of the displacement vector, all with respect to the sheath surface. For $\boldsymbol{\varepsilon}_{\mathbf{pl}}$ a cold plasma [2] description is used. Due to the $D_n$ dependence of the sheath width the SBC is a non-linear equation, preventing a direct inversion of the underlying set of equations. A hybrid implementation of a SBC in the TOPICA code [3] was reported in [4], plasma properties were introduced for the calculation of the sheath parameters ($\Delta_{sh}$, $\varepsilon_{pl}$ and $\varepsilon_{sh}$), but the wave propagation was calculated using a vacuum Green's function. In the present paper a realistic finite density plasma is assumed to surround the antenna, and a cold plasma description assesses the impact of a magnetized dielectric medium on the antenna near-fields. The COMSOL Multiphysics [5] package was used for the RF modelling.

## Two-dimensional models

### Coaxial cable

The SBC was imposed on the outer surface of a 50cm piece of coaxial cable (oriented along the z-direction). The coaxial port excitation was in TEM mode and the input power was 100 kW. An iterative approach, similar to the one in [4], was used to solve the non-linear set of equations. In the first step of the problem the **B** and **E**-fields are calculated using COMSOL, assuming the outer boundary (sheath surface) to be a perfect electric conductor (PEC). The resulting normal component of **D** with respect to the sheath surface is used as input in MATLAB [6] for solving the non-linear SBC equation. The MATLAB outputs are the sheath properties ($\Delta_{sh}$, $\nu_{sh}$, the power dissipated in the sheath) and the tangential component ($E_t$) to be imposed on the sheath surface. In the following COMSOL step, a new RF problem is solved with the imposed $E_t$ replacing the PEC boundary condition. The iterative process continues by exporting $D_n$ from COMSOL and solving the SBC in MATLAB. The iterations are stopped when the total power dissipated in the sheath does not change anymore from one step to the next. Resulting sheath widths, losses ($\nu_{sh}$) and dissipated power are shown in figure 1. In the simulations the plasma density was increased from 0 at the level of the port up to $1\text{x}10^{16}$ $\text{m}^{-3}$ at z=0.5m (the end of the coaxial line), $T_e$=10 eV, $B_t$ = 3.4T and frequency f = 50 MHz.

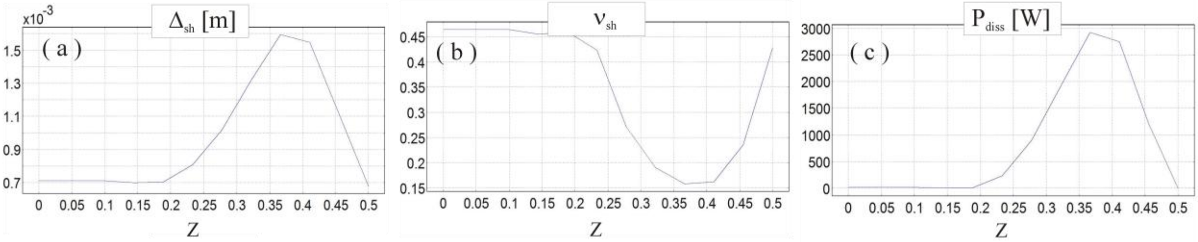

Figure 1: (a) $\Delta_{sh}$, (b) $\nu_{sh}$ and (c) dissipated power along the 50 cm length of the coaxial cable

It was found that the iterative process fails when plasma density and/or input power becomes too high, the main reason being the creation of strong gradient, resulting in large values for $E_t$. Also sheath widths increase to non-physically high values. This not only poses a practical problem of numerical accuracy, but suggests the SBC relies on too crude a model to capture the sheath boundary layer dynamics for realistic parameters. For future simulations either a direct approach (non-linear solvers) or a different description of the sheath will be necessary.

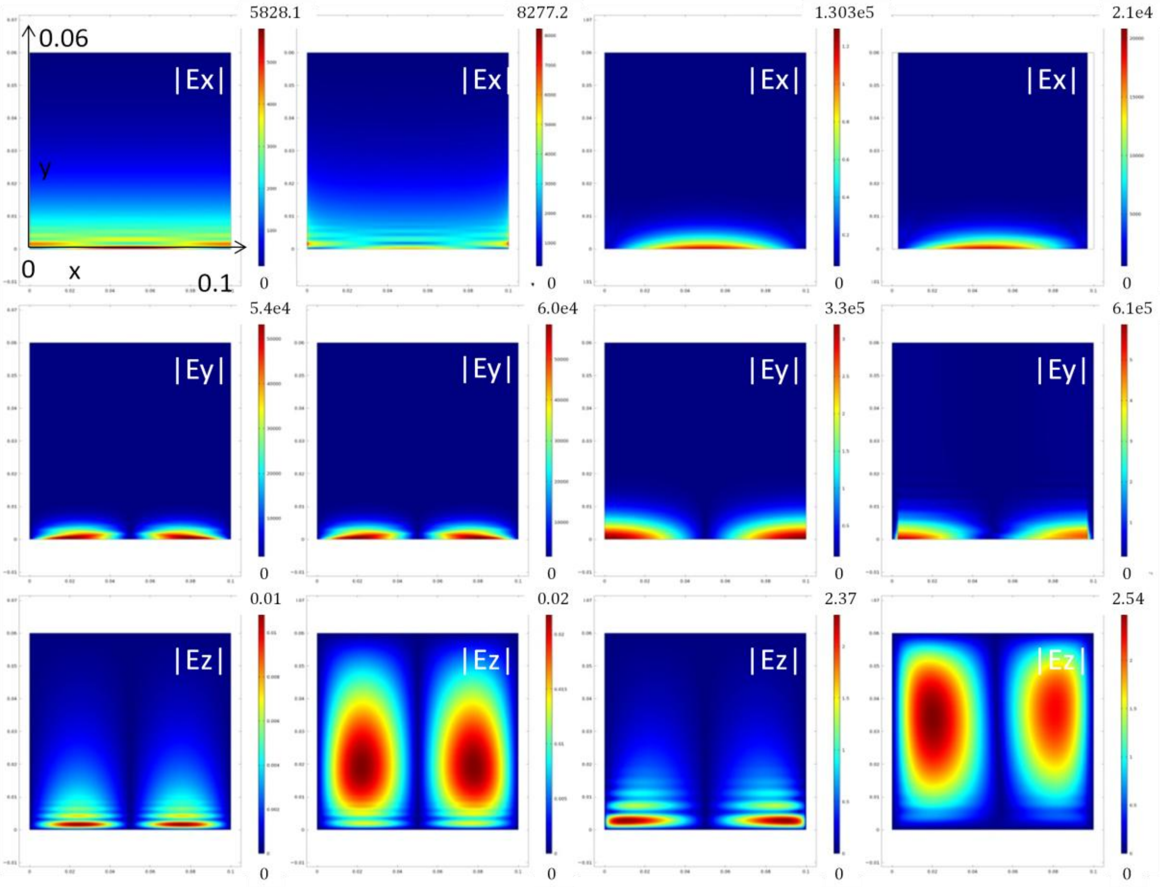

Figure 2: Electric field components for slow wave excitation in a rectangular box

## Rectangular box

In a rectangular box model (10 cm x 6 cm) and with (x,y,z) parametrizing the toroidal, radial and poloidal directions in the tokamak, respectively, the SBC condition was tested for predominantly slow and fast wave excitation. The port is taken at y=0.0 m and a sinusoidal electric field $E_\alpha=\sin(\pi x/0.1)$ is imposed at the port purely in the x-direction $\mathbf{E} = (E_\alpha, 0, 0)$ for the slow wave or purely in the z-direction $\mathbf{E} = (0, 0, E_\alpha)$ for the fast wave. The sheath width $\Delta_{sh}$ and losses $\nu_{sh}$ are fixed to 1 mm and 0.5 (dimensionless parameter, relative permittivity) respectively, to allow a direct solution in COMSOL, avoiding the iterations and thus the interfacing with MATLAB. Input power was 100 kW, $T_e$ = 10eV, $B_t$ = 3.4T and f = 50MHz. The density was increased from 0 to $2\text{x}10^{17}\text{m}^{-3}$ from the port up to y = 0.06 m. In figures 2 and 3 the electric field components are plotted for slow and fast wave excitation respectively. In the first column PEC boundary conditions were used. In the second column a sharp drop in density towards the boundaries x=0.0 m and x=0.1 m has been introduced to mimic the electron poor sheath region. For the third column the SBC was applied for x=0.0 m and x=0.1 m. In column 4 a lossy vacuum zone of 2 mm was introduced and PEC boundary conditions were applied at the outer edges. It can be seen that results of columns 3 and 4 are indeed similar. Comparison between columns 1 and 3 shows the effect of the sheath, $|E_y|$ is increased especially at the sheath boundaries. The comparison between the columns 2 and 3 shows that a strong drop in $n_e$ towards the PEC boundaries does not have the same effect as the SBC description. Perhaps the simulation domain or mesh size is not sufficient to capture all small wavelength effects.

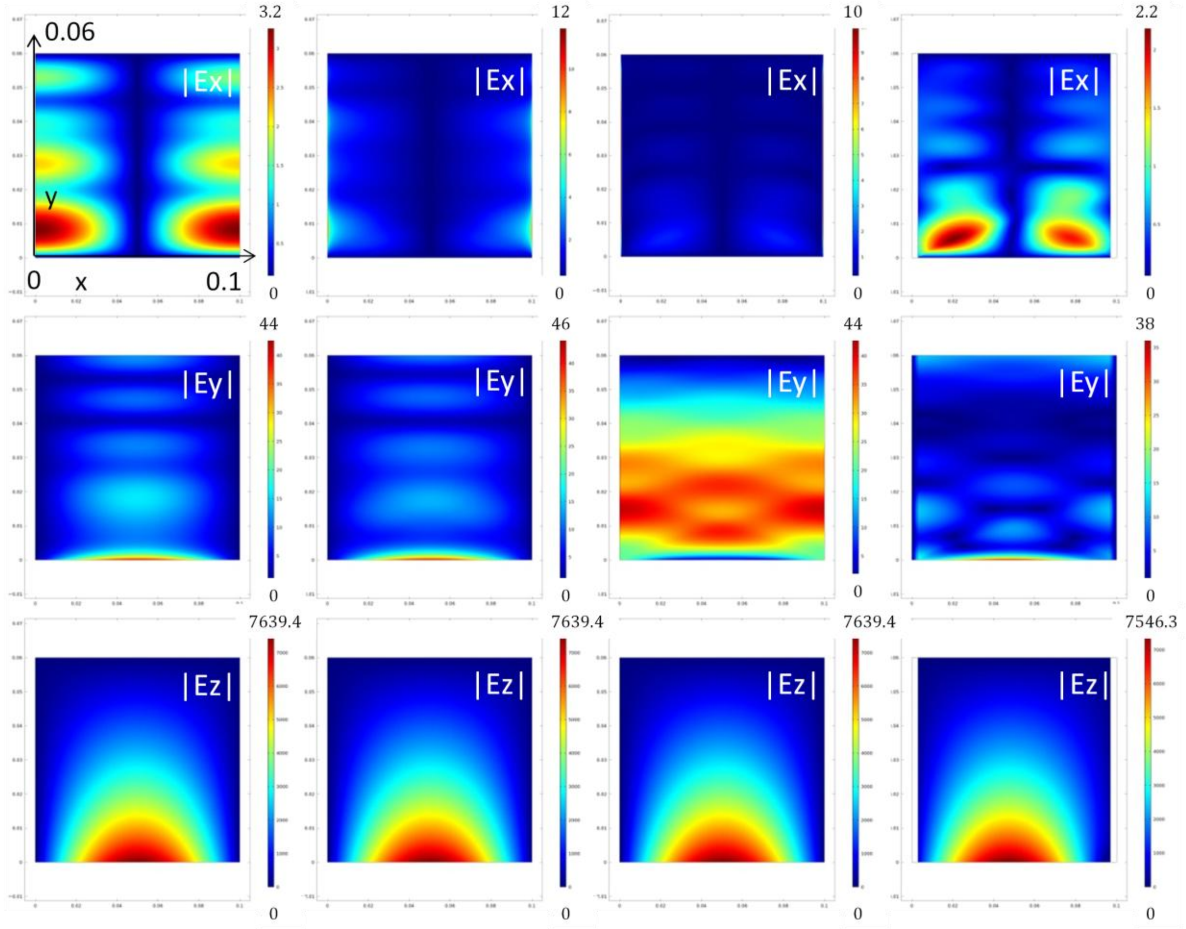

Figure 3: Electric field components for fast wave excitation in a rectangular box

## Extension of the sheath description

Ultimately, the actual 3D geometry of the lauching structure needs to be accounted for. Therefore, the implementation of the SBC on a 3D single strap model was tested; the input power at the coaxial port is 100W, $n_e$ gradually increases from 0 at x=0.265m (at the boundary between coax and antenna box) to $5x10^{15}m^{-3}$ at x=0.0 m; the other parameters were fixed ($T_e$ = 10eV, $B_t$ = 3.4T, f = 50MHz, $\Delta_{sh}$ = 1 mm and $\nu_{sh}$ = 0). The grid size was limited by computer performance, but as can be seen in fig.4, a dense grid is required to capture the detailed field structure close to metallic objects in order to avoid numerical artefacts to be mistaken for physical effects.

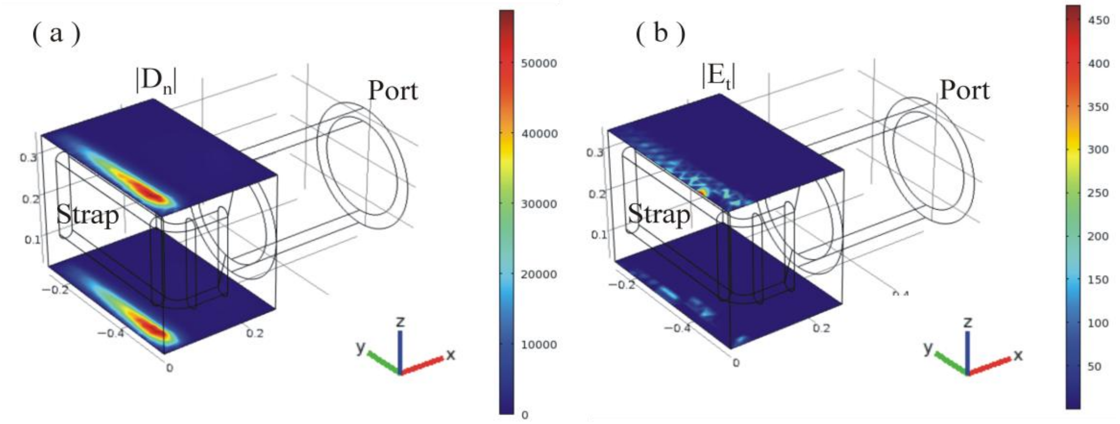

Figure 4: (a) $|D_n|$ and (b) $|E_t|$ on the surface with SBC imposed

CPU constraints will be a serious limitation for this kind of sheath modelling. Moreover, the SBC approach does not account for the interplay between antenna near-field and plasma density, in spite of the fact that experimentally RF induced density variations have been observed, in particular in RF scenarios with bad power absorption in the plasma [7]. In addition, in order to understand the power losses in the sheath, a detailed description of the ion and electron behaviour in the sheath layer itself is necessary. The SBC approach cannot take these effects into account. A two-fluid approach could be a good candidate for a more complete and detailed sheath description. A coupled set of 14 equations (8 of which are independent) needs to be solved: the continuity equations and equations of motion for the densities and velocities of the two fluids (electrons and ions), and Maxwell's equations to account for the RF effects. The two-fluid description is numerically much heavier than the here presented SBC approach, but potentially allows to describe the coupling between the plasma and the sheath in a more selfconsistent way. Preliminary results with 1D and 2D models have shown the existence of wave solutions with very different scale lengths (sub-millimeter range in the sheath up to meter range in the plasma).

## References

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