Effect of poloidal asymmetries on impurity peaking in tokamaks

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Introduction Accumulation of impurities in the core of fusion plasmas has debilitating effect on fusion reactivity; consequently a significant effort has been spent to find conditions in which core accumulation can be avoided. Additional central heating has experimentally shown to give a flattening effect on impurity density profiles in the core, but the reasons are still not properly understood. Recent work noted that impurity cross-field transport driven by electrostatic turbulence depends on the poloidal distribution of the impurities [1, 2], and that poloidal asymmetries may be a contributing factor to the avoidance of impurity accumulation. In this work the emphasis will be on the effect of radio frequency (RF) heating in the plasma core and in particular the study of inboard accumulation. The $\mathbf{E}\times\mathbf{B}$ drift of the impurities in the presence of poloidal asymmetries and its impact on impurity transport is studied. Impurity self-collisions are modeled with a Lorentz operator and the gyrokinetic (GK) equation is solved using a variational approach.

Model A mechanism that produces in-out asymmetries in minority ICRH heated plasma cores is the increase of the minority density (we consider hydrogen) on the outboard side which gives rise to an electric field that pushes the impurities to the inboard side [4]. Each particle species can be assumed to follow a Boltzmann distribution $n_\alpha = n_{\alpha 0}\exp(-e_\alpha\phi_E/T_\alpha) \approx n_{\alpha 0}(1-e_\alpha\phi_E/T_\alpha)$, except the minority ions which are strongly affected by the RF heating. e_α is the charge and T_α is the temperature of the species, and ϕ_E is the equilibrium potential. In order to get a simple approximate expression for the poloidally varying potential it is assumed that the linear expansion in $Ze\phi_E/T_z$ of the Boltzmann distributed impurities is valid, which is a reasonable approximation for experimentally relevant values of $Ze\phi_E/T_z$. This implies that the poloidal variation of the density on a flux surface $\tilde{n}_\alpha = n_\alpha - n_{\alpha 0}$ is given by $\tilde{n}_\alpha/n_{\alpha 0} \simeq -e_\alpha\phi_E/T_\alpha$. Assuming that the poloidal variation in the potential ϕ_E is produced by the poloidally asymmetric distribution of the heated minority ions, using quasineutrality we obtain

$$\frac{n_z}{n_{z0}} = \exp\left(-\frac{Ze\phi_E}{T_z}\right) = \exp\left(-\frac{Z\hat{n}_H/n_{e0}}{(T_z/T_i)(n_{i0}/n_{e0}) + (T_z/T_e) + (n_{z0}Z^2/n_{e0})}\right). \quad (1)$$

Here, $\hat{n}_H = \hat{n}_H(\theta)$ represents the fraction of the hydrogen minority density which feels the RF resonance and does not follow a Boltzmann distribution and ϕ_E is normalized so that

$n_{i0} + n_{H0} + Zn_{z0} - n_{e0} = 0$. Since the exponent in Eq. (1) is negative, a maximum in $\hat{n}_H$ corresponds to a minimum in n_z, hence outboard minority ion accumulation pushes the impurity ions to the inboard side.

Applying ICRH with hydrogen minority species and with the resonance layer at the low field side not intersecting the studied flux surface, the poloidal variation of the potential is expected to be sinusoidal to first order [5]. If the radial variation of ϕ_E is ignored (i.e. toroidal rotation neglected) we can motivate the following *ansatz* for the equilibrium potential

$$Ze\phi_E/T_z = -\kappa \cos(\theta - \delta). \tag{2}$$

For ICRH driven inboard impurity asymmetries $\delta = \pi$. We might expect that ECRH will result in an outboard ($\delta = 0$) accumulation of impurities, and accordingly we will present results with both $\delta = 0$ and $\delta = \pi$. The asymmetry strength, κ, depends on the ICRH resonant magnetic field strength $b_c = B_c/B_0$ (B_c and B_0 are the magnetic field strengths at the resonance position and the magnetic axis), on the minority temperature anisotropy $\alpha_T = T_\perp/T_\|$ (Fig. 1) and minority concentration n_{H0}/n_e. We will refer to ”in-out” and ”out-in” asymmetries as the situations when the maximum of the poloidally varying impurity density is located at the high-field and low-field sides of the plasma, respectively.

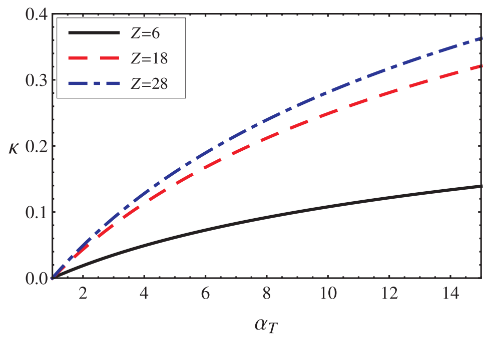

Figure 1: κ as a function of α_T for carbon, argon and nickel with $n_{H0}/n_{e0} = 0.07$, $n_{z0}/n_{e0} = 2\times10^{-3}$, $b_c = 0.91$, $T_i = T_z = 0.85T_e$ and $r/R_0 = 0.1$.

We assume that the electrostatic fluctuations responsible for the cross-field transport do not significantly affect the processes causing the poloidal asymmetries. The equilibrium electrostatic potential will be ordered $e\phi_E/T_\alpha \ll 1$, but we allow for $Ze\phi_E/T_z = \mathcal{O}(1)$. We consider an axisymmetric, large aspect-ratio torus with circular magnetic surfaces. The parallel dynamics and the trapping of impurities due to $\nabla_\| B$ and $\nabla_\| \phi_E$ are neglected, and for simplicity also finite Larmor-radius effects are omitted. The non-adiabatic part of the perturbed distribution function g_z can be obtained from the linearized gyrokinetic (GK) equation,

$$-i(\omega - \omega_{Dz} - \omega_E)g_z - C[g_z] = -i\frac{Zef_{z0}}{T_z}\left(\omega - \omega_{*z}^T\right)\phi, \tag{3}$$

where the notation is standard (see definitions in [3]), except for the new quantity, ω_E, which stems from the $\mathbf{E}\times\mathbf{B}$ drift in the equilibrium electrostatic field

$$\omega_E = -\frac{k_\theta}{B}\frac{s\theta}{r}\frac{\partial \phi_E}{\partial \theta}. \tag{4}$$

If $n_z Z^2/n_e$ is of order unity or larger, impurity-impurity collisions dominate over collisions between impurities and other species; thus it is sufficient to consider only the impurity self-collisions. Since the motion of impurities is slow, momentum conservation can be neglected, and we model impurity collisions, $C[g_z]$, by a Lorentz operator.

The Lorentz operator makes the distribution more isotropic in velocity space in a diffusive way. The GK equation (3) contains anisotropy in the magnetic drift term, which can be written in terms of Legendre polynomials $P_l(\xi)$ where $\xi = x_\parallel/x$ denotes the cosine of the pitch-angle and $x = v/v_{Tz}$ represents velocity normalized to the thermal speed $v_{Tz} = (2T_z/m_z)^{1/2}$. Since $P_l(\xi)$ also are eigenfunctions of the Lorentz operator it is convenient to write g_z as a truncated Legendre polynomial series and derive an approximate variational solution $g_z(x,\xi) = \sum_n g_n(x) P_n(\xi) \approx g_0(x) P_0(\xi) + g_2(x) P_2(\xi)$.

Using the quasilinear particle flux for impurities $\Gamma_z \simeq -(k_\theta/B)\mathrm{Im}\left[\int d^3v\, g_z\, \phi^*\right]$, the normalized zero-flux impurity density gradient a/L_{nz}^0 (the peaking factor) can be obtained from the requirement that the flux surface average of the particle flux vanishes $\langle\Gamma_z\rangle = 0$. Here a is the outermost minor radius, L_{nz} is the density scale length and $\langle\ldots\rangle = (1/2\pi)\int_{-\pi}^{\pi}(\ldots)\,d\theta$.

By employing the constant energy resonance approximation $[v_\perp^2 + 2v_\parallel^2 \to 4(v_\perp^2 + v_\parallel^2)/3]$ in ω_{Dz} and expanding in the smallness of $1/Z$, an approximate analytical solution for the peaking factor can be constructed. Returning to the GK equation (3) we note that $\omega_{Dz}/\omega, \omega_{*z}^T/\omega \propto 1/Z$, while ω_E can be as large as $|\omega|$, seemingly independent of Z. However our ordering $Ze\phi_E/T_z \sim \mathcal{O}(1)$ requires that ω_E/ω is formally $\sim 1/Z$. Keeping only terms to the first order in $1/Z$ we find that collisions disappear when taking the density moment, and an approximate expression for the impurity peaking factor is given by

$$\frac{a}{L_{nz}^0} = 2\frac{a}{R}\langle\cos\theta + s\theta\sin\theta\rangle_\phi + s\kappa\frac{a}{r}\langle\theta\sin(\theta-\delta)\rangle_\phi, \tag{5}$$

where $\langle\cdots\rangle_\phi = \left\langle \cdots \mathcal{N}|\phi|^2/[(\omega_r-\omega_E)^2+\gamma^2]\right\rangle / \left\langle\mathcal{N}|\phi|^2/[(\omega_r-\omega_E)^2+\gamma^2]\right\rangle$ and $\mathcal{N}(\theta) = \exp[\kappa\cos(\theta-\delta)]$. The second term in Eq. (5) stems from the $\mathbf{E}\times\mathbf{B}$ drift, and shows the explicit dependence on s and κ.

Simulations The perturbed electrostatic potential and eigenvalues were obtained by linear electrostatic gyrokinetic initial-value calculations with GYRO [7], assuming that they are unaffected by the presence of a weak poloidal variation of the electrostatic potential and the poloidally asymmetrically distributed *trace* impurity species. In the simulations the following local profile and magnetic geometry parameters were used: $r/a = 0.3$, $R/a = 3$, $k_\theta\rho_s = 0.3$, $q = 1.7$, $a/L_{ne} = 1.5$, $T_i/T_e = T_z/T_e = 0.85$, $a/L_{Te} = 2$, $a/L_{Ti} = a/L_{Tz} = 2.5$, $s = 0.22$, $\rho_s/a = 0.0035$ and $\hat{\nu}_{ei} = 0.0058\,c_s/a$. Nickel ($Z = 28$) impurity was assumed to be present in trace quantities, in the sense that $Zn_z/n_e \ll 1$ ($n_z/n_e = 2\times10^{-3}$ in the simulations). However note that $Z^2n_z/n_e \sim 1$. Figure 2 shows how the peaking factor for nickel varies with asymmetry strength and magnetic shear.

Conclusions The main results of the paper are summarized as follows. A poloidally asymmetric equilibrium electrostatic potential $Ze\phi_E/T_z$ of order unity can yield a significant reduction of the impurity peaking factor. The asymmetry and magnetic shear are the two most important parameters that govern the peaking of moderate and high-Z impurities. This dependence is illustrated in Fig. 2, and its importance can be understood from the approximate solution in Eq. (5) where κ and s appear as explicit factors. Figure 2 also indicates that the $\mathbf{E}\times\mathbf{B}$ drift frequency, in the poloidally varying electrostatic potential, is a major contributing factor to the reduction of the peaking factor, since the change is small when ω_E is neglected. Furthermore it is clear that, for $s \geq 0$, in-out asymmetries lead to a decrease in peaking factor, while out-in asymmetries increase it. Other plasma parameters, such as collisionality, ion and electron temperature gradients and electron density gradient do not influence the peaking factor significantly. Experimentally these results could be checked by magnetic shear scans in discharges with low field side ICRH.

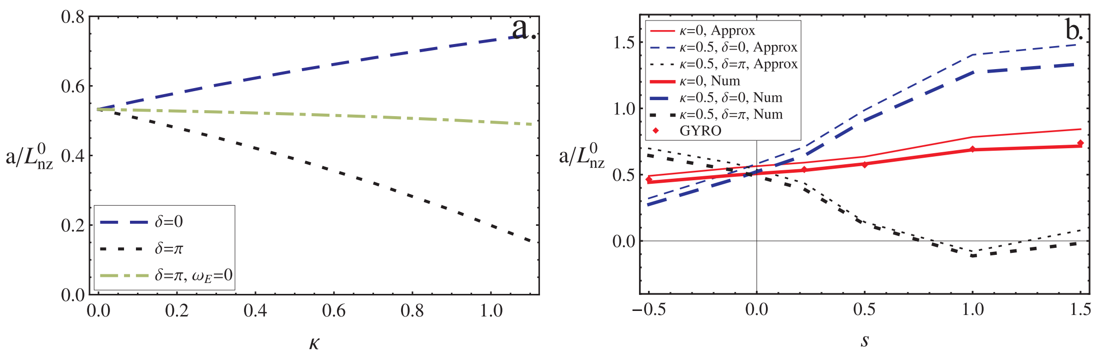

Figure 2: Peaking factor for nickel as a function of asymmetry strength (a) and shear (b). Symmetric impurity density (red, solid), out-in asymmetry (blue, dashed), and in-out asymmetry (black, dotted). Symmetric case benchmarked to GYRO results (red diamonds). In (a) green dash-dotted curve represents in-out asymmetry with ω_E neglected. In (b) the analytical approximation is given for comparison.

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