A natural bound to the stability of the boundary layer in the H-mode regime

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This work examines the role of the particular state consisting of large (virtually infinite) values of the *effective Larmor radius* parameter $\rho_{eff} = \rho_s (1 - v_{*i}/u)^{-1/2}$ on the saturation of the H-mode structure and the possible limitation to the confinement.

One can broadly identify two phases of the periodic events within the stationary confinement in tokamak. The first consists of the formation of the layer of sheared poloidal rotation, characteristic to the H-mode. The onset of poloidal rotation is a fast transition consisting of collection and condensation of angular momentum carried by drift waves. The second phase refers to the progressive onset of a particular saturation regime of this rotation, inducing weaker stability against vortex nucleation and filamentation, with loss of H-mode confinement. This sequence is repeated and, in the absence of external action (like the RMP) it establishes a limiting regime, a kind of thermodynamical barrier that may be not favorable for reactors. The sheared poloidal rotation (a vorticity sheet near the last closed magnetic surface) suppresses the transport and induces formation of the pedestal with strong gradient of the pressure [1]. Then the ion diamagnetic velocity v_{i*} increases and approaches the plasma poloidal velocity u. The dynamics rely on a parameter $\rho_{eff} = \rho_s (1 - v_{i*}/u)^{-1/2}$ that tends to infinity, which means suppression of the Ertel 's theorem or equivalently decoupling of density and vorticity (on meso- and macroscopic scales). The layer of vorticity becomes unstable and breakes-up by nucleating a periodic array of vortices (filaments). This second phase appears as a principial obstacle in maintaining H-confinement since the increase of ρ_{eff} is unavoidable.

Although the present work is mainly concerned with the second phase we include a discussion of the first (build-up of the rotation layer).

It is usually assumed that plasma rotation is due to three concurent mechanisms: direct ion loss, Reynolds stress due to drift instabilities, Stringer spin-up induced by asymmetries of the transport rate. We can mention two others. (a) the high convective transport rate (due to strong radial gradients of temperature) spontaneously transform the tilting of the drift wave eddies into ordered flow oriented transversally on the direction of convection: this corresponds to the well-known second bifurcation of the Rayleigh-Benard system, with the "wind" parallel to the plates corresponding to our H-mode rotation layer. (b) a distinct mechanism consists of continuous and exponentially accelerating process of collection of the angular momentum carried by drift wave structures and its condensation into the vorticity sheet.

We ennumerate the steps which we take in constructing the argument regarding the formation and the suppression of the rotation layer in H-mode.

(1) It is assumed that the outer region of plasma (large minor radius) is dominated by drift-type instabilities.

(2) These instabilities can generate relatively stable structures which are vortices. We estimate the average density of the presence of such strong vortices. First we assume that turbulence is dominately two-dimensional and that we can neglect the stretching of vortices along toroidal direction (*i.e.* we assume small ρ_s compared to the scale of flow). This amounts to reduce the description of vortices to the Euler fluid model, instead of Hasegawa-Mima. A field-theoretical formulation of the Euler fluid reveals the connection between the distribution of vorticity and the intrinsic geometry of a surface in $3D$ space, which evolves to the state of Constant Mean Curvature, and relates the physical variables of the fluid to the two principal curvatures of the surface. A strongly localised (quasi-singular) vortex corresponds, in this mapping, to points on the surface where the two curvatures are approximately equal. The exact equality defines the so-called *umbilic points*. The statistical properties of *umbilic points* have been calculated for a Gaussian random surface in $3D$ [2] and we map it back into the statistics of strong vortices in turbulence. It results an average of ≈ 3 vortices on a surface of eddy size.

(3) Due to the gradient of the background vorticity the strong, isolated vortices are migrating towards the maximum or the minimum of the vorticity, a process which transports angular momentum in a very localized form. The mechanism is described by Schecter and Dubin [3]. At the base it is the mixing of the vorticity of the background gradient in the region just around the *clump* or the *hole* of the isolated vortex of positive or negatice circulation. This modifies one of the contributions to the canonical momentum P_θ and, since this one is conserved, other component must compensate. In consequence the positive-circulation vortex (clump of vorticity) moves against the gradient, *i.e.* toward the maximum of the vorticity sheet. The negative-circulation vortex (hole) has the opposite behavior.

(4) Arriving at the layer of initial poloidal rotation, the vortices are melting into the rotating layer, contributing to the angular momentum and sustaining the rotation. This process is the inverse of the generation of isolated vortices in a Bose-Einstein (BE) condensate. When the BE condensate is rotated and the angular momentum exceeds a threshold a vortex is generated [4]. Here we invoke the reverse process, with isolated vortices coalescing and building-up or

enhancing a rotation flow. Absent in the case of the BE condensate, this hypothesis is reasonable here due to the process of separation of vortices (clumps or holes) on a background of gradient of vorticity. We can see this mechanism as a sort of *discrete-events* Reynold stress, but substantially enhanced by the self-induced motion of the vortices. The model is the Kardar-Parisi-Zhang equation but in k-space.

(5) The collection of angular momentum is equivalent to an instability of increasing the rotation in the layer. This is because any "quantum" of vorticity contributed by a vortex joining the vorticity sheet will increase the gradient of the background vorticity, enhancing the migration of other vortices. The velocities of the individual vortices ascending the gradient of vorticity are determined in relation with their positions with respect to the maximum of the vorticity.

(6) The poloidal rotation suppresses the drift instabilities and reduces the transport accross the layer and leads to the formation of the pedestal, increasing the gradient of the pressure.

(7) The gradient of the pressure increases the diamagnetic velocity of the ions.

(8) When the relative magnitudes of the ion diamagnetic velocity v_{i*} and the poloidal rotation velocity u are equal, the *effective Larmor radius* becomes infinite. To see the consequences, we note that the increase in magnitude and shear of the poloidal velocity is due to the radial polarization current, equivalently, it is made possible by the compressibility of the ion polarization velocity $divV_{i,pol} \neq 0$.

$$-\frac{u}{v_{th,i}}\frac{1}{\rho_i}\left(1-\frac{v_{i*}}{u}\right)V_{Ex}+\nabla_\perp\cdot\mathbf{V}=0$$

where $V_{Ex}=\left(-\nabla\phi\times\widehat{\mathbf{e}}_z\right)|_x$, $y'\equiv y-ut$ and the equation for the potential is

$$\left(\frac{\partial}{\partial y'}+\left[\left(-\nabla\phi\times\widehat{\mathbf{e}}_z\right)\cdot\nabla\right]\right)\nabla_\perp^2\phi=\left(\frac{v_{th,e}}{v_{th,i}}\right)^2\frac{1}{\rho_s^2}\left(1-\frac{v_{i*}}{u}\right)\frac{\partial\phi}{\partial y'}$$

When $\rho_{eff}\rightarrow\infty$ the divergence of the ion polarization velocity is suppressed, the left hand side is zero and the equation becomes the Euler equation, with no intrinsic length (conformal invariant). The variables $n\sim\phi$ and $\omega=\nabla_\perp^2\phi$ are decoupled.

(9) The H-mode rotation layer (which is a vorticity sheet) is robust because of the strong shear of the poloidal rotation. An instability of drift-type has an eigenfunction which is shifted relative to the resonance surface. This implies enhanced magnetic shear damping via nonlinear coupling of radial harmonics. According to *Carreras et al* [5] the equation for a density perturbation is of the form

$$\frac{\partial^2\widetilde{n}_{\mathbf{k}}}{\partial x^2}-\frac{1}{\rho_s}\left(1+k_y^2\rho_s^2-\frac{\omega_{*e}}{\omega-\omega_E}\right)\widetilde{n}_{\mathbf{k}}=i\left(\frac{D_0k_y^2}{\omega-\omega_E}-\frac{\Delta_{\mathbf{k}}^2S^2}{4\rho_s^2}\right)\widetilde{n}_{\mathbf{k}}-i\frac{1}{\Delta_k^4}\left(x+i\xi_{\mathbf{k}}\right)^2\widetilde{n}_{\mathbf{k}}$$

where x is the radial coordinate, $\omega_E=k_yu$, $\Delta_{\mathbf{k}}$ is the modified width of the mode, S is normalized shear flow frequency parameter and $\xi_{\mathbf{k}}$ is the shift on x induced by the sheared flow. We observe that for long wavelength perturbations ($k_y^2\rho_s^2 \ll 1$) the propagator at zero-frequency $\omega \sim 0$ of the operator in the left hand side becomes singular as $\rho_{eff} \to \infty$, since

$$\frac{1}{\rho_s}\left(1+k_y^2\rho_s^2-\frac{\omega_{*e}}{\omega-\omega_E}\right) \approx \frac{1}{\rho_s^2}\left(1-\frac{v_{*e}}{|u|}\right) = \frac{1}{\rho_{eff}^2} \to 0$$

This means that the dynamics is algebraic (not exponential) and no eigenemode can be defined. The mechanism that protects the layer against the instabilities is operational until ρ_{eff} remains finite,while at high values of ρ_{eff} the zero frequency (algebraic) and long poloidal deformations destabilises the layer.

(10) When $\rho_{eff} \to \infty$ the layer is unstable to breaking-up into vortices that look as filaments. Indeed it has been shown [6] that the vorticity layer, which is also a layer of high local values of the current density is torn apart by a Chaplygin-type (*anomalous polytropic*) instability. At that moment the equation looks like the Flierl-Petviashvili equation $\Delta\phi = -\overline{k}_0^2\phi + \gamma\phi^3$ since the term with ϕ^2, is zero due to $d\left(v_i^{dia}\right)/dr = 0$. The coefficient in the first term is $\overline{k}_0^2 = \frac{v_i^{dia}}{u} - 1 \geq 0$ and $\gamma \equiv \frac{\rho_s}{L_n}\frac{d^2 v_0^{dia}}{dr^2}\frac{1}{6u^3}$. The solution is [7]

$$\phi\left(y'\right) = \sqrt{\nu}\frac{2m}{1+\sqrt{\nu}} sn\left(\frac{2m}{1+\sqrt{\nu}}y';\nu\right)$$

where *sn* is the Jacobi elliptic function with real elliptic parameter $0 \leq \nu \leq 1$ and is determined from $(1+\nu)\left(\frac{2m}{1+\sqrt{\nu}}\right)^2 \equiv \frac{\overline{k}_0^2}{\gamma/2}$. Since the elliptic function *sn* is a series of *kinks* (tanh) the solution is a succession of radial positive and negative velocities which create a sequence of periodic filaments distributed equidistant along the poloidal circumference.

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