Helical features in nonlinear 3D MHD modeling of Pinch configurations

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Abstract. We study the MHD behaviour of Pinch configurations (both RFP and Tokamak-like) when providing a helical boundary condition for the magnetic field in MHD modeling. The topological properties of the obtained magnetic field configurations are also discussed.

Introduction. The development of MHD helical regimes in pinch configurations (i.e. helical configurations resulting from nonlinear saturation of resistive-kink/tearing modes) provide partial or complete nonlinear stabilization of modes otherwise linearly unstable with respect to the axis-symmetric state. This is seen in 3D nonlinear modeling of the Reversed Field Pinch (RFP). There, the development of Quasi Single Helicity (QSH) or Single Helicity (SH) regimes makes the "spurious" modes (i.e. Fourier components different from the main helical one) significantly decrease in amplitude or vanish [1]. This is also seen in the nonlinear modeling of Ultra Low q pinches, since potentially dangerous resonant modes are totally inhibited in their growth if any contiguous resonant mode is already present [2]. In addition to this -rather general- dynamical "stabilization effect", helical regimes bring ordered magnetic topologies in the RFP. This occurs in particular due to separatrix expulsion [3], when Single Helical Axis (SHAx) states develop (i.e. magnetic topologies characterized by bean-shaped magnetic flux surfaces without magnetic islands). In such conditions, chaos healing occurs despite the presence of remnant spurious modes (contrary to a *naïve application* of Chirikov criterion reading "*the larger the amplitudes and the number of Fourier components, the more chaotic the dynamics*"). Such dynamical and topological effects find clear experimental evidence [4, 5]. In this paper we show new simulations which further extend the above recalled results concerning the impact of MHD helical regimes on dynamical and topological aspects. We show that a finite radial magnetic field at the edge of plasma domain on a selected Fourier component helps helical self-organization (which spontaneously occurs upon first on-axis resonant modes [6]). It may also allow self-organization of RFP upon helical states close to the spontaneous one, even included internally non-resonant modes, which extends results obtained in [7]. In the Ohmic Tokamak case, a small helical perturbation of the edge radial field with (m=1,n=1) periodicity changes the dynamical character of the internal resistive kink mode from a sawtoothing behaviour to a snake-like stationary regime. Concerning chaos resilience properties, we focus here on the helical RFP. We show that helical topology features are not simply understood in terms of Chirikov criterion applied on remnant Fourier components obtained as “secondary modes” in standard cylindrical coordinates. In fact, phase changes of dominant or secondary Fourier components, or removal of one of them, may give chaos (once again against *naïve application* of Chirikov criterion).

Impact of helical boundary conditions on dynamics. We show here the effect of helical boundary conditions on the MHD dynamics in both the RFP and the Ohmic Tokamak. In the RFP case, a proper reference on a selected MHD mode results in a QSH state with the corresponding helicity. There is a threshold in the value of $b_r(a)$ under which the reference is not able to influence the natural QSH state for a MHD simulation with given S, P and Θ parameters. As an example, we show a SpeCyl simulation characterized by a Lundquist number $S=3\times10^4$ and a magnetic Prandtl number $P=10^3$ both constant during the simulation. The pinch parameter of the equilibrium is $\Theta=1.5$ and a wide spectrum of MHD modes is used (see [6] for SpeCyl code details). The MHD dynamics (Fig. 1a) is characterized by the emergence as dominant mode of the strongly unstable (1,-8) mode, marginally non-resonant with respect to the axisymmetric q-profile. The non linear interaction between MHD modes leads to the establishment of the (1,-10) as dominant one of the ensuing QSH regime. After t=3000 τ_A (solid vertical line in Fig. 1a) finite edge references are applied to the (1,-8) mode.

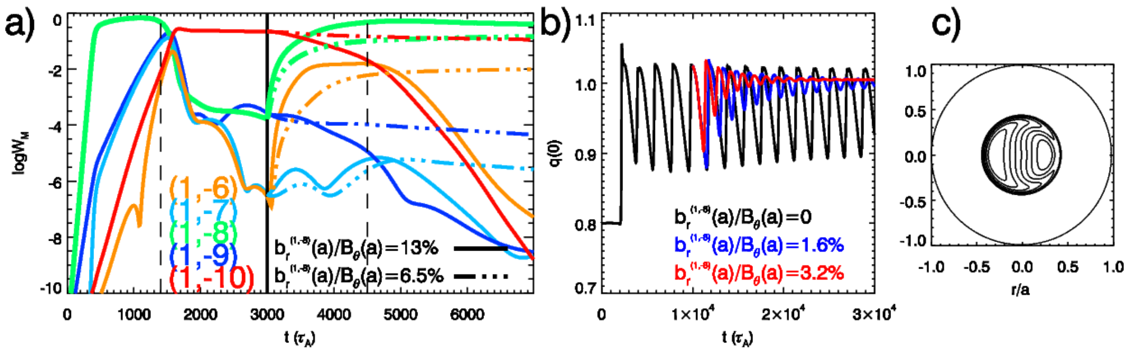

Figure 1: *Impact of helical boundary conditions for RFP and circular Tokamak. RFP case:* ***a)*** *modes’ energy evolution. At t=3000* τ_A *a finite reference on the (1,-8) mode (green line) is applied. The behaviour of the system with respect to different references is shown. Tokamak case:* ***b)*** *evolution of the on-axis safety factor under different edge amplitudes of the (1,1) mode,* ***c)*** *contour plot of helical flux function in the final state of the case with the highest helical perturbation of panel b).*

Fig.1a shows the system behavior for two different edge references: $b_r^{(1,-8)}(a)/B_\theta(a)\sim6\%$ represents a threshold (dotted curves in the plot) under which the system is not able to “recover” a (1,-8) mode QSH state. This is instead obtained raising the value of the edge reference to 13%. Further simulations, not shown here, indicate that the value of the threshold varies with the wave number of the selected mode; in particular it increases for lower *n*s (non-resonant modes). A somehow similar impact of helical boundary conditions in the Ohmic

Tokamak case is illustrated in Fig. 1b. It shows the temporal evolution of the on-axis value of the safety factor q for a set of SpeCyl simulations with $S=10^5$ and P=30 and varying amplitudes of the (1,1) perturbation on the edge radial field. In the case without helical boundary conditions (also shown in [8]) the internal kink mode undergoes clear quasi-periodic sawtooth oscillations. However, as soon as helical boundary conditions are applied, the bifurcation to a snake-like stationary solution occurs. The final state is characterized by incomplete reconnection (i.e., a magnetic island is present in the core as shown in Fig. 1c for the case with highest helical perturbation) and is reminiscent of the equilibrium states with helical core computed in [9]. Note that the bifurcation to a snake-like solution may also occur without the help of helical boundary conditions, when dissipation is increased [8].

Impact of helical boundary conditions on RFP topology. The magnetic field topology, studied with the field line tracing code NEMATO [10], evidences amazingly strong chaos resilience of the initial (1,-8) QSH phase, as previously highlighted in [11]. However, the following analysis shows that such magnetic order seems indeed to rely on a rather "peculiar" character of the spectrum of modes obtained during the initial phase of QSH emergence. As a first example we consider the two MHD states (dashed vertical lines) highlighted in Fig.1a. Even though these states have similar axisymmetric properties and energy distribution of the modes, the corresponding Poincaré plots exhibit very different topological features. The $1400\tau_A$ snapshot shows the perfectly conserved magnetic surfaces characterizing the first QSH phase. It was previously indicated that such chaos resilience relies on the presence of a strong helical component, since by excluding the dominant mode, the chaos formed by remnant modes emerges [11]. Magnetic chaos emerges also if we eliminate the secondary (1,-7) mode from the computed spectrum as seen in Fig. 3a. This behaviour is not explainable in terms of magnetic island overlap, as a lower number of perturbations to the system is present. Magnetic order is also lost by simply changing mode phase with respect to the initial QSH phase in SpeCyl simulation, which is characterized by strict mode-mode phase locking. In fact, in Fig. 3b a π phase was added to the (1,-7) mode that almost completely destroys the conserved magnetic surfaces of Fig. 2a. The state is fragile also versus π variation (flipping) of the phase of the dominant mode (1,-8),

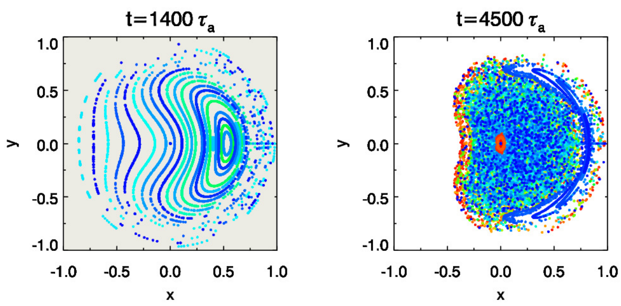

Figure 2: *Poincaré plots of the two times highlighted by dashed vertical lines in Fig. 1a. Despite similar MHD modes energy, two different topological states are found.*

the one building up the QSH (SHAx) state, see Fig. 3c. Note that such flipping leaves unchanged the (helical) q of the state, successfully used in other cases to discuss chaos development [8]. Interestingly, such effect on magnetic order can be verified upon simple hamiltonian three wave model suitable for accurate solution [12].

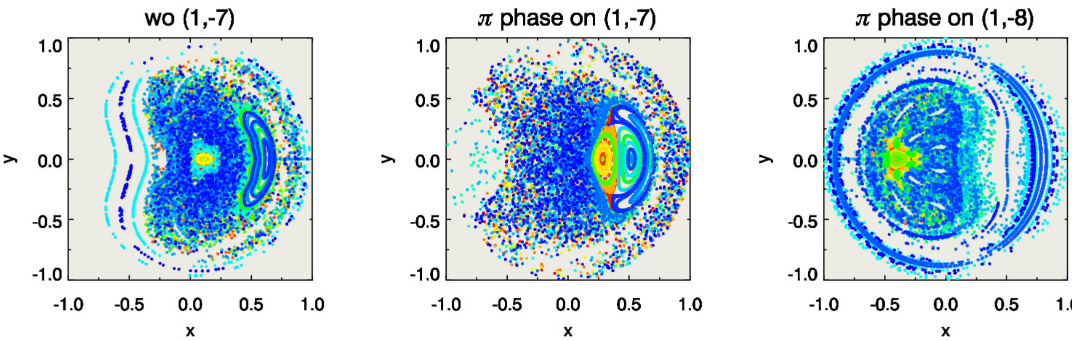

Figure 3: *Fragility of the resilient MHD state of Fig. 2a. Three cases: magnetic topology a) without the (1,-7) secondary mode, b) adding a π phase to the (1,-7) mode and c) adding a π phase to the (1,-8) mode.*

In summary, the initial QSH regime of our 3D nonlinear MHD simulation is successfully recovered dynamically by suitable application of helical boundary condition on radial magnetic field, similarly to experimental results [13]. However, the amazing magnetic order obtained during the first spontaneous QSH formation is not recovered anymore. Indeed, the peculiar topological features of that initial state appear rather fragile and not straightforwardly understandable in terms of "common" tools like overlapping conditions on cylindrical Fourier components, or shape of q profile. In this respect, the study of "helical" tools for the RFP is ongoing and will be the subject of next publications. In particular, we believe the analysis in terms of helical flux coordinates should provide the key tool for correct interpretation of present results. We finally note that the magnetic order fragility highlighted above, may provide some hints in interpreting the one observed for the helical thermal structure in RFX-mod experiment [14].

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