Self-Consistent ICRH Modelling

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Introduction

A code, SELFO-light, for self-consistent modelling of the power deposition and ion distribution functions for ICRH (Ion Cyclotron Resonance Heating) has been developed for tokamaks. The code package combines the full wave solver LION [1, 2], which uses a FEM (Finite Element Method) discretization in both the radial and the poloidal directions in toroidal geometry, with a 1D time dependent FEM Fokker-Planck solver for calculating pitch angle averaged distribution functions of resonant species. The spatial dispersion of waves in the ICRF (Ion Cyclotron Range of Frequency) complicates the calculations of the wave field, since the plasma response depends on the direction of propagation. The conventional way to solve this problem is to Fourier decompose the wave field. Novel methods suitable for FEM are presented for including higher order Larmor radius effects, upshift of the parallel wave number and transit time magnetic pumping. The Fokker-Planck code includes sources and sinks to allow modelling of NBI and fusion products [3]. A benchmarked formula for ICRF heating is used to calculate the parallel ion temperatures. Self-consistency is obtained by modifying, at every time step, the susceptibility tensors of the resonant ion species used in the wave code to be consistent with the changes in the pitch angle and flux surface averaged distribution functions and changes in their parallel temperatures [4]. The flux surface averaged quasi-linear operators used in the Fokker-Planck code are calculated from the wave fields. Fast wave current drive is calculated from the local power deposition on electrons using a bench-marked formula including trapped electrons [5].

SELFO-light

In the standard version of SELFO-light the equilibrium is calculated with the CHEASE code [6] and the wave field with the LION code [1, 2]. To make the wave fields consistent with the non-thermal distribution functions the susceptibility tensors corresponding to bi-Maxwellian distribution functions of the different resonant ion species are modified by using

amplification factors. The quasi-linear RF-operators used in the Fokker-Planck code are calculated by averaging the wave field and quantities obtained from the wave solver over subvolumes delimited by magnetic flux surfaces in which density, temperature and power are assumed to be constant. Changes in the density profiles determined by transport solvers or measurements are achieved by using thermal sources and sinks in the Fokker-Planck solver.

Four time scales, $\tau_4 > \tau_3 > \tau_2 > \tau_1$, are used in order to get the coupled power for each toroidal mode to agree with the absorbed power. On τ_4 the equilibrium quantities, RF-power, mode spectra, NBI power and injection energy may change. On τ_3 new wave field calculations are done including modifications of the susceptibility tensors of the resonant species and their parallel temperatures. The Fokker-Planck solver is called on the time scale τ_2, after each call the electric wave fields are normalized so that the absorbed power agrees with the coupled power. The time scale τ_1 is determined by the convergence of the Fokker-Planck solver.

The wave field is calculated with the LION code solving the equation

$$\left(\nabla\times\nabla\times\mathbf{E}_\perp\right)_\perp - \left(\frac{\omega}{c}\right)^2 \left(\boldsymbol{\varepsilon}\left(f(\mathbf{v},\mathbf{x})\right)\right)_\perp \cdot \mathbf{E}_\perp = i\mu_0\omega\mathbf{J}_{\perp ant} \ , \qquad (1)$$

where $\mathbf{E}_\perp = (E_\mathrm{n}, E_\mathrm{b})$, E_n and E_b are the electric field components normal and tangential to the magnetic flux surfaces, respectively, $\mathbf{J}_\mathrm{ant}$ is the antenna current and $\boldsymbol{\varepsilon}$ is the dielectric tensor. The dielectric tensor is defined from local unform bi-Maxwellian susceptibility tensors to all orders in the ratio between the ion Larmor radius and the perpendicular wavelength by inclusion of the Bessel functions. Since the LION code does not calculate the parallel electric field, the electron damping due to TTMP/ELD is approximated by half of the TTMP damping [7]. The current driven is calculated from the electron absorption using a formula [8] taking into account trapped electrons.

The effects of a non-thermal distribution function, $f_\mathrm{n}(\upsilon)$, due to ion cyclotron absorption are included by calculating a local equivalent parallel temperature and amplification factors for the susceptibility tensor elements from the 1D time-dependent distribution functions [4]. The amplification factors $a_{ik}^{(n,l)}$ and the equivalent parallel temperatures are for each resonant species, n, calculated in each subvolume for each harmonic, l, in order to take into account the presence of a flux surface intersecting more than one harmonic: $a_{ik}^{(n,l)} = \dfrac{\chi_{ik}^{(n,l)}\left(f_n\right)}{\chi_{ik}^{(n,l)}\left(f_{M,n}\right)}$, where

$\chi_{ij}^{(nl)}(f_n) \approx \frac{\omega_p^2}{8\omega\omega_c}\int_0^\infty d_{ij}^{(nl)} \frac{\partial f_n}{\partial \upsilon}\upsilon^2 d\upsilon$, $f_{M,n}(\upsilon)$ is the corresponding Maxwellian distribution function $d_{nn}^{(n,l)} \approx 0.25\left|J_{l-1}+J_{l+1}\right|^2$, $d_{bb}^{(n,l)} \approx 0.25\left|J_{l-1}-J_{l+1}\right|^2$ and $d_{nb}^{(n,l)} \approx i0.25\left(J_{l-1}^2 - J_{l+1}^2\right)$.

The predominant effect of spatial dispersion comes from the parallel wave number, which is approximated by $k_\parallel = n_\phi/R$. Other important spatial dispersive effects are higher order FLR terms and TTMP damping. Methods to correct these have been developed suitable for FEM codes. The spatial dispersive effects caused by FLR depend in leading order only on $k_\perp$, the higher order terms are important for describing how the E_+ and E_- absorption adds up and depend on the direction of $\mathbf{k}_\perp$, which can be taking into account by locally rotating the susceptibility tensor with an angle θ resulting in

$$\chi_\perp = \begin{pmatrix} T+\sigma\cos 2\theta & -i\chi_{xy}-\sigma\sin 2\theta \\ i\chi_{yx}-\sigma\sin 2\theta & T-\sigma\cos 2\theta \end{pmatrix}, \qquad (2)$$

where $T = (\chi_{xx} + \chi_{yy})/2$, $\sigma = (\chi_{xx} - \chi_{yy})/2$, χ_{xx} , χ_{yx} and χ_{yy} are calculated in a coordinate system where $k_y = 0$. The angle θ is given by $\theta = (\alpha_- - \alpha_+)/2$ where α_+ and α_- are defined by $e^{i\alpha+}|E_+| = 0.5\,(E_n + i\,E_b)$ and $e^{i\alpha-}|E_-| = 0.5\,(E_n - i\,E_b)$, the wave field components E_n and E_b are obtained from the previous iteration.

The TTMP damping is first calculated assuming the plasma to be local uniform with $\mathbf{k}_\perp$ normal to the magnetic flux surfaces. The electron damping is then corrected by multiplying the *bb*-component of the electron susceptibility tensor with $\left|(\nabla\times\mathbf{E}_\perp)_\parallel\right|^2 / \left|k_\perp E_b\right|^2$ where $(\nabla\times\mathbf{E}_\perp)_\parallel$ is calculated from the solution.

The upshift, applicable to solutions with strong single pass damping, is calculated consistent with the stationary phase method. The change in energy ΔW due to wave-particle interactions is obtained by integrating the change in energy of the particle along the drift orbit

$$\Delta W = Ze\int_0^{\tau_B} dt\upsilon_\perp \exp\left\{\ln E_+(t_-) + \int_{t_-}^{t}\left[\mathrm{Re}\left(\mathbf{v}_g\cdot\nabla\ln E_+\right)\right]dt' + i\vartheta\right\}, \qquad (3)$$

where $\mathbf{v}_g$ is the guiding centre velocity and the phase ϑ is given by $\vartheta = \int_{t_-}^{t}\left(\mathrm{Im}\left[\mathbf{v}_g\cdot\nabla\ln E_+\right] + n\omega_{ci} - \omega\right)dt'$. For decorrelated interactions the resonances correspond to points where ϑ becomes stationary. An equivalent local $k_\parallel$ can then be defined so that the resonance condition is given by the stationary phase point, neglecting the drift and the variation of $\upsilon_\parallel$ at the resonance one obtains

$$k_{\|} = \frac{B_\phi}{B}\frac{n_\phi}{R} + \frac{B_\chi}{B}\,\mathrm{Im}\left[\frac{1}{E_+}\frac{\partial E_+}{\partial \chi}\right]. \qquad (4)$$

The wave field is then calculated by means of iteration replacing $k_{\|} = n_\phi/R$ in the susceptibility tensor with Eq. (4).

Flux surface averaged distribution functions are calculated for each subvolume and resonant ion species with a time dependent 1D cubic FEM Fokker-Planck code [3]

$$\frac{\partial F_n}{\partial t} = \frac{\partial}{\partial \upsilon}\left[-\alpha F_n + \frac{1}{2}\frac{\partial}{\partial \upsilon}\left(\beta F_n\right)\right] + \frac{\partial}{\partial \upsilon}\left[H_n\left(\frac{k_\perp \upsilon}{\omega_{ci}}\right)\left(\frac{\partial F_n}{\partial \upsilon} - \frac{2F_n}{\upsilon}\right)\right] + S_n(\upsilon) + D(F_n), \qquad (5)$$

where $H_n\left(\bar{k}_{\perp,ik}^{(n,l)}\upsilon/\omega_{ci}\right) = \left\|\bar{E}_+\right| J_{n-1} + \left|\bar{E}_-\right| J_{n+1}\Big|^2$ is the diffusion coefficient in the quasi-linear operator, $F_n(\upsilon) = 4\pi\upsilon^2 f_n(\upsilon)$, where $f_n(\upsilon)$ is the local distribution function of the n:th species, α and β are Chandrasekhar´s Coulomb collision operators. The argument of the Bessel functions is $\bar{k}_{\perp,ik}^{(n,l)}\upsilon/\omega_{ci}$, where the perpendicular wave number, $\bar{k}_{\perp,ik}^{(n,l)}$, and wave field components, $\bar{E}_\pm$, are averaged with respect to power over each subvolume for each species and each harmonic. The source, S_n, and loss, D, are used for modelling NBI, fusion reaction products and changes of the resonant ion densities.

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