Secondary magnetic flows phenomena in turbulent plasma

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I. It is generally recognized that nonlinear interaction of short-scale drift-type fluctuations in turbulent plasma can spontaneously generate and sustain the large-scale strongly anisotropic secondary flows with additional symmetry, so called zonal flows and streamers that play an important role in different areas of plasma research. Generation of such flows is commonly attributed to the effect of Reynolds stress produced by small scale fluctuations, using the free energy stored in density and temperature gradients. The mechanism behind can be connected to the well known inverse cascade guaranteed in two (and quasi-two)-dimensional fluids by the conservation of energy and enstrophy. On the other hand, transport and amplification properties of large-scale magnetic fields are widely investigated because of their importance in different physical phenomena. One impressive effect of large, strong magnetic fields is the release of high-energy bursts in solar flares. It is interesting to combine the phenomena of excitation of large scale structures and strong quasi-steady magnetic fields, and thus develop a nonlinear theory capable of describing the generation of such large-scale magnetic fields by small scale turbulence and their mutual interactions.

Here we investigate the generation of large-scale magnetic fields in magnetic electron drift (MED) mode turbulence. The small scale turbulent fluctuations are drift type modes excited in a non-uniform plasma in the frequency range between the electron and ion plasma frequencies and are fed by density and temperature gradients through the first order baroclinic vector. These modes are of interest in, e.g., laser fusion experiments, where they are thought to be responsible for the very strong self-generated magnetic fields observed since 1970s. Moreover, phenomena occurring in such time scales may even be more important as a source of secondary magnetic field structures related, for example, to the reconnection of magnetic field lines. To understand the nonlinear dynamics of these modes, we employ a self-consistent spectral two field model. Note that this model does not deal with flows in the original sense, since it is not flow of the particles, but rather magnetic structures that are elongated along one direction and periodic with a long wavelength along the other direction as well. Following this similarity, we call the corresponding large-scale structures "zonal magnetic fields, or magnetic streamers" as it has been adopted in the literature. With this model, we focus on the generation of large scale magnetic fields, stability of these structures and its further nonlinear evolution along with simulation study of the MED mode turbulence.

II. We consider a nonuniform unmagnetized plasma, and fluctuations on a space scale much smaller than that of the equilibrium density and temperature inhomogeneities, which are taken to be in the *x*-direction. The time scale is faster than the ion and slower than the electron plasma frequency, and hence we consider an unpolarized electron fluid and immobile ions. Then, starting from the momentum equation together with Maxwell's equations and energy equation, the model equations for MED mode turbulence can be reduced to a pair of coupled non-linear equations for magnetic field, $B(x, y)\mathbf{z}$ say, and perturbed electron temperature T

$$\frac{\partial}{\partial t}\left(B-\lambda^2\nabla^2 B\right)+\beta\frac{\partial T}{\partial y}-\lambda^4\frac{e}{mc}\left\{B,\nabla^2 B\right\}=0\,, \tag{1a}$$

$$\frac{\partial}{\partial t}T+\alpha\frac{\partial B}{\partial y}+\lambda^2\frac{e}{mc}\left\{B,T\right\}=0\,, \tag{1b}$$

Here α and β are coefficients proportional to the inverse length scales of the density and temperature inhomogeneities and $\lambda = c/\omega_{pe}$ is the electron skin depth, the curl brackets denote the Poisson brackets and are defined as $\{c,d\}\equiv\left(\nabla c\times\nabla d\right)\cdot\mathbf{z}$. Linear analysis of Eqs.(1) shows that there is purely growing solutions for $\alpha\beta<0$, so that the underlying MED mode turbulence is driven by gradients of density and temperature. Eqs.(1) have two conserved quantities corresponding to that of the energy and enstrophy integrals. By analysis of the spectral properties of the MED modes in the weakly nonlinear approximation, one can show that the presence of these two integrals necessitates the **double energy cascade** as the key property of the MED mode turbulence.

The nonlinear transfer of wave energy from small scales towards the long wave length region (the so-called ‘‘**inverse cascade**’’) is a cause of spontaneous generation and sustainment of large-scale structures. So, *the MED mode turbulence is capable of generating the large scale wing of the wave spectrum*. During this flow generation, thermodynamic free energy stored in gradients is converted into kinetic energy of magnetic flows by fluctuation induced Reynolds stress and thus these gradients constitute the energy source for the magnetic structures. To prove this statement, we use the ansatz of multi-scale expansion between the spatio-temporal scales of the flows and those of micro-turbulence. The magnetic field and temperature are then decomposed into a large-scale slowly varying component and a small scale component. It is important to note that the conserved energy integral contains both small-and large-scale components. It means that the whole wave spectrum and the interaction between different regions of the spectrum have to be included into considerations. To describe this interaction we will separate the whole turbulent spectrum into two parts, one describing large-scale structures with a wave vector denoted by $\mathbf{q}$ and the other describing small-scale turbulence with a wave vector denoted by $\mathbf{k}$. We therefore have the relation $|\mathbf{q}|<<|\mathbf{k}|$. Note that the both $\mathbf{q}$ and $\mathbf{k}$ describe the same spectrum but different parts of it. Using these decompositions yields the **set of *q*-th Fourier components** of Eqs.(1). It is seen from these equations that a small scale turbulence can indeed drive large-scale structures characterized by B_q and T_q via the **magnetic Reynolds stress** $\sum k_x k_y B_k B_{-k}$. In order to describe the nonlinear evolution of the total wave spectrum in a self-consistent way, we have to determine the “loop-back”, i.e. the response of small scales to large-scale structures changes. To this end, it is relevant to consider the evolution of MED mode micro-turbulence in a medium which is slowly modulated by large scale structures. This can be conveniently done using a **wave kinetic equation** for the generalized wave action density $N_k(\mathbf{r},t)$ in $\mathbf{r}-\mathbf{k}$ space. An appropriate action-like invariant takes the form $N_k = 4\alpha\left(1+k^2\lambda^2\right)|B_k|^2/\beta$. The set of q-th Fourier components of Eqs.(1) and corresponding wave kinetic equation constitute the basic system used to describe the nonlinear evolution of the MED mode turbulence in a self-consistent way.

III. We are looking for a general criterion for the generation of large scale fields depending on the form of the wave spectrum. We will focus our attention on the zonal magnetic fields, i.e. structures elongated perpendicular to the direction of plasma inhomogeneity. Introducing the large scale "vector" $\overline{B} \equiv \left(e\lambda^2/4m\right)\left(B_q - \sqrt{\beta\alpha^{-1}}T_q\right)\exp\left(i\mathbf{qr}\right)$ and performing some calculations with equations (1) yields the equation of motion for $\overline{B}$. To solve this equation, we first replace the magnetic field Fourier component with the wave spectrum using the definition of the wave action density N_k. Then we decompose N_k into an equilibrium and perturbed part, $N_k = N_0 + \tilde{N}_k$, and assume $N_k \sim \exp\left(-i\Omega t + i\mathbf{pr}\right)$. So that the equation of motion for $\overline{B}$ finally becomes

$$\gamma^{zf} - i\Omega^{zf} = -K_q^2 \int q_x^2 k_x \left(\partial N_0 / \partial k_x\right) M_k R\left(\Omega, p\right) d^2\mathbf{k} \tag{2}$$

here M_k and K_q are real coefficients. The response function $R\left(\Omega, p\right) = i / \left(\Omega - p\mathrm{v}_g + i\gamma^N\right)$ where γ^N is small and positive, v_g is the group velocity of the MED mode. The strongest interaction between the small scale turbulence (medium) presented by its wave spectrum N_0, and the zonal fields can expected when the reaction of the medium (Ω in the response function) is in resonance with the perturbation (represented by v_g in the response function), that is, when $\Omega \simeq p\mathrm{v}_g$ and thus $R\left(\Omega, p\right) \sim 1/\gamma^N > 0$. Because of strong interaction, we can assume $\Omega^{zf} \simeq 0$. The corresponding regime of zonal field generation is called "kinetic" because of its obvious similarities with Landau damping in the kinetic wave theory. The result of these considerations is the criterion for instability $\gamma^{zf} > 0 \Leftrightarrow k_x\left(\partial N_0 / \partial k_x\right) < 0$. This result is opposite to the condition for the Langmuir turbulence, where the slope of the velocity distribution function must be positive for positive velocities.

We derive rather general criterion for excitation of large scale magnetic fields by small scale turbulence, which depends on the equilibrium spectrum distribution. For explicit integration of the equation (2), one has to consider a specific form of the equilibrium wave spectrum N_0. Assuming a monochromatic wave packet, $N_0^k = N_0 \delta\left(\mathbf{k} - \mathbf{k}_0\right)$, and performing the integration by parts yield the dispersion relation for ZF

$$\left(\Omega - q\mathrm{v}_g\right)^2 = K_q^2 q^2 k_{0y}^2 N_0 \left(\partial \mathrm{v}_g / \partial k_x\right) / \left(k_{0y}\sqrt{\alpha\beta}\right) \tag{3}$$

So that, requirement for the instability is $N_0\left(\partial \mathrm{v}_g / \partial k_x\right) / \left(k_{0y}\sqrt{\alpha\beta}\right) < 0$. Note that this condition is similar to the well known Lighthill criterion for the modulational instability.

IY. The long term dynamics of the large scale magnetic fields generated and strengthened through instabilities can be determined by looking for the time evolution equation of the large scale "flow velocity" $\mathbf{v}_f$. To derive this equation, we will decompose the wave spectrum into equilibrium, resonant and nonresonant first order and nonresonant second order perturbed part, $N_k = N_0 + \tilde{N}_k^r + \tilde{N}_k^{(1)} + \tilde{N}_k^{(2)}$. Now performing calculations of resonant and nonresonant parts and summing up corresponding results yields the evolution equation for "flow velocity" $\mathbf{v}_f$

$$\partial_t \partial_x \mathrm{v}_f = D\partial_x^3 \mathrm{v}_f + u\partial_x^2 \mathrm{v}_f + b\partial_x^2 \mathrm{v}_f^2 \tag{4}$$

Here D, u, and b are some integral coefficients which depend on the equilibrium spectrum distribution. We now look for the stationary solutions of Eq.(4), propagating with constant velocity u_{0x} in the x direction, $v_f(x - u_{0x}t)$. The simplest stationary solution for Eq.(4) with the imposed boundary conditions corresponding to a solitary wave with different asymptotic values, i.e. a “switching” or “kink” soliton, is given by

$$2v_f = \{v_1 + v_2 + (v_1 - v_2)\tanh[b(v_1 - v_2)x/2D]\}$$

This solution describes the transient region between two different values of the flow v_1 to v_2. We note that it is different from the stationary vortex solution found earlier. Thus, cooperative effects of the wave motion, steepening and instability give a possibility of forming stationary or moving kink solitons in between the surfaces of two different flow velocities.

Y. A simulation study of the Eqs. (1) for different sets of parameters has been performed. The simulation code is based on a pseudospectral method to resolve derivatives in space with periodic boundary conditions, with random fluctuations as initial conditions.

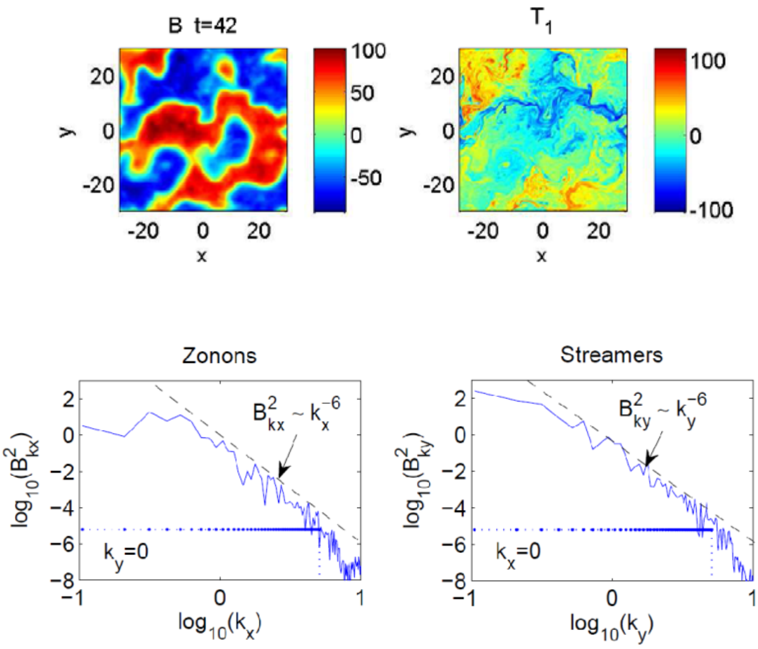

Figure 1: Linearly unstable regime. Top panels: Magnetic field and temperature fluctuations
Bottom panels: Zonon and streamer energy spectra of the magnetic field.

In the unstable regime ($\alpha\beta < 0$), displayed in Fig. 1, we could observe magnetic field generation and the formation of large scale magnetic structures, accompanied by small-scale turbulence visible in the temperature fluctuations. The energy spectra are non-Kolmogorov and concentrated to streamers at small wave numbers.

In the linearly stable regime ($\alpha\beta > 0$), we observe small-scale turbulence and the formation of zero-frequency zonal flows (zonons). The energy spectra are strongly anisotropic with magnetic wave energy concentrated at zonons.