Dynamical and statistical properties of two-dimensional turbulence in pure electron plasmas

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The experimental conditions for electron plasmas confined in Penning-Malmberg traps [1] can be chosen in such a way that the cold non-relativistic guiding center approximation is valid and the transverse dynamics of the plasma is well described by the drift-Poisson equations [2]. These equations are isomorphic to the 2D Euler equations for an ideal fluid, whose vorticity correponds, up to a costant of proportionality, to the electron plasma density. Therefore, highly magnetised pure electron plasmas have been extensively used to study 2D fluid turbulence. In the last years, the dynamics and statistics of 2D turbulence in electron plasmas have been investigated through different methods, such as Fourier transforms [3], wavelets [4, 5], and Proper Orthogonal Decomposition [6].

In this work, the properties of 2D turbulence in pure electron plasmas are studied by analysing the results of experiments performed with the Penning-Malmberg trap ELTRAP [7]. A low density ($n = 10^{12} - 10^{13}$ m^{-3}) and temperature ($T = 1 - 10$ eV) electron plasma is generated by a thermoionic spiral cathode heated with a constant current and negatively biased with respect to a grounded grid. Different spatial distributions of the electrons reaching the grid can be obtained by varying the voltage drop across the filament and the source bias. The plasma is contained in a set of cylindrical electrodes, with radius $R_W = 4.5$ cm. The radial confinement is provided by an axial magnetic field B (up to 0.2 T), while the axial trapping is obtained through a suitable negative potential applied on two cylinders. The system is investigated through an inject/hold and manipulate/dump cycle, and the time evolution is monitored by means of an optical diagnostic system. After being injected into the device, the electrons are trapped for a given time and then dumped onto a phosphor screen. The light emitted by the screen is imaged by a 12 bit charge-coupled device (CCD) camera, so that the light intensity in each point is proportional to the axially averaged electron density $n(x,y,t)$, which represents also the vorticity $\zeta(x,y,t)$ of the 2D fluid. The shot-to-shot reproducibility of the initial conditions is very high (the maximum relative variation of the measured charge at given position and time is typically less than 0.1 %), so that the time evolution of the plasma can be studied by keeping the injection parameters fixed and increasing the trapping time. The sequences studied in this work consist of $N = 250$ frames with a trapping time step of 2 μs. The plasma density evolution for the considered sequences is shown in Fig. 1. The first frame in both sequences (trapping time $t =$ 2 μs) reflects the shape of the initial density distribution. Two types of initial conditions for the electron density, namely annular and spiral configurations, have been considered.

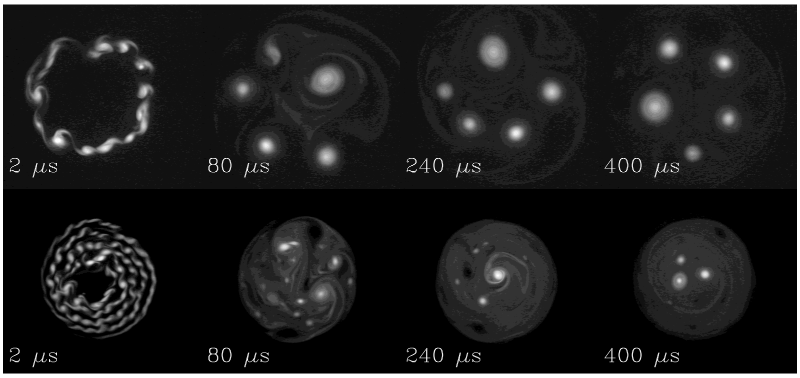

Figure 1: Snapshots of the plasma density for the two analysed sequences. The trapping time is indicated at the bottom left corner of each frame.

In order to investigate the intermittency properties of 2D turbulence in electron plasmas we study the scaling behavior of vorticity increments in both x and y directions

$$\begin{aligned}\Delta\zeta_l^{(x)}(x,y,t) &= \zeta(x+l,y,t)-\zeta(x,y,t)\ ,\\ \Delta\zeta_l^{(y)}(x,y,t) &= \zeta(x,y+l,t)-\zeta(x,y,t)\ .\end{aligned}$$

This can be done through the analysis of the Probability Density Functions (PDFs) and the structure functions of the increments. One of the signatures of intermittency is the change of the PDF shape with the scale l. Fig. 2 shows the PDFs of the standardised vorticity increments $\Delta\zeta_{l,\mathrm{st}}^{(x)} = (\Delta\zeta_l^{(x)} - \langle\Delta\zeta_l^{(x)}\rangle)/\sigma_{\Delta\zeta_l^{(x)}}$ along the x direction for the two initial conditions, at two time instants and for three spatial separations (similar results are found for increments along y).

For the annular initial condition the PDFs show a central core and large increment tails at all spatial separations and do not evolve significantly over time. The nearly Gaussian core corresponds to background fluctuations, while the tails are due to differences between high density values in the vortices and low density values in the background. For the spiral initial conditions the PDFs at $t = 2$ μs are nearly Gaussian at all the considered separations. However, a clear time evolution of the PDFs is found in this case. Tails at large increments increase with time, especially at small scales. The deviation from the Gaussian shape increases going from large to small scales, which indicates the occurrence of intermittency in the turbulent dynamics.

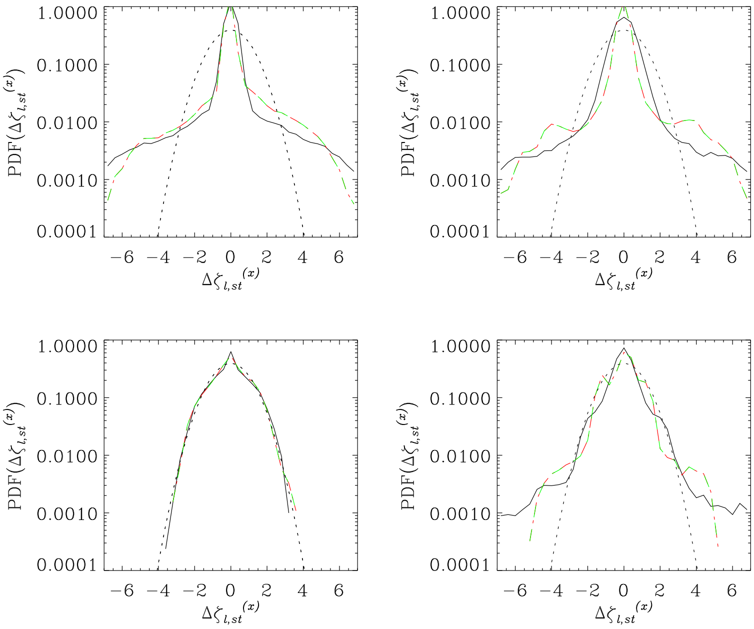

Figure 2: PDFs of the standardised vorticity increments $\Delta\zeta_{st}^{(x)}$ along the x direction for the annular (top row) and spiral (bottom row) initial conditions at two time instants: $t = 2$ μs (left column) and $t = 240$ μs (right column). The spatial separations l are: $l = 0.98$ mm (black solid curve), $l = 4.9$ mm (red curve), $l = 9.8$ mm (green curve). The Gaussian PDF with zero mean and $\sigma = 1$ is also shown for comparison (black dashed curve). Similar results are found for increments along the y direction.

A more quantitative measure of the intermittency is given by the flatness $F(l)$. Structure functions are defined as the moments of field increments, that is, $S_p(l) = \langle \Delta\zeta_l^p \rangle$, where $\langle\cdot\rangle$ denotes spatial averages. $F(l)$ is given by the ratio of the 4th order moment to the square of the 2nd order moment, $F(l) = S_4(l)/[S_2(l)]^2$. The flatness is 3 for Gaussian PDFs, while it increases as l decreases in the presence of intermittency. The flatness of the vorticity increments along the x direc-

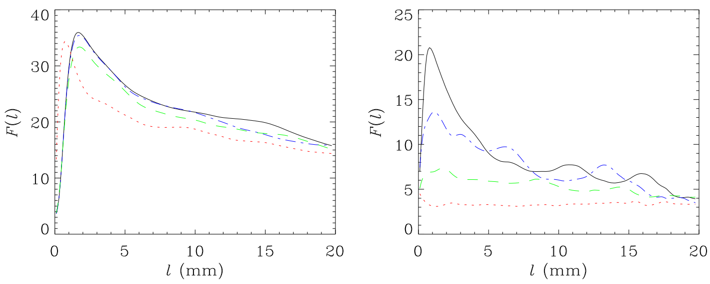

Figure 3: Flatness of the vorticity increments along the x directions for the annular (left panel) and spiral (right panel) initial conditions at different trapping times: 2 μs (red curve), 80 μs (green curve), 240 μs (blue curve), 400 μs (black curve). Similar results are found for increments along the y direction.

tion at different trapping times is shown in Fig. 3 for the two initial conditions.

For the annular initial condition, $F(l)$ grows as l decreases down to $l \approx 2$ mm, but it almost does not change with time, in agreement with the PDFs behavior. Therefore, the growth of $F(l)$ must be attributed to the shape of the initial condition and not to the presence of intermittency. The decrease of $F(l)$, observed for $l < 2$ mm, to the Gaussian value 3 is due to the instrumental noise fluctuations, which dominate at very small spatial scales. For the spiral initial condition, $F(l) \approx 3$ at all scales for $t = 2$ μs, as it could be expected from the Gaussian shape of the PDFs. Then $F(l)$ starts to grow going from large to small scales, down to $l \approx 1$ mm, and this growth becomes stronger and stronger with time. At variance with the annular case, the observed increase of the flatness at small scales is not a trivial effect already present in the initial conditions, but it represents the manifestation of the intermittency arising from the turbulent dynamics of the plasma. The decrease of the flatness to the Gaussian value 3 for $l < 1$ mm can be attributed also in this case to the instrumental noise fluctuations of the optical diagnostics.

In conclusion, for the annular initial condition it is found that the statistics of the vorticity increments does not change significantly with the scale and with time. This means that intermittency is absent and the scaling behavior of the increments is basically determined by the initial conditions. On the other hand, for the spiral initial condition, the vorticity increments show nearly Gaussian statistics at all spatial scales during the very early stages of the plasma evolution, while the growth of large increment tails in the PDFs and of the flatness at small scales indicates the development of intermittency arising from the plasma turbulent dynamics.

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