Oscilliton solitary waves in pair plasmas with dust

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Introduction

Nonlinear theory is presented for solitary waves propagating parallel to the magnetic field in a cold plasma with charged dust grains or heavy ions. The particular case of an electron-positron plasma is investigated. For linear waves in such pair plasmas, the imbalance of charge has been shown to lead to phenomena normally associated with ion-cyclotron effects or Hall currents in electron-ion plasmas, such as absorption at the generalized Alfvén resonance [1]. A class of nonlinear waves propagating along the magnetic field in an electron-ion plasma have been shown to have a structure of oscillations within a solitary wave envelope, with an associated rotation of the wave magnetic field, known as oscillitons [2]. The oscillatory property is due to the mass difference of the two charged species (or the difference in the Hall currents of the species). If the two species have the same mass and density (a charge-balanced pair plasma), the oscillations and field rotation are absent in the solitary wave envelope [3].

Here we introduce a difference in the density (and therefore the total charge) of the two oppositely charged equal mass species, with the remainder of the charge on a very heavy third species (dust grains or heavy ions). An analysis based on that of [3] is used to derive the relevant nonlinear wave equation, using the approximation that the heavy species velocities must be negligible compared with the lighter species velocities. Exact solutions based on the analysis of [4] are presented, and show that the charge imbalance plays the same role as a difference in mass: oscillations occur within the solitary wave envelope, and a rotation of the magnetic field occurs.

Nonlinear equations

We consider a cold magnetized plasma, composed of protons (or positive ions or positrons) and electrons, plus a heavy species that retains part of the charge, positive or negative. The equilibrium magnetic field is in the x-direction. As in [3] we consider solitary wave structures propagating along the magnetic field with a velocity V in the plasma frame. We transform x to the coordinate $\xi = x - Vt$. Define the light species charge imbalance parameter $\eta = Z_d n_{d0}/n_{p0}$, with $n_{e0,p0,d0}$ the equilibrium electron, positive ion and dust number densities respectively, and $Z_d e$ the dust particle charge . Using charge neutrality and particle flux conservation we have

$n_{e0} = n_{p0}(1+\eta)$, and for the light species x velocities, to first order in η,

$$V - v_{ex} = (V - v_{px})(1 + \eta v_{px}/V). \tag{1}$$

We make the simplifying assumption that both η and the wave amplitude are small, $v_{px} << V$. so the parallel velocities of the lighter species are equal.

The conservation equations for the 3 species are, for non-relativistic motion,

$$\sum_j n_{j0} m_j v_{jx} - \frac{B_\perp^2}{2\mu_0 V} = 0, \tag{2}$$

$$\sum_j n_{j0} m_j \mathbf{v}_{j\perp} + \frac{B_0}{\mu_0 V}\mathbf{B}_\perp = 0, \tag{3}$$

$$\sum_j n_{j0} m_j [(v_{jx} - V)^2 + v_{j\perp}^2 - V^2] = 0, \tag{4}$$

$$\sum_j \frac{n_{j0} m_j^2 v_{j\perp}^2}{q_j} = 0, \tag{5}$$

The change in $v_{dy,z}$ relative to the change in $v_{py,z}$ is of order $Z_d m_p/m_d$, so that for $m_d \gg Z_d m_p$ the transverse dust velocity components remain negligible compared with the proton and electron velocities. This is also true for the x-components of velocity. We therefore set the dust velocity components to zero where possible. The kinetic energy change of the dust can then be neglected. A further simplifying assumption is that the transverse momentum in the dust is negligible.

We represent the velocities and magnetic field as polar decompositions, eg $\mathbf{v}_e = v_{e\perp}(\mathbf{e}_y \cos\alpha_e + \mathbf{e}_z \sin\alpha_e)$, with α_e and α_p the electron and ion phases, and define $\phi = \alpha_p - \alpha_e$. The transverse equations of motion of the 2 lighter species then may be written

$$\frac{dv_{p\perp}}{d\xi} = -\frac{\mu_0 V e}{B_0}(n_{e0} n_{p0})^{1/2} v_{p\perp} \sin\phi, \tag{6}$$

$$\frac{dv_{e\perp}}{d\xi} = -\frac{\mu_0 V e}{B_0}(n_{e0} n_{p0})^{1/2} v_{e\perp} \sin\phi, \tag{7}$$

$$\frac{d\alpha_p}{d\xi} = \frac{eB_0}{m_p(V - v_x)} - \frac{\mu_0 V e}{B_0}(n_{p0} + (n_{e0} n_{p0})^{1/2}\cos\phi), \tag{8}$$

$$\frac{d\alpha_e}{d\xi} = -\frac{eB_0}{m_e(V - v_x)} + \frac{\mu_0 V e}{B_0}(n_{e0} + (n_{e0} n_{p0})^{1/2}\cos\phi). \tag{9}$$

Assuming that the phases are approximately locked, as is the case in the absence of dust [3], i.e. $\alpha_p + \alpha_e = 2\beta$, where β is the phase of the transverse magnetic field, we find from the conservation equations and equations of motion, neglecting the dust contribution,

$$B_\perp = -\frac{\mu_0 V}{B_0}(n_{p0}m_p v_{p\perp} + n_{e0}m_e v_{e\perp})\cos(\alpha_p - \beta), \tag{10}$$

$$\frac{d\beta}{d\xi} = \frac{eB_0}{(V-v_x)}\frac{(1/m_p - (1+\eta)^{1/2}/m_e)}{(1+(1+\eta)^{1/2}}, \tag{11}$$

$$\frac{d\phi}{d\xi} = \frac{eB_0}{(V-v_x)}(\frac{1}{m_p}+\frac{1}{m_e}-\frac{\mu_0 n_{p0}}{B_0}(2+\eta+(1+\eta)^{1/2}\cos\phi). \tag{12}$$

As in [3], the equations of parallel motion may be used, via a compatibility condition, to arrive at an invariant:

$$\cos\phi = \frac{4(1+\mu)^2}{(1+(1+\eta)^{1/2})^2 M(2M-u_x)} - 1, \tag{13}$$

with the normalizations $V_A^2 = B_0^2/\mu_0 n_{p0}(m_p+m_e)$, $\zeta = \xi\Omega_e\sqrt{\mu}/V_A$, $M = V\sqrt{\mu}/V_A$, $u_x = v_x\sqrt{\mu}/V_A$, and $\mu = m_e/m_p$.

The equation for the parallel velocity becomes

$$\frac{du_x}{d\zeta} = \frac{(1+\mu\eta/(1+\mu))^2}{(1+\eta)^{1/2}}\frac{M[(M-u_x)^2-M^2]}{(1+\mu)(M-u_x)}\sin\phi. \tag{14}$$

The envelope of the solitary solution is therefore only slightly modified by η.

Solution

Following [4], we transform to the new independent coordinate τ:

$$\frac{d\zeta}{d\tau} = u_x - M, \tag{15}$$

so τ is the time for a fluid particle to go a distance ζ. Then we find to dominant order in η,

$$\left(\frac{du_x}{d\tau}\right)^2 = \left(1+\frac{(5\mu-3)}{2(1+\mu)}\eta\right)u_x^2\left[2M(2M-u_x)-(1-\eta/2)(1+\mu)^2\right]. \tag{16}$$

A solution exists of the form $u_x = A\ \mathrm{sech}^2(\lambda\tau)$, where

$$\lambda^2 = \left(1+\frac{(5\mu-3)}{2(1+\mu)}\eta\right)(M^2 - \frac{1}{4}(1-\eta/2)(1+\mu)^2) \tag{17}$$

and $A = 2\lambda^2/M$. We also have

$$\zeta = \frac{2\lambda}{M}\tanh(\lambda\tau) - M\tau. \tag{18}$$

The normalized y component of the transverse magnetic field is, for $\mu = 1$ and small η,

$$B_y = 2(1+\eta/4)\lambda\ \mathrm{sech}(\lambda\tau)\cos(\eta\tau/4), \tag{19}$$

with

$$\lambda^2 = M^2 - 1 + M^2\eta/2, \tag{20}$$

so for the oscillatory behaviour to be prominent, as in the Figure below, we require $\lambda^2 > 0$ and $\lambda < \eta/4$, leading to

$$1 - \eta/4 < M < 1 - \eta/4 + \eta^2/8. \tag{21}$$

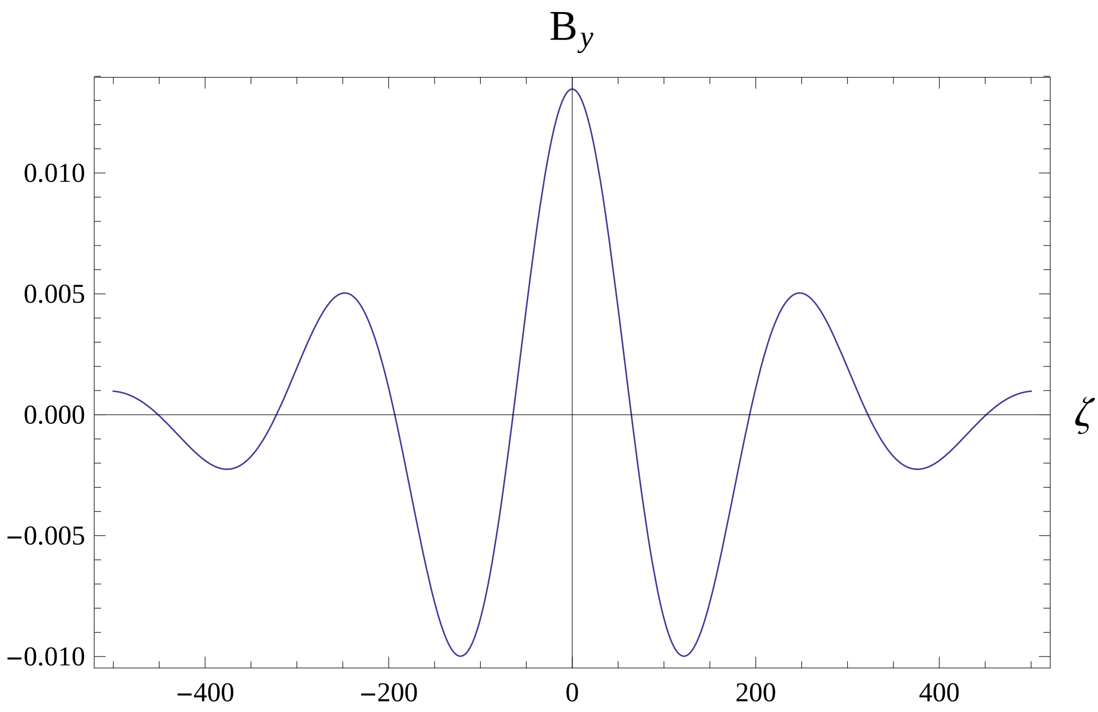

Figure 1: For nonzero η, the normalized component B_y of the transverse magnetic field is shown against the normalized ζ coordinate, for $m_e = m_p$, $\eta = 0.1$, $M = 0.9747$.

Conclusions

A nonlinear equation for solitary waves in a pair plasma with some charge residing on heavy dust particles has been derived, under the assumption of negligible dust velocities. An anaytic solution has been found illustrating the existence of oscillations within the solitary wave envelope, which do not occur if the dust is absent.

References

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