

Physics driven scaling laws for similarity experiments

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Similarity experiments are conceived to check on existing tokamak facilities, characteristics of scenarios found on other devices or planned for new machines. The possibility of doing similarity experiments is linked to the physics to be tested and it gives in any case partial views which can be found in integrated way only on the planned devices. The paper presents scaling laws obtained to check pedestal physics, MHD limits or L-H transitions and ELM behaviour, as well as bulk plasma confinement. The scaling laws are given in terms of plasma density (n) and temperature (T), current (I_p) and magnetic field (B), versus major radius (R) and aspect ratio ($A=R/a$, R =major radius, a =minor radius). For obtaining the scaling of heating power (P_{heat}) the IPB(y,2) confinement scaling is used leading to a strong dependence of P_{heat} upon the geometry and aspect ratio. The paper is organized as follows: in sec.1 a short introduction to scaling laws for similarity experiments; in sec.2 the derived scaling laws useful to test physics hypothesis limited to bulk plasma confinement, pedestal confinement and stability and beta limits; in addition, notes on scaling for edge similarity are outlined; in sec.3 general trends detected in the scaling laws and conclusions are given.

1. Introduction

Similarity in dimensionless parameters [1-4] can be used to extrapolate from existing to planned tokamak devices. To be completely rigorous, this would require identity in not only the known set of dimensionless plasma physics parameters (ρ^*_T , v^* , β_p) but also similarity of plasma cross-section shape (including identity of aspect ratio A), heating power deposition, and poloidal to toroidal field ratio (measured by q_{95}). In this rigorous form only few combinations of devices are in principle capable of truly equivalent operation, with the closest approach being Alcator C mod in conjunction with existing “mid-size” devices like DIII-D or ASDEX Upgrade. A broader range of comparisons and extrapolations are however possible, if assumptions concerning the dominant physics effects are made to reduce the imposed

$$\text{aspect ratio } A = \frac{R}{a} = \frac{\text{major radius}}{\text{min radius}};$$

$$\text{beta toroidal } \beta_T \approx \frac{nT}{B_T^2};$$

$$\text{beta poloidal } \beta_p \approx \frac{nT}{B_p^2} = \frac{n a^2}{I_p^2};$$

$$\text{rho star toroidal } \rho_i^* \approx \frac{V_{Ti}}{a^* \omega_{cTi}} \approx \frac{T^{1/2}}{a B_T}$$

$$\text{rho star poloidal } \rho_i^* \approx \frac{V_{Ti}}{a^* \omega_{cPi}} \approx \frac{T^{1/2}}{I_p}$$

$$q = \text{safety factor} \approx \frac{R B_T}{A^2 I_p}$$

$$\nu^* = \text{collisionality} \approx n R q T^{-2} A^{3/2}$$

Table I – Dimensionless parameters used to derive scaling laws. (n = plasma density, T = plasma temperature, $B_{T,p}$ = toroidal (poloidal) magnetic field, I_p = plasma current, $\omega_{cTi}/2\pi$ = ion toroidal cyclotron frequency, $\omega_{cPi}/2\pi$ = ion poloidal cyclotron frequency)

plasma. On this basis, scaling laws can be derived of dimensional physics parameters (n , T , I_p , B_T) upon the major radius R and the aspect ratio A . Three hypotheses (named hyp1, 2 and 3) are analyzed corresponding to a selection of dimensionless parameters and related to 1. Confinement of bulk plasma (hyp1); 2. Pedestal confinement and ELM physics (hyp2); 3. MHD stability and beta limits (hyp3). To derive the scaling law of the input heating power the ELMy H-mode IPBy2 scaling [6] $\tau_E IPB(y,2) \propto I_p^{0.93} B^{0.15} n^{0.41} P_{heat}^{-0.69} R^{1.97} (a/R)^{0.58} M^{0.19} k^{0.78}$ is

used together with the relation $P_{heat} \propto \frac{Ra^2 knT}{\tau_E}$.

2. Derivation of the scaling laws

2.1. Scaling laws for bulk plasma confinement (hyp1)

We assume that bulk plasma confinement can be described by the toroidal beta, collisionality

$$n \approx R^2 A^2$$

$$T \approx R^{1/2} A^{7/4}$$

$$I_p \approx R^{1/4} A^{1/8}$$

$$B \approx R^{5/4} A^{15/8}$$

$$P_{heat} \approx R^{0.74} A^{4.36}$$

Table II Scaling for bulk plasma similarity

and normalized ion Larmor radius. The motivation of this choice is related to the H-mode IPB(y,2) scaling law itself, which can be expressed as $\tau_E \propto \tau_{Bohm} \rho^{*-0.70} \beta^{-0.90} \nu^{*-0.01}$ ($\tau_{Bohm} = a^2 B/T$). This choice has been used in studies of core transport similarity between JET and JT-60U [7] in particular for the optimized scenario (monotonic magnetic shear) and also for some pedestal identity

study [8]. Taking fixed the previous parameters, the scaling obtained is reported in Table II. To get some insight on the consequences of these dependences we could consider similarity

experiments between devices with equal major radius, but different aspect ratio. Moving from low to high aspect ratio all the plasma parameters must be increased (only the plasma current I_p remains nearly constant) : the heating power and magnetic field (as deduced from $IPB(y,2)$ confinement) must be increased by $\Delta P_{heat} / P_{heat} = 40\%$ and $\Delta B / B = 17.5\%$ for an increase of aspect ratio of $\Delta A / A = 10\%$.

2.2. Scaling laws for pedestal and ELM dynamics(hyp2)

The pedestal width (Δw) has been recently characterized [9] by the scaling with the beta poloidal ($\Delta w \approx \beta_p^{1/2}$), as well as by the bootstrap current ($I_{bs}/I_p \approx A^{-1/2} \beta_p$), while the ELM

$$\begin{aligned} n &\approx R^2 A^2 \\ T &\approx R^{1/2} A^{7/4} \\ I_p &\approx R^{1/4} A^{7/8} \\ B &\approx R^{5/4} A^{23/8} \\ P_{heat} &\approx R^4 A^{0.87} \end{aligned}$$

Table III scaling laws for pedestal similarity

dynamics has a strong dependence upon the pedestal collisionality[10]. This suggests of taking as dimensionless parameters for pedestal similarity the beta poloidal, the poloidal Larmor radius and the pedestal collisionality as well as the safety factor. The derived scaling laws are given in Table III. To test pedestal similarity between devices with the same major radius and moving from low to high aspect ratio the heating power, magnetic field and plasma current must be increased by $\Delta P_{heat} / P_{heat} = 8.7\%$, $\Delta B / B = 28.5\%$ and $\Delta I_p / I_p = 9\%$ for an increase of aspect ratio of $\Delta A / A = 10\%$.

2.3. Scaling laws for MHD stability and beta limit(hyp3).

The beta limit and MHD stability can be characterized by the beta toroidal or beta normalized and by the poloidal normalized Larmor radius which is a scale length linked to the pedestal

$$\begin{aligned} n &\approx R^2 A^4 \\ T &\approx R^{1/2} A^{11/4} \\ I_p &\approx R^{1/4} A^{11/8} \\ B &\approx R^{5/4} A^{27/8} \\ P_{heat} &\approx R^{0.76} A^{6.14} \end{aligned}$$

Table IV scaling law for MHD stability and beta taking fixed (q, β_T, ρ_p, v^*)

pressure gradient and its stability. Taking the set (q, β_T, ρ_p, v^*) as the set of dimensionless parameters to be fixed, the scaling obtained is shown in Table IV. Using this scheme to test pedestal similarity between devices with the same major radius and moving from low to high aspect ratio the heating power, magnetic field and plasma current must be increased by $\Delta P_{heat} / P_{heat} = 61\%$, $\Delta B / B = 33.7\%$ and $\Delta I_p / I_p = 13.7\%$ for an increase of aspect ratio of $\Delta A / A = 10\%$. If instead the similarity is done taking fixed the set (q, β_N, ρ_p, v^*) the scaling obtained is slightly different and it is shown in Table V. In this case the relevant change of dependence is in the heating power needed to scale the scenario:

$$\begin{aligned} n &\approx R^2 A^3 \\ T &\approx R^{1/2} A^{9/4} \\ I_p &\approx R^{1/4} A^{9/8} \\ B &\approx R^{5/4} A^{25/8} \\ P_{heat} &\approx R^{3.97} A^{3.5} \end{aligned}$$

Table V scaling law for MHD stability and beta taking fixed (q, β_N, ρ_p, v^*)

the power must be increased by 35% (instead of 61%) increasing the aspect ratio by 10%.

2.4. Edge similarity

Following ref.[5] the plasma edge region differ from the core since atomic physics effects

$$n \approx R^{-1} A^{3/2}$$

$$I_p \approx A^{-1}$$

$$B \approx R^{-1} A$$

$$P_{heatsep} \approx R A^{1/2}$$

Table VI scaling law for edge similarity

taking fixed (q, T, ρ^*, v^*)

play an important role. Therefore assuming that binary collisions are dominant, the temperature(T) can be assumed as an important parameter for similarity while beta, being quite low, can be neglected. In this case the set of parameters kept constant in the similarity can be (ρ^*, T, v^*, q) . The scaling obtained is shown in Table VI. In this case the $P_{heatsep} \approx n T^{3/2} R^2 A^{-1}$ is the heating flux through the separatrix. To be noted that the parameter P/R has some dependence on the aspect ratio. The Greenwald density $n_G = I^2/a^2 \approx R^{-2}$ and the Power threshold for L-H transition[11] ($P_{thrL-H} \approx n^{3/4} B R^2$) scales as $P_{thrL-H} \approx R^{1/4} A^{17/8}$.

3. General trends detected in the scaling laws and conclusions.

The general trend detected can be expressed saying that the aspect ratio (A) has an important role in scaling for the P_{heat} needed to realize the scaled scenarios : $P_{heat} \sim R^\alpha A^\eta$, where $1 \leq \eta \leq 6$. In particular high exponents ($3.5 \leq \eta \leq 6$) in the aspect ratio A are obtained for experiments scaled to study MHD limits and bulk plasma confinement (hyp1 and 3). The scaling of equivalent $Q = P_{fusion}/P_{heating}$ results in similar behaviour $Q \sim R^\alpha A^\eta$, $1 \leq \eta \leq 5$. Other general trends are related with the dependence of plasma parameters (n, T, I_p , B) : $(n, T, B) \sim R^\alpha A^\eta$, $2 \leq \eta \leq 4$, while $I_p \sim R^\alpha A^\eta$, $\eta \sim 1$. In conclusion, strong dependence upon the aspect ratio is derived on similarity for bulk, pedestal and edge plasmas. In particular, scaling for similarity experiments between devices with equal major radius require substantial increase of the heating power whenever moving from low to high A.

References

- [1] B B Kadomtsev - Tokamak Plasma : a complex physical system - IOP Publishing(1992)
- [2] J W Connor and J B Taylor , Nucl Fusion 17(1977)1047
- [3] T C Luce, CC Petty and J G Cordey- Plasma Phys Controlled Fusion 50(2008) 043001
- [4] K Lackner - Comments Plasma Phys Contr Fusion 13(1990)163
- [5] K Lackner - Comments Plasma Phys Contr Fusion 15(1994)359
- [6] ITER Physics Basis NF 39(1999)2208
- [7] X.Litaudon et al. NF 51(2011)073020
- [8] G. Saibene et al. NF 46(2004)A195
- [9] P.B.Snyder et al. Phys Plasmas 16(2009)056118
- [10] M.N.A.Beurskens et al. Plasma Phys Contr Fus 51(2009)12405
- [11] F Ryter NF 37(1996)1217