Nonlinear dust-acoustic solitary waves in strongly coupled plasmas

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Introduction

In 1986 Ikezi [1] predicted that a dusty plasma can enter the strongly coupled regime due to the high charge number and low temperature of the dust. Here, the coupling parameter, $\Gamma \gg 1$ where we have

$$\Gamma = \frac{Z_d^2 e^2}{4\pi\varepsilon_0 k_B T_d a_d}, \tag{1}$$

with Z_d, T_d and $a_d = n_d{}^{-1/3}$ being the charge number, temperature and mean interparticle distance of the dust particles, respectively. There have subsequently been many approaches used to theoretically study dust-acoustic waves in strongly coupled plasmas, but the model on which this paper is based is the fluid approach presented by Gozadinos *et al.* [2], which was developed to numerically model crystalline dusty plasmas under microgravity conditions. In their paper they formulated an equation of state for this regime given by

$$P_\star \simeq \frac{N_{nn}}{3}\Gamma k_B T_d n_d (1+\kappa)\exp(-\kappa), \tag{2}$$

where N_{nn} is the number of nearest neighbours that determine the dusty plasma's structure and κ is the lattice parameter, defined as the mean interparticle distance $n_d^{-1/3}$, divided by the dynamical Debye screening length, λ_D such that

$$\kappa = \frac{1}{\sqrt[3]{n_d}\lambda_D}, \qquad \lambda_D = \sqrt{\frac{\varepsilon_0 k_B T_i T_e}{e^2(n_i T_e + n_e T_i)}}, \tag{3}$$

where T_s and n_s are the temperature and number density of species $s = e, i$, respectively. This model, although originally developed for crystalline plasma structures, has recently been applied as an approximation to the equation of state for strongly coupled plasmas near to the liquid-crystal phase transition. This theory is seen to be in excellent agreement with the experimental observations of the linear wave mode, as elegantly demonstrated by Yaroshenko *et* *al.*, for example Fig. 5 in Ref. [3]. By considering the form of Eqns. (1) and (2), an effective electrostatic 'temperature' was defined such that

$$k_B T_\star = \frac{N_{nn} Z_d^2 e^2}{12\pi\varepsilon_0} \sqrt[3]{n_d}(1+\kappa)\exp(-\kappa). \quad (4)$$

In doing so, they demonstrated that this model predicts the transition to an effective thermal mode at high wavenumbers, which was observed in experimentally obtained dispersion curves, that the dust kinetic temperature, $k_B T_d$ alone was not able to explain. It was shown that the electrostatic repulsion of similarly charged dust particles produces an effect similar to that of a temperature, with a magnitude greater than $k_B T_d$ by typically a few orders of magnitude [3].

In this paper, we investigate nonlinear dust-acoustic solitary waves by a derivation of the Korteweg-de Vries (KdV) equation, accounting for strong coupling between the dust particles using the electrostatic temperature approach of Yaroshenko *et al.* [3]. We note that Eqns. (3)-(4) show that the electrostatic temperature is a function of the dust, ion and electron densities. For Maxwellian distributed electrons and ions, whose densities are functions of electrostatic potential, Φ such that $n_e = n_{e0}\exp(e\Phi/k_B T_e)$ and $n_i = n_{i0}\exp(-e\Phi/k_B T_i)$, we therefore have $T_\star \equiv T_\star(n_d, \Phi)$. In the vicinity of the wave, n_d and Φ are seen to vary, so $T_\star$ is also a dynamically varying quantity. Uniquely, in the derivation of the KdV equation, we account for this by introducing the concept of electrostatic temperature perturbations.

Fluid Model

We use a normalised fluid model such that

$$\frac{\partial n}{\partial \tilde{t}} + \frac{\partial}{\partial \tilde{x}}(nu) = 0, \quad (5)$$

$$n\left(\frac{\partial u}{\partial \tilde{t}} + u\frac{\partial u}{\partial \tilde{x}}\right) = n\frac{\partial \phi}{\partial \tilde{x}} - \frac{\partial (nd)}{\partial \tilde{x}}, \quad (6)$$

$$\frac{\partial^2 \phi}{\partial \tilde{x}^2} \approx (n-1) + c_1\phi + c_2\phi^2 + c_3\phi^3, \quad (7)$$

where d is the normalised electrostatic temperature. The coefficients in Poisson's equation are calculated to be

$$c_1 = 1, \qquad c_2 = -\frac{(1-\mu)(1-\mu\theta^2)}{2(1+\mu\theta)^2}, \qquad c_3 = \frac{(1-\mu)^2(1+\mu\theta^3)}{6(1+\mu\theta)^3},$$

where we have $\mu = n_{e0}/n_{i0}$ and $\theta = T_i/T_e$. We note that the c_3 coefficient is not normally required in the derivation of the KdV equation. However, this coefficient appears in the derivation of the electrostatic temperature perturbations, so its definition is presented here. In Eqns. (5)-(7) we have normalized the temperature, length, time, velocity, density and electrostatic potential terms by $T_0 = \frac{Z_d^2 n_{d0} T_i T_e}{n_{i0} T_e + n_{e0} T_i}$, $\lambda_{D0} = \sqrt{\frac{\varepsilon_0 k_B T_0}{n_{d0} Z_d^2 e^2}}$, $\omega_{pd}^{-1} = \sqrt{\frac{\varepsilon_0 m_d}{n_{d0} Z_d^2 e^2}}$, $v_0 = \sqrt{\frac{k_B T_0}{m_d}}$, $n_0 = n_{d0}$ and $\Phi_0 = \frac{k_B T_0}{Z_d e}$, respectively.

Reductive Perturbation Method

To obtain the KdV equation, in which we balance nonlinearity with dispersion, we first stretch the space and time coordinates in Eqns. (5)-(7) in the style of Washimi and Taniuti [4] such that $\tilde{\xi} = \varepsilon^{1/2}(\tilde{x} - v\tilde{t})$ and $\tilde{\tau} = \varepsilon^{3/2}\tilde{t}$, where ε is a small parameter. We then expand the dynamic quantities about their equilibrium values such that

$$n = 1 + \varepsilon n_1 + \varepsilon^2 n_2, \qquad u = \varepsilon u_1 + \varepsilon^2 u_2, \qquad \phi = \varepsilon\phi_1 + \varepsilon^2\phi_2, \qquad d = d_0 + \varepsilon d_1 + \varepsilon^2 d_2,$$

where the normalised equilibrium electrostatic dust temperature, d_0 is expressed as

$$d_0 = T_{\star 0}/T_0, \qquad k_B T_{\star 0} = \frac{N_{nn} Z_d^2 e^2}{12\pi\varepsilon_0}\sqrt[3]{n_{d0}}(1+\kappa_0)\exp(-\kappa_0), \qquad \kappa_0 = \frac{1}{\sqrt[3]{n_{d0}}\lambda_{D0}}.$$

The normalised electrostatic temperature perturbations d_1 and d_2 are found to be

$$d_1 = d_{11}n_1 + d_{12}\phi_1, \qquad d_2 = d_{21}n_2 + d_{22}\phi_2 + d_{23}n_1^2 + d_{24}n_1\phi_1 + d_{25}\phi_1^2$$

with the coefficients d_{ij} being

$$d_{11} = d_{21} = \frac{d_0}{3}\frac{1+\kappa_0+\kappa_0^2}{1+\kappa_0}, \qquad d_{12} = d_{22} = -d_0 c_2 \frac{\kappa_0^2}{1+\kappa_0}, \qquad d_{23} = \frac{d_0}{18}\frac{\kappa_0^3 - 3\kappa_0^2 - 2\kappa_0 - 2}{1+\kappa_0},$$

$$d_{24} = -\frac{d_0}{3}c_2\frac{\kappa_0^2(\kappa_0 - 1)}{(1+\kappa_0)}, \qquad d_{25} = -\frac{d_0}{2}\left(3c_3 - c_2^2\kappa_0\right)\frac{\kappa_0^2}{1+\kappa_0}.$$

Korteweg-de Vries Equation and Parametric Investigation

To lowest order, the expansion of Eqns. (5)-(7) gives an equation for the linear phase velocity such that $v = \sqrt{1 + d_0 + d_{11} - d_{12}}$, as well as relations between the perturbations such that $n_1 = -\phi_1$ and $u_1 = -v\phi_1$. Taking the expansion to the next lowest order leads to the KdV equation such that

$$\frac{\partial\phi_1}{\partial\tilde{\tau}} + \tilde{A}\phi_1\frac{\partial\phi_1}{\partial\tilde{\xi}} + \tilde{B}\frac{\partial^3\phi_1}{\partial\tilde{\xi}^3} = 0 \tag{8}$$

in which we have

$$\tilde{A} = -\frac{(1 + 2v^2 + 2\alpha c_2 + 2\gamma)}{2v}, \qquad \tilde{B} = \frac{\alpha}{2v},$$

where

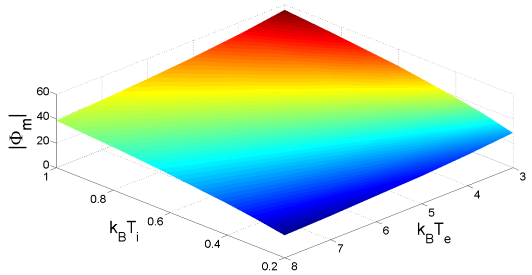
$$\alpha = 1 - d_{12}, \qquad \gamma = d_{11} - d_{12} + d_{23} - d_{24} + d_{25}.$$

Eqn. (8) can be solved by separation of variables, giving a solution of

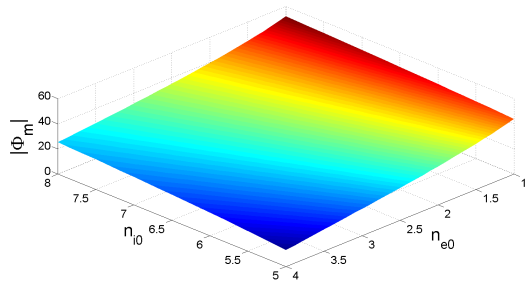
$$\phi_1(\tilde{\xi},\tilde{\tau}) = \phi_m \mathrm{sech}^2\left[\frac{\tilde{\xi} - \tilde{U}\tilde{\tau}}{\tilde{\Delta}}\right]. \tag{9}$$

where $\phi_m = \frac{3\tilde{U}}{\tilde{A}}$, $\tilde{\Delta} = \sqrt{\frac{4\tilde{B}}{\tilde{U}}}$ and $\tilde{U}$ are the amplitude, width and normalised velocity of the nonlinear solitary wave in the moving reference frame, respectively.

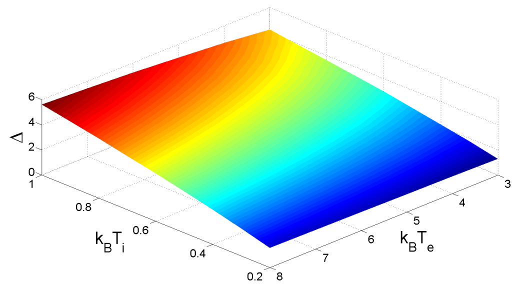
By transforming back to the laboratory frame, in Figure 1 we show how the amplitude, $|\Phi_m|$ and width, Δ of the solitary waves vary with equilibrium plasma conditions. Here, we have based the parameters on those measured by Bandyopadhyay *et al.* [5], with the dust charge number being consistently calculated across the parameter range using an orbit motion limited approach [6].



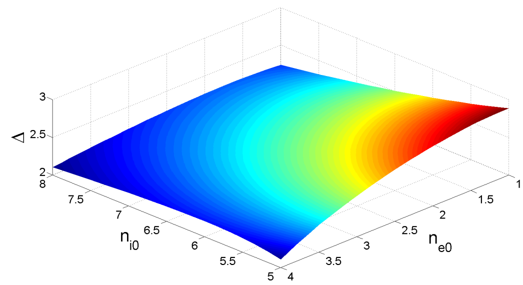
(a) The amplitude of the solitary wave, $|\Phi_m|$ (mV) as a function of the electron and ion temperatures (eV)



(b) The amplitude of the solitary wave, $|\Phi_m|$ (mV) as a function of the electron and ion densities ($\times 10^{13}$ m^{-3})



(c) The dependence of the width of the solitary wave, Δ (mm) on the electron and ion temperatures (eV)



(d) The dependence of the width of the solitary wave, Δ (mm) on the electron and ion densities ($\times 10^{13}$ m^{-3})

Figure 1: The attributes of solitary waves travelling at 2 mm/s above sound speed. In all plots, the dust particle mass, $m_d = 1 \times 10^{-13}$ kg, the dust particle radius, $r_d = 0.2$ μm, $m_e/m_i = 1.37 \times 10^{-5}$ and $N_{nn} = 12$. For (a), (c), we have $n_{i0} = 7 \times 10^{13}$ m^{-3} and $n_{e0} = 4 \times 10^{13}$ m^{-3} and for (b), (d), we have $k_B T_i = 0.3$ eV and $k_B T_e = 8$ eV.

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