Stability of electron-driven fishbones

A. Merle\textsuperscript{1} (antoine.merle@cea.fr), J. Decker\textsuperscript{1}, X. Garbet\textsuperscript{1}, R. Sabot\textsuperscript{1}, Z. O. Guimarães-Filho\textsuperscript{2}

\textsuperscript{1} CEA, IRFM, F-13108 Saint-Paul-lez-Durance, France.
\textsuperscript{2} Aix-Marseille Univ., IIFS-PIIM UMR 7345, F-13397, Marseille, France.

Introduction

In tokamaks, MHD instabilities such as electron-driven fishbone modes are frequently observed in the presence of electron heating and current-drive [1, 2, 3]. These modes provide a good test bed for the linear theory of fast-particle driven instabilities as they exhibit a very high sensitivity to the details of both the equilibrium and the electronic distribution function. In Tore Supra, electron-driven fishbones are observed during LHCD-powered discharges in which a high-energy tail of the electronic distribution function is created [4]. Although the destabilization of those modes is related to the fast particle population, the modes are observed at relatively low frequency. A basic estimate of resonant electrons energy assuming resonance with the toroidal precession frequency of barely trapped electrons falls in the thermal range [4]. In this work, we analyze the stability of internal kink modes in the presence of energetic electrons using the code MIKE which has been recently modified to fully account for the resonance of modes with passing particles. For circulating electrons, the parallel motion contributes to the resonance condition. Our analysis with ECRH-like distribution functions shows that a relatively low frequency is compatible with a mode that is mostly driven by energetic barely passing electrons, even in the absence of a wide $q \sim 1$ region [5].

The linear dispersion relation

The classic MHD energy principle can be modified to account for the contribution of fast particles. If the fast particle beta is negligible compared to the global plasma beta and there is a position $r_s$ such that $q = 1$ at $r = r_s$ (or $q$ reaches a minimum value close to 1 at $r = r_s$), then the structure of the $(m = -1, n = 1)$ internal kink mode is not affected by the presence of fast particles, and the usual step-function for the radial MHD displacement is recovered. The fishbone dispersion relation is obtained [2, 6]

$$\delta \hat{W}_f + \delta \hat{W}_h(\omega) = \delta I(\omega),$$

where $\delta \hat{W}_f$ and $\delta \hat{W}_h$ account for the contribution of the plasma bulk and plasma hot component, respectively, and $\omega$ is the complex frequency of the mode. $\delta I$ is called the inertial term and its form depends on the relevant physics inside the inertial ($q = 1$) layer. In this paper, the case of a
single inertial layer is considered. If the magnetic shear \( s = r q' / q \) at \( r = r_s \) does not vanish then the expression for \( \delta I \) is,

\[
\delta I = i |s| \sqrt{\frac{\omega (\omega - \omega_i)}{\omega_A}} \sqrt{1 + \left( \frac{0.5 + 1.6}{\sqrt{\epsilon_i}} \right)}
\]

(2)

where diamagnetic effects [7] are included as well as kinetic effects of thermal ions [8], and where \( \omega_i \) is the ion diamagnetic frequency, \( \omega_A \) the Alfvén frequency, all those quantities being evaluated at the position \( r_s \) of the inertial layer.

The fast particle contribution to the fishbone dispersion relation can be written \( \delta \hat{W}_h(\omega) = \delta \hat{W}_k(\omega) + \delta \hat{W}_{f,h} \) where \( \delta \hat{W}_k \) accounts for all resonant effects between the fast particles and the mode and \( \delta \hat{W}_{f,h} \) is the contribution of fast particles to the interchange term of the usual MHD energy. Their expressions are

\[
\delta \hat{W}_k = -\frac{\pi}{2} \frac{\mu_0}{B_0^2} \langle E^2 \Omega_d \omega \frac{\partial F_h}{\omega - q (\omega_b \delta_p - \omega_d)} \rangle_{\mathbf{x},\mathbf{p}}, \quad \delta \hat{W}_{f,h} = \frac{\pi}{2} \frac{\mu_0}{B_0^2} \langle R_p \Omega_d E \frac{\partial F_h}{\partial r} \rangle_{\mathbf{x},\mathbf{p}}
\]

(3)

with \( \langle \alpha \rangle_{\mathbf{x},\mathbf{p}} = V^{-1} \int d^3 \mathbf{x} d^3 \mathbf{p} \alpha F_h \). In equation (3) \( \mathbf{x} \) is the position in real space, \( \mathbf{p} \) in momentum space, the integral is limited to the space inside the \( q = 1 \) surface of total volume \( V = 2 \pi^2 r_s^2 R_p \), \( F_h \) is the distribution function of fast particles of mass \( m_h \) and charge \( e_h \), \( \omega_b \) and \( \omega_d \) are the bounce-frequency and toroidal drift frequency of fast particles, \( \tilde{\Omega}_d \) is defined by \( \omega_d = (q E \tilde{\Omega}_d) / (e_h B_0 R_p r) \) where \( E \) is the energy of the particle, \( \omega_s = q / (e_h B_0 r) \). Finally \( \delta_p \) is equal to 1 for passing particles and 0 for trapped particles. Expression (3) was obtained by neglecting the effect of collisions on the perturbed electronic distribution.

Expression (3) implies that the resonance occurs when the frequency of the mode is close to the toroidal drift frequency of the particles and that the source of the instability lies in the radial gradient of the distribution function. Unlike their ion counterpart, electron-driven fishbones go unstable for an inversed radial profile of the electronic distribution function \( \partial_r F_s > 0 \). Resonant electrons must have a reversed toroidal drift \( \tilde{\Omega}_d < 0 \). Hence only barely trapped or passing electrons can resonate [2].

The term associated to \( \delta_p \) comes from the parallel part of the usual resonance condition of the Landau effect \( \omega = \langle \mathbf{k} \cdot \mathbf{v} \rangle \) where the brackets stand for orbit-averaging. For trapped particles, the parallel velocity averages to 0 over one poloidal orbit, while for passing particles one has \( \langle v_{||} \rangle = q R_p \omega_b \) and \( k_{||} = (q - 1) / (q R_p) \). This term is usually neglected based on the argument that \( q \) is close to 1 (\( k_{||} \) small) or restricting the study to barely passing particles (\( \omega_b \) small).
**Influence of different classes of particles**

A previous study [5] shows that particles with different pitch-angles have different influence on the stability of the mode. For trapped particles, one can distinguish deeply trapped particles which have a negative value of \( \omega_d \) and which are stabilizing and barely trapped particles which have a reversed toroidal drift motion and are destabilizing. For these particles, the frequency of the mode at threshold is close to \( \omega_d \). Passing particles can be differentiated by the value of the parameter \((q - 1) \omega_b / \omega_d\). If it is much lower than one then their destabilizing influence is similar to barely trapped particles, if it is comparable to one their influence is destabilizing but at frequencies lower than \( \omega_d \). If it is greater than one then they destabilize the mode as well, but mostly through the non-resonant part of \( \delta \hat{W}_h \). Particles with \((q - 1) \omega_b / \omega_d \gg 1\) have almost no influence on the mode.

**Simulation Parameters**

To model ECRH-heated plasmas, the fast electron distribution function is chosen to have a Maxwellian momentum dependence with an anisotropic temperature, 

\[
f(p, \xi_0, r) = \tilde{f}(r) \exp \left( -\frac{p^2}{2m_e k_B T(\xi_0)} \right) \tag{4}\]

with \( \xi_0 = v_{[\theta=0]} / v \). The function \( T(\xi_0) \) is an extension of the 2-temperature model such that 

\[
\frac{1}{T(\xi_0)} = \frac{\xi_0^2}{T_\parallel} + \left( 1 - \xi_0^2 \right) / T_\perp \text{ (see [5] for details).}
\]

For the safety factor profile, we construct a parabolic profile between \( r = 0 \) and \( r = r_i \), followed by a plateau between \( r = r_i \) and \( r = r_s \). The value at the center is noted \( q_0 \), the value of the plateau \( q_i \) and is close to 1. For \( r > r_s \), \( q \) rises up to the edge, the magnetic shear at \( r = r_s^+ \) is noted \( s \). This type of profile has been proposed for sawtoothing plasmas where partial reconnection can occur and a plateau in \( q \) appear near \( q = 1 \). It is of interest in this study since electron-fishbones have been observed in-between sawteeth on various tokamaks such as HL-2A[9].

**Results**

We study the evolution of the frequency and growth rate of the mode when the fraction of fast particles \( n_h / n_e \) is increased. We start from a situation where the mode is stable in the absence of energetic electrons (\( \delta \hat{W}_f > 0 \)).

To highlight the effect of the \( \langle k \parallel v \rangle \) term in the resonance condition, we compare the results of simulations based on equations (1), (2) and (3) with the ones obtained by setting \( k \parallel = 0 \) in the computation of \( \delta \hat{W}_h \). The simulations are run for two different values of \( r_i / r_s \). The first one corresponds to a wide plateau with \( r_i / r_s = 0.5 \), the second to a narrow one with \( r_i / r_s = 0.95 \).
Figure 1 shows the results. When $k_{\parallel}$ is set to 0, the value of $n_h$ at threshold is strongly increased, as well as the frequency of the mode. Another remarkable result when finite $k_{\parallel}$ effects are included is that the destabilization of the mode persists when the width of the $q \sim 1$ plateau is reduced. In this case, the main effect is a depletion of the population of barely passing particles with $(q - 1) \omega_b/\omega_d \ll 1$ which had a similar influence as barely trapped particles.

**Summary**

The original fishbone dispersion relation is extended to account for the transit frequency in the resonance with passing particles in the zero-orbit width limit and in circular concentric magnetic geometry. The inclusion of finite $k_{\parallel}$ effects breaks the symmetry of the resonance condition for passing particles. Using the MIKE code with analytical unidirectional distributions, we show that the finite $k_{\parallel}$ effects lead to both a reduced mode frequency and a lower fast-particle density at the threshold. This result is not limited to cases where the $q$-profile exhibits a wide $q \sim 1$ region.

**References**


