FIELD LINE ESCAPE PATTERNS IN TOKAMAKS WITH POLOIDAL DIVERTOR

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Abstract

Introduction

Computational codes that solve Grad-Shafranov equation and describe magnetic surfaces in tokamaks with poloidal divertors, are time-consuming and generate a great computational cost. Thus, to study the field line dynamics near the separatrix, this procedure is quite hard and a simple model can be introduced to reproduce the main field line dynamical properties.

The magnetic field of two infinite wires was introduced in 1978 to calculate the width of the scrape-off layer of a divertor tokamak [1]. Equilibrium models using two and three wires were used to study the chaotic layer formation due to error fields in single-null (one X-point) [2] and double-null (two X-points) [3] divertor tokamaks, respectively. The particle drift orbits were obtained in [4] for a model of three circular coils in the presence of non-axisymmetric magnetic field perturbations. A comparison between the chaotic layer formation using a two-wire model and a discrete map was investigated in [5,6]. A single-null equilibrium plasma model, composed by three infinite wires, was used in [7] in order to implement a method of canonical mapping near the separatrix in the presence of magnetic perturbations created by pairs of loop coils with opposite flowing currents.

In this work we use a set of five parallel infinite wires conducting electric currents to describe equilibrium magnetic surfaces of double-null divertor tokamaks. This number of wires is sufficient to reproduce the cross-section of an arbitrary choice of magnetic configuration, depending on the wires positions and their currents. We show the model versatility by creating surfaces with similar topology of ITER surfaces. This simple model reproduces quite well ITER like magnetic topology capturing some aspects of the magnetic field line dynamics of the plasma region near the separatrix, but not in the plasma core.

To study the field line escape patterns near the separatrix, we introduce perturbations on the equilibrium created by resonant magnetic perturbations by error fields due to asymmetries on the machine [2,3]. We solve numerically the perturbed magnetic field line differential equation to investigate the effect of magnetic perturbations on the stochastic layer formation and deposition patterns at the divertor plates.

Model

The magnetic field equations, produced by a set of \( N \) infinite wires, is written in rectangular coordinates as
\[
\begin{aligned}
\mathbf{B}_i(x,y) &= \sum_{n=1}^{N} \frac{\mu I_n}{2\pi} \left( \frac{y-y_i}{(x-x_i)^2 + (y-y_i)^2} \right), \\
\mathbf{B}_j(x,y) &= \sum_{n=1}^{N} \frac{\mu I_n}{2\pi} \left( \frac{-y-y_i}{(x-x_i)^2 + (y-y_i)^2} \right),
\end{aligned}
\]

where \( I_n \) and \((x_n, y_n)\) are the electric current and position of the \( n \)-th wire. In this work we use \( N=5 \) wires: one of them to represent the plasma current \((I_P)\) and the other four used to reproduce a desirable plasma shape. The position \((x_P, y_P)\) of \( I_P \) coincides with the magnetic surface axis.

The \( z \)-component of the toroidal field, generated by the external coils, is given by \( B_z = R_0 B_0/x \), where \( R_0 \) is the major radius of the torus (or geometric axis) and \( B_0 \) is the toroidal field at the geometric axis. To model the toroidal geometry, we must introduce a periodicity in the \( z \)-direction. The magnetic surfaces are defined as the set of points of constant \( \Psi \), where

\[
\Psi(x,y) = \frac{\mu I}{2\pi} \ln \left( \prod_{n=1}^{N} \frac{r}{r_n} \right)
\]

is the magnetic flux passing through a plane extending from the magnetic axis out to the generic point \((x,y)\) along \( z \)-direction. The distance of the generic point \((x,y)\) from the position \((x_n, y_n)\) of the conductor \( I_n \) is defined by

\[
r_n = \left( (x-x_n)^2 + (y-y_n)^2 \right)^{\frac{1}{2}}.
\]

Interpreting the \( z \)-coordinate as the independent variable we can write the differential equation of the field lines \( B_0 x \, dl = 0 \) as

\[
\frac{dx}{dz} = \frac{B_x}{B_z}; \quad \frac{dy}{dz} = \frac{B_y}{B_z}.
\]

To study the magnetic configuration with the wires model we integrate numerically eq. (4).

### Numerical Results

The major parameters of ITER equilibrium are listed in ref. [8]. In our model, to represent magnetic surfaces of ITER configuration, we choose the parameters of currents and positions of wires listed in Table 1.

<table>
<thead>
<tr>
<th>( n )</th>
<th>( x_n )</th>
<th>( y_n )</th>
<th>( I_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (plasma)</td>
<td>6.41 m</td>
<td>0.513 m</td>
<td>15 MA</td>
</tr>
<tr>
<td>2</td>
<td>3.72 m</td>
<td>-7.58 m</td>
<td>15.9 MA</td>
</tr>
<tr>
<td>3</td>
<td>3.2 m</td>
<td>8.6 m</td>
<td>16.28 MA</td>
</tr>
<tr>
<td>4</td>
<td>2.45 m</td>
<td>0.513 m</td>
<td>-5.69 MA</td>
</tr>
<tr>
<td>5</td>
<td>10 m</td>
<td>0.515 m</td>
<td>-4.6 MA</td>
</tr>
</tbody>
</table>

Table 1: Wire positions and current values of our model.

Wire 1 represents the plasma current and its position coincides with the magnetic surfaces axis. The major role of wires 2 and 3 is to create the lower and upper X-points, respectively. The positions of lower and upper X-point are in different separatrixes. The negative currents in wires 4 and 5 compress the left and right sides of magnetic surfaces and allow us to control the elongation of surfaces. Although each wire has a principal influence in specific characteristic of the surfaces, the equilibrium configuration depends on the parameters values of ITER parameters.
as a whole. Using these values we obtain the magnetic surfaces of Figure 1(a) with
the surfaces internal to the active separatrix. Figure 1(b) (extracted from ref. [8])
shows the correspondent magnetic configuration of ITER. We can verify a good
agreement between the model and the ITER magnetic surfaces.

![Figure 1](image1)

We use a model of error fields proposed in [7] for the perturbing magnetic field:
\[
\mathbf{B}^{(p)} = \varepsilon B_0 \nabla \chi
\]  
where
\[
\chi = R_0 \exp \left[ \frac{(x - x_p)}{R_0} \right] \cos \left[ \frac{z}{R_0} \right]
\]

In Figure 2a, for perturbation parameter \( \varepsilon = 2 \times 10^{-4} \), we observe the formation of
a chaotic layer. The initial conditions are in the region internal to the ideal separatrix.
The magnetic field lines are no longer closed and eventually reach the inner part of
the chamber following the lines leaving the X-point. These lines finish at the divertor
plate. In Fig. 2b and 3 it is possible to observe KAM islands immersed in the chaotic
sea. These islands play and important role on the field line escape.

![Figure 2](image2)

Figure 2: (a) Stochastic Layer due the Error Field. The red square indicates the amplification shown in
Fig. 2b.
Conclusions

The versatility of our model, with five wires, is exemplified recreating quite well an equilibrium magnetic configuration close to ITER equilibrium with a double-null divertor. Considering predicted error field perturbations, we obtain numerically the magnetic field lines to study the characteristics of the chaotic layer of one single-null divertor.

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REFERENCES