Comparison of turbulent transport models using 1.5D trasnport code TASK/TR

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Introduction

In tokamak plasmas anomalous transport caused by turbulence is dominant for radial transport. Several transport models have been proposed to explain the turbulent transport. The strong non-linearity of turbulent transport models sometimes causes numerical instability in transport simulation as a stiff problem. In this case, we have to take a very small time step for stable calculation, and the simulation requires a very long computation time. Recently Pereverzev has proposed a numerical scheme which substantially improves the numerical convergence [1].

The purpose of this study is to introduce the improved numerical scheme into the TASK/TR module and to examine several transport models, including stiff ones, by comparing the calculated temperature profiles with experimentally observed ones.

TASK/TR

In our simulation of tokamak plasmas, we use the TR module of the integrated transport analysis code TASK[2]. TASK is an integrated transport analysis code which is being developped in Kyoto University, and it has several modules to calculate other phenomenon in fusion plasmas, such as transport, MHD equilibrium and wave propagation.

The TASK/TR module assumes 1-D diffusive transport on the grounds that the flux surface variation of physical quantities will typically be small. The basic equations of the TASK/TR module are three diffusion equations: that of density, energy and poloidal magnetic flux with respect to the normalized radius ρ. TASK/TR has been revised and implemented using the finite element method (FEM).

In the present analysis, we use the following turbulent transport models: CDBM model [3], GLF23 model [4], mixed Bohm/gyro-Bohm model [5], MMM95 model [6], and MMM7_1 model [7].

Stable numerical scheme

Some turbulent transport models, such as GLF23, are numerically stiff and unstable for large step. When using these models in transport simulations, we have to reduce the size of the time step considerably, which make the calculation very time-consuming. Recently, Pereverzev has proposed a stable numerical scheme for stiff transport models.

We consider a simple diffusion equation in cylindrical coordinates

$$\frac{\partial T}{\partial t} = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\chi\frac{\partial T}{\partial \rho}\right) + S. \tag{1}$$

In Fig.1 the solid line shows the gradient-flux relation of a typical stiff transport model. The flux increases abruptly when the absolute value of the gradient of T exceeds a threshold η_{cr}. If we use the linear approximation from the origin χ_{eff} for non-linear iteration, the solution oscilates and the iteration does not converge.(See Fig.1(left).) This makes the calculation very time-consuming and unstable.

Now, we consider the modified equation

$$\frac{\partial T}{\partial t} = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\hat{\chi}\frac{\partial T}{\partial \rho} - \rho\bar{V}T\right) + S, \tag{2}$$

where $\hat{\chi} = \chi_{\mathrm{eff}} + \bar{\chi}$, $\bar{V} = \bar{\chi}(\partial T/\partial\rho)/T$, and $\bar{\chi}$ is an arbitrary additional diffusive coefficient. This equation is identical to Eq.(1) mathematically. If we evaluate $\hat{\chi}$ at $t + \Delta t$ implicitly while $\bar{V}$ is calculated at t ,where δt is the size of the time step, and choose appropriate value of $\bar{\chi}$, the iteration converges rapidly and the problem becomes stable. The gradient-flux relation in Eq.(2) is shown by the dashed line in Fig.1(right). However, it should be noted that there is a numerical error inevitably, until the system reaches at steady state.

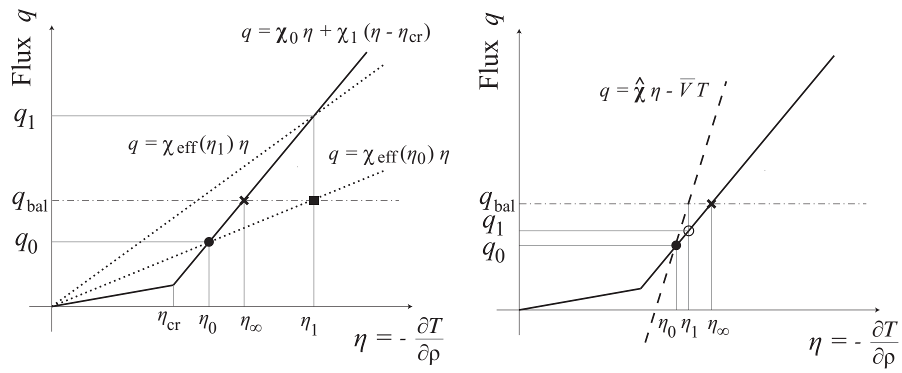

Figure 1: The concept of stable numerical scheme

The numerical scheme for FEM

Implementing the new stable scheme in the transport formulated by FEM requires careful thought for the modification of the equation in FEM. In order to obtain a steady state solution form Eq.(2) which is close enough to the one obtained from Eq.(1), the additional term in the equation in FEM must vanish at steady state.

Now, we consider a typical transport equation

$$\frac{\partial}{\partial t}(V'X) = \frac{\partial}{\partial \rho}\left[V'\langle|\nabla\rho|^2\rangle\chi\frac{\partial X}{\partial \rho} - V'\langle|\nabla\rho|\rangle VX\right]. \tag{3}$$

The first term of the right-hand side is a diffusion term and the second is a convective term. Other terms are omitted for simplicity. X is the variable to be solve and, V', $\langle|\nabla\rho|\rangle$, and $\langle|\nabla\rho|^2\rangle$ are metric quantities. Supposed that the diffusion coefficient χ and the convective velocity V are evaluated on a half-integer grid; then the RHS of this transport equation can be discritised by FEM as follows:

$$\frac{\partial}{\partial t}(V'\mathbf{X}_i) = -\overleftrightarrow{R}_\chi \mathbf{X}_i + \overleftrightarrow{R}_V \mathbf{X}_i, \tag{4}$$

where $\mathbf{X}_i$ is a variable vector corresponding to the i-th element. $\overleftrightarrow{R}_\chi$ and $\overleftrightarrow{R}_V$ are element matirices arising from the first and the second term of the RHS respectively as follows:

$$\overleftrightarrow{R}_\chi = \begin{bmatrix} \frac{\chi_i}{2h_i}(G_{2i}+G_{2i+1}) & -\frac{\chi_i}{2h_i}(G_{2i}+G_{2i+1}) \\ -\frac{\chi_i}{2h_i}(G_{2i}+G_{2i+1}) & \frac{\chi_i}{2h_i}(G_{2i}+G_{2i+1}) \end{bmatrix}, \tag{5}$$

$$\overleftrightarrow{R}_V = \begin{bmatrix} -\frac{V_i}{6}(2G_{1i}+G_{1i+1}) & -\frac{V_i}{6}(G_{1i}+2G_{1i+1}) \\ \frac{V_i}{6}(2G_{1i}+G_{1i+1}) & \frac{V_i}{6}(G_{1i}+2G_{1i+1}) \end{bmatrix}, \tag{6}$$

where $G_1 = V'\langle|\nabla\rho|\rangle$, $G_2 = V'\langle|\nabla\rho|^2\rangle$ and h_i is the width of each element. When the arbitrary additional diffusive coefficient $\bar{\chi}_i$ is added to χ_i, the corresonding $\bar{V}_i$ must be added to V_i. As discussed above, these addtional terms must vanish in Eq.(4) at steady state. Therefore, Eq.(4), (5),and (6) give a relation,

$$\overleftrightarrow{R}_\chi \mathbf{X}_i = \overleftrightarrow{R}_V \mathbf{X}_i$$

$$-\frac{\bar{\chi}_i}{2h_i}(G_{2i}+G_{2i+1})X_i + \frac{\bar{\chi}_i}{2h_i}(G_{2i}+G_{2i+1})X_{i+1} = \frac{\bar{V}_i}{6}(2G_{1i}+G_{1i+1})X_i + \frac{\bar{V}_i}{6}(G_{1i}+2G_{1i+1})X_{i+1} \tag{7}$$

Assuming an expression for $\bar{V}$ as follows,

$$\bar{V}_i = \bar{\chi}_i \frac{X^p_{i+1} - X^p_i}{h_i X_{\mathrm{ave}\,i}}, \tag{8}$$

where superscript p denotes the previous time step, we can determined the expression for $X_{\mathrm{ave}\,i}$ as

$$X_{\mathrm{ave}\,i} = \frac{\bar{\chi}^p_i}{\bar{\chi}_i} \frac{(2G_{1i}+G_{1i+1})X_i + (G_{1i}+2G_{1i+1})X_{i+1}}{3(G_{2i}+G_{2i+1})} \tag{9}$$

with the use of Eq.(7) and the steady state condition $X_i = X^p_i$.

Simulation results

We have carried out preliminary calcualtions for reversed magnetic shear operation using trasnport models described above. Plasma parameters are major radius $R_0 = 3.0$m, minor radius $a = 1.0$m, toroidal field $B_0 = 3$T, elipticity $\kappa = 1.5$. The plasma current is ramped-up from 1MA at 1sec to 3MA at 2sec, and RF-like heating starts at 2sec. The results are shown in Fig.2. Weak ITB profiles are found in the results with CDBM model and MMM95 model.

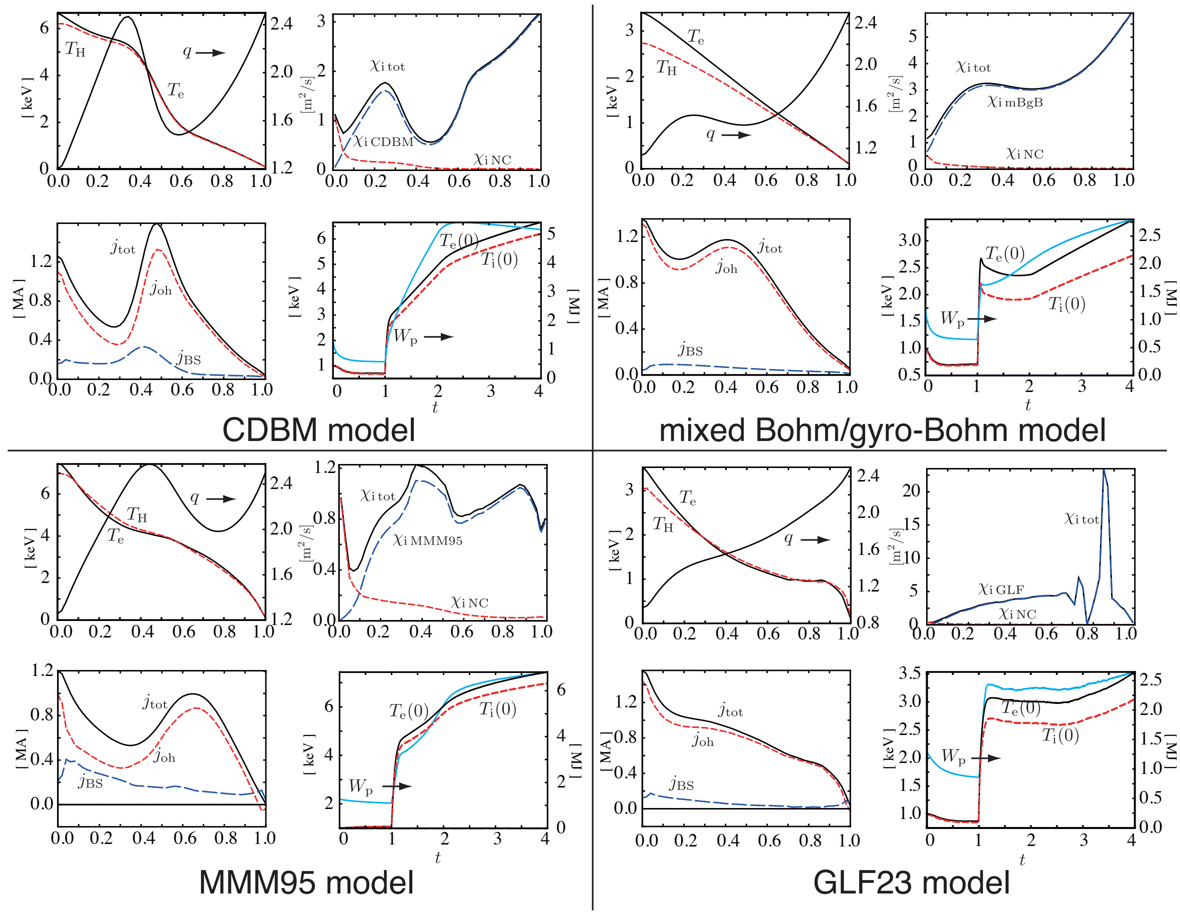

Figure 2: Comparison of profiles from each turbulent transport model

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