

Progress on a code for numerical calculation of 3D MHD equilibria with flow and FLR effects

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Introduction

Flow and nonsymmetric effects can play an important role in plasma equilibrium and energy confinement, causing profile pedestals and the loss of good flux surfaces. Further, in the presence of a profile pedestal, FLR effects may be important. We have previously developed a concept for a code to calculate such equilibria, called KITES (Kyoto ITERative Equilibrium Solver) [1]. The algorithm is based largely on the PIES code [2], extended to include the additional physics, but there are two significant changes: (1) using physical coordinates instead of magnetic coordinates to handle the topology analysis and magnetic differential equations; and, (2) using the vector potential as an unknown rather than the magnetic field. These differences are expected to make the code more robust at the cost of speed.

Progress on the KITES code has been promising. The model for handling the magnetic field topology has been completed, allowing the work-in-progress code to be used to calculate force-free equilibrium. Here, we present the algorithm and sample results for a simple case.

Algorithm

The force-free equilibrium problem is determined by specifying the shape of the vacuum vessel, the applied vector potential $\mathbf{A}_{\text{applied}}$, a function relating the induced plasma current to the magnetic field, and an initial guess for the plasma vector potential $\mathbf{A}_{\text{plasma}}$. The code primarily uses a cylindrical (R, ϕ, Z) coordinate system, with Fourier decomposition in ϕ for some tasks. The code iterates over the following steps until the difference between successive guesses for $\mathbf{A}_{\text{plasma}}$ is sufficiently small:

1. Calculate the magnetic field $\mathbf{B} = \nabla \times (\mathbf{A}_{\text{applied}} + \mathbf{A}_{\text{plasma}})$.
2. Calculate the topology of \mathbf{B} and calculate the corresponding field line label ζ .
 - 2.1. Find the magnetic axis.
 - 2.2. Examine several trial lines to get a general sense of the topology of the field.
 - 2.3. Accurately find any separatrices.
 - 2.4. Determine ζ on each surface and interpolate between surfaces.
3. Calculate the updated plasma current $\mathbf{J}_{\text{plasma}}^{(+)}$ based on \mathbf{B} , ζ , and the given function.
4. Calculate the updated guess for the plasma magnetic field $\mathbf{A}_{\text{plasma}}^{(+)} = [\nabla \times]^{-2} \mathbf{J}_{\text{plasma}}^{(+)}$.

Step 2 is the most complicated and deserves further attention. For a given starting point, we follow the field line using a 4th order Runge-Kutta method in the ϕ direction along with 4th order interpolation of $R\mathbf{B}/B_\phi$ in R and Z . To find the magnetic axis, we start at the geometric axis, follow the field line for many circuits, then use an steepest-descent method to bring the spread of points on the plane $\phi = 0$ down to near zero.

For a given field line, we determine the behavior of that line by looking at the puncture points with $\phi = \text{constant}$ planes. For a given set of puncture points on a plane, for each point, we calculate the minor radius r and poloidal angle θ relative to the magnetic axis on that plane, then order the points in θ using a merge sort. If the function $r(\theta)$ has an unacceptably large $\Delta r/\Delta\theta$, we take this to indicate that it is not a good flux surface. Otherwise, if all $\Delta\theta$ are sufficiently small, we take the function $r(\theta)$ to represent a good flux surface. If neither of these criteria are met, we either follow the line further or give up on determining the behavior of the line, depending on assorted criteria.

Between a known good surface and a known bad surface, we search for a separatrix. We iteratively guess the location of the separatrix using a bisection method, erring towards the known bad surface: it is typically much faster to determine that a bad surface is bad than to determine that a good surface is good.

On each good flux surface, we take ζ on that field line to be:

$$\zeta(\text{field line}) = \sqrt{V_{\text{good}}(\text{field line})/V_{\text{good}}(\text{LCFS})} \quad (1)$$

where $V_{\text{good}}(\text{field line})$ is the total volume of good surfaces inside that surface and $V_{\text{good}}(\text{LCFS})$ is the total volume of good surfaces inside the last closed flux surface. For magnetic islands and stochastic regions, ζ is equal to the value on the separatrix.

Step 4 deserves some comments. We take the boundary condition $\mathbf{A}_{\text{plasma}}^{(+)}(\infty) = 0$ and calculate $\mathbf{A}_{\text{plasma}}^{(+)}$ using the Biot-Savart law. Using a Fourier representation in ϕ , this process requires a double integral over all R and Z , but only a single integral over the modes. Step 4 would be the dominant cost of each iteration for an (R, Z) mesh of size 500×500 or larger, but step 2 is the dominant cost for an (R, Z) mesh of size 200×200 or smaller.

Sample Calculation

We now present results from a sample calculation. First, define $s(R, Z)$, $f(R, Z)$, and $\theta(R, Z)$:

$$s \equiv \sqrt{(R-4)^2 + Z^2}, \quad f \equiv \left[s^2 (1-s^2)^2 \right]^2, \quad \theta \equiv \text{atan2}(Z, R) \quad (2)$$

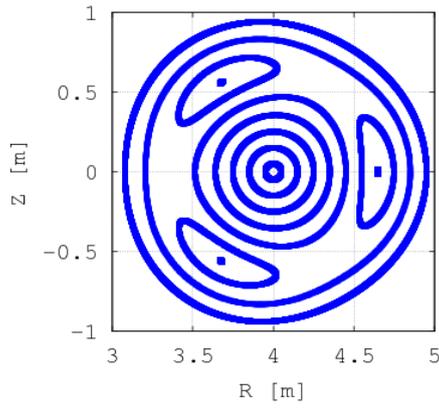


Figure 1. Topology of $\mathbf{A}_{\text{applied}}$ at $\phi = 0$.

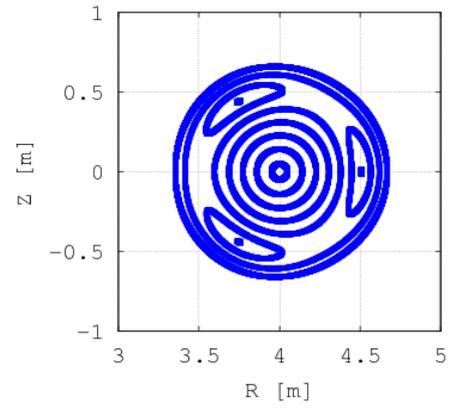


Figure 2. Topology of $\mathbf{A}_{\text{applied}}$ at $\phi = \pi$.

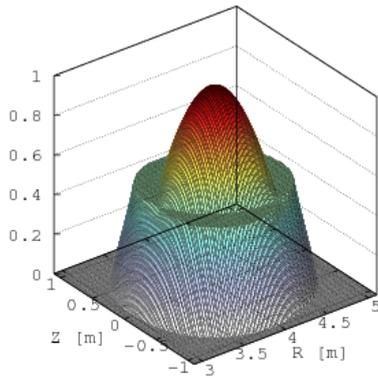


Figure 3. $1 - \zeta$ of $\mathbf{A}_{\text{applied}}$ at $\phi = 0$.

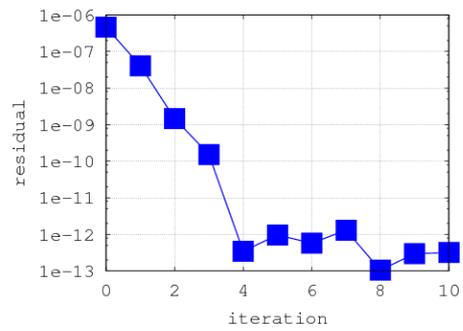


Figure 4. Convergence of calculation.

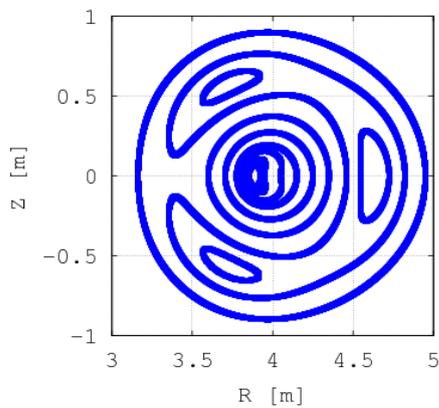


Figure 5. Topology of converged \mathbf{A} at $\phi = 0$.

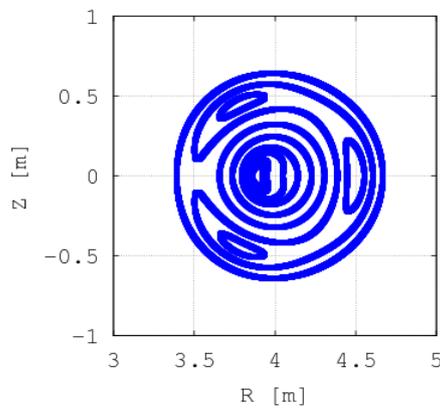


Figure 6. Topology of converged \mathbf{A} at $\phi = \pi$.

The components of the applied vector potential were:

$$A_{\text{applied}, R} = [Z - 0.2s^2 \sin(\theta) \cos(\phi)] / R \quad (3)$$

$$A_{\text{applied}, Z} = [0.2s^2 \cos(\theta) \cos(\phi)] / R \quad (4)$$

$$A_{\text{applied}, \phi} = [10^{-3}f + 5 \times 10^{-3}f^2 \cos(3\theta)] / R \quad (5)$$

The applied vector potential produces magnetic islands and a plasma minor radius which varies with ϕ . The topology of the applied field is shown in Figs. 1 and 2. The value of ζ on the plane $\phi = 0$ for the applied field is shown in Fig. 3. Note that $1 - \zeta$ is plotted, for clarity. Also note that this vector potential requires an applied current within the plasma apart from the induced current. The function for the induced plasma current was:

$$\mathbf{J}_{\text{plasma}}^{(+)} = 5 \times 10^{-5} (1 - \zeta^2)^2 \mathbf{B} \quad (6)$$

The vacuum vessel had a square cross section from $R = 3 \sim 5\text{m}$ and $Z = -1 \sim 1\text{m}$. The computational domain extended 0.1m outside of this range in each direction, and used a 101×101 grid in (R, Z) . 10 modes (0 through 9) were used for Fourier representation in the toroidal direction. The initial guess for the plasma vector potential was zero.

Using a single core of an Intel Core i5-2400 3.1GHz processor, step 2 required about 97 seconds per iteration and step 4 required about 5 second per iteration; the per-iteration costs of other parts of the code were negligible. The calculation converged from an initial residual of $\sim 10^{-6}$ to $\sim 10^{-12}$ in 4 iterations; the convergence is shown in Fig. 4. The equilibrium field topology is shown in Figs. 5 and 6. Observe that the flux surfaces have been significantly deformed.

Conclusions

We have completed modules for determining the magnetic field topology and calculating the field line label. The work-in-progress KITES code has been used to calculate 3D force-free equilibria. The next major milestone is the calculation of equilibrium with pressure, which requires solving a magnetic differential equation along each field line.

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References

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