Optimization of the LHD-type planar-axis stellarator configuration using a small number of Fourier modes

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The three-dimensional structure of stellarator has a large variety of free parameters for magnetic configurations. Consequently, lots of different types of stellarators were built in the world and many of them are still in operation producing valuable data for the confinement physics studies. Owing to the high performance of recent computers, more complicated designs of stellarator configuration have been made based on the analysis of Fourier modes of the geometrical shape of plasma boundary to achieve a further optimization of the confinement property, which are called the advanced stellarators. However, new problems were pointed out that the more advanced configuration design requests the more complicated coil shape, which sometimes cause bad effects on the coil manufacturing and the construction cost.

In this paper, we made an optimization of planar-axis stellarator configuration in the limited range of Fourier modes that were obtained in the process of analyzing the real configurations in the LHD experiments. LHD is a largest stellarator experiment in the world with superconducting magnets and has been producing a large amount of technical and scientific data which are valuable for both stellarator and tokamak researches. From the experiments, we selected three nominal magnetic configurations (with different magnetic axis positions: R_{ax}) having clearly different confinement properties. For analyzing the boundary shape, we use the Fourier decomposition of the three-deminsional torus boundary used in the VMEC equilibrium solver [1], which is expressed as following formulas:

$$R(\theta, \phi) = \sum_{m,n} rbc(m, n) \cdot \cos(m\theta - n\phi)$$
$$Z(\theta, \phi) = \sum_{m,n} zbs(m, n) \cdot sin(m\theta - n\phi)$$

When the plasma pressure and the plasma current profiles are given, we can calculate three-dimensional equilibria using these coefficients of boundary shape. The difference in the stability and confinement properties of LHD magnetic configurations with different magnetic axis positions should be characterized by these coefficients. We analyzed three configurations with magnetic axis positions: R_{ax} = 3.6 m (inward shifted configuration), 3.75 m (standard configuration) and 3.9 m (outward shifted configuration). The inward shifted configuration with a stronger vertical field gives favorable orbits of the trapped particles and the outward shifted configuration gives deeper magnetic well which generally contributes to the better MHD stability. As we are interested in finding which Fourier modes are essential for giving the different characteristics of LHD configurations, we first modify three configurations to have the same major radius and the same aspect ratio in order to eliminate the effects of these parameters. It can be done by unifying the *rbc*(0, 0), *rbc*(1, 0) and *zbs*(1, 0) coefficients, taking the values from $R_{ax} = 3.75$ m configuration (standard one) [2]. Now we have three LHD configurations of the same toroidal geometries (major radius and the aspect ratio). We then try to find which Fourier modes are effective to give very different characteristic of three configurations with different magnetic axis positions. It is done by eliminating the modes of small amplitudes step by step until the characteristics of the favorable orbits and the magnetic well will disappear. The final result is shown in Fig. 1 as variations of amplitudes of essential Fourier modes for three configurations, which are necessary to give the typical characteristics of LHD configurations for inward and outward axis shift.

Inward shifted configuration

Outward shifted configuration

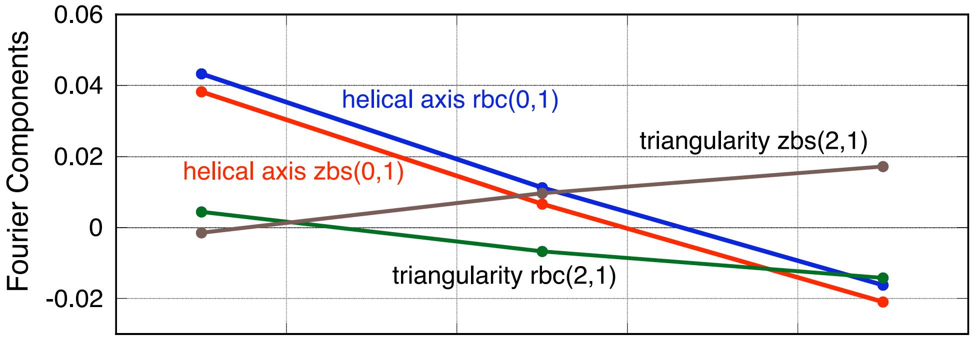


Fig. 1 Essential Fourier components for three nominal LHD configurations. Fourier components expressing major radius, aspect ratio and helicity are not shown. Cross sections of magnetic surfaces are shown for inward and outward shifted configurations for emphasizing the effects of triangularity components (components of m= 2 are doubled for both cases).

Small amplitudes of two sets of Fourier components are essential in determining the confinement properties of LHD configurations. The *rbc*(0, 1) and *zbs*(0, 1) work to introduce the helical axis structure in the LHD configurations. Since m is 0 for these components, it does not change boundary shapes at all. The important characteristics of a good confinement of trapped particles in the inward shifted configuration is given by the helical axis structure. The triangularity, namely D-shape structure works to give a good MHD property for outward shifted configuration. The cross sections in Fig. 1 is exaggerated by artificially increasing the Fourier components with m=2 by two times (for both inward and outward shifted cases). The appearances of (0, 1) mode and (2, 1) mode in the configuration control with the magnetic axis shift is understandable from the viewpoint of the so-called sigma optimization. In terms of the Fourier modes of magnetic field ripple structure, these two modes are sidebands of the fundamental helical mode (1, 1). They must be in the appropriate polarities to make the magnetic axis shift more effective.

However, we try to combine these two Fourier modes with an opposite phase to find a new configuration (denoted by NC hereafter) having the favorable trapped particle orbits and the magnetic well simultaneously. As is shown in Fig. 1, the helical axis structure gives the favorable orbits for the inward shifted configuration without the assistance of the triangularity shape. We then apply the triangularity shape on the inward shifted configuration with the helical axis structure. Figure 2 shows a comparison of the profiles of specific volumes (magnetic well) as a function of the normalized minor radius (r/a) for two vacuum configurations of inward shifted (denoted by IS hereafter) and NC configurations. They have both magnetic hill for the vacuum configurations but the hill is moderated for NC. The confinement property of trapped particles is evaluated by calculating the effective ripples using NEO code [3]. Figure 3 shows the profiles of effective ripples

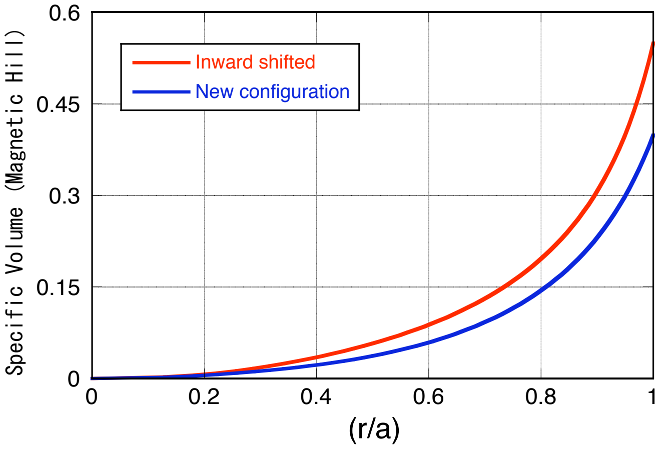


Fig. 2 Profiles of specific volume of IS and NC.

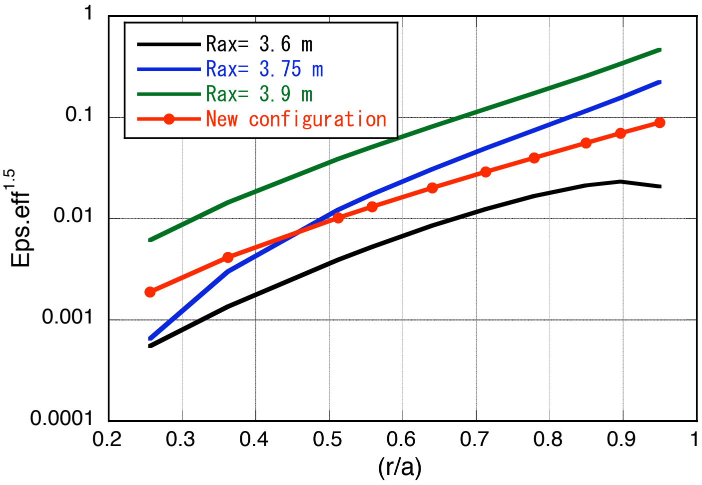
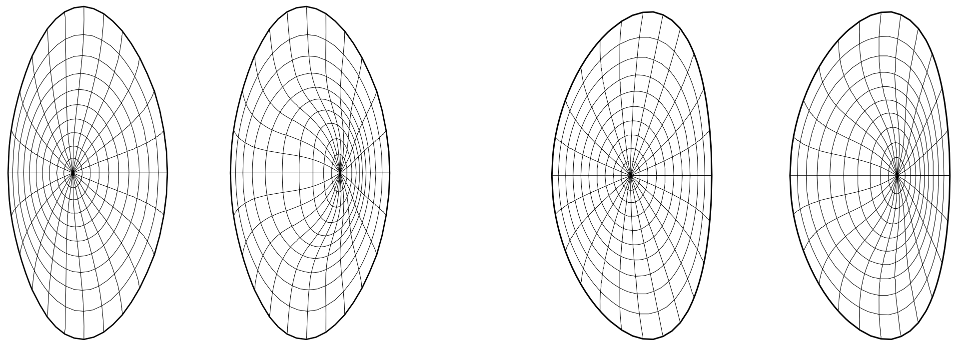


Fig. 3 Profiles of effective ripples for three LHD configurations and NC.

($\mathrm{Eps}_{\mathrm{eff}}{}^{1.5}$) for three configurations of LHD and NC. The neoclassical confinement of NC is between the R_{ax} = 3.75 m and 3.6 m configurations of LHD.



IC vacuum IC 2% beta NC vacuum NC 2% beta

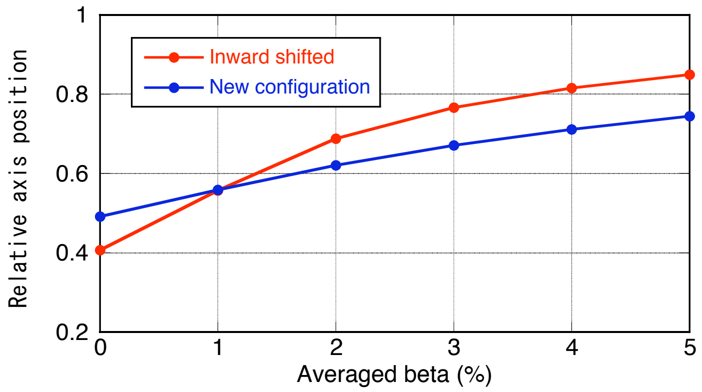


Fig. 4 Comparisons of vacuum and 2% beta equilibria for IC and NC and variations of relative axis positions agaist averaged beta.

Important characteristics of NC is the change of confinement properties in finite beta equilibria. A big problem of LHD configurations is a large Shafranov shift produced in high beta equilibria. Such a large Shafranov shift contributes to the MHD stability due to the magnetic well created by the magnetic axis shift. However it increases the deviation of magnetic surfaces from the trapped particle orbits and deteriorates the neoclassical confinement. Figure 4 shows the changes of equilibria between the vacuum and 2% averaged beta equilibria for IS and NC. The change of magnetic surface shapes for 2% beta is smaller for NC. Bottom graph shows the relative positions of magnetic axes (the position of axis between the innermost and outermost positions of LCMS of vertically elongated cross section) for IS and NC as a function of averaged beta. For NC, the shift is smaller compared to IS.

References

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