Multi-scale turbulence and electron heat transport

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A theoretical study of electron heat transport based on test particle approach is developed. Test particle approach means essentially to consider a given turbulence (described by a known spectrum or Eulerian correlation). The heat transport coefficient is evaluated using the “test particle expression”, as time integral of the Lagragian velocity correlation (LVC) or as derivative of electron mean square displacement. The LVC has to be determined for given Eulerian correlation (EC) of the fluctuating potential. The LVC determines the time dependent diffusion coefficient and it is also a measure of the statistical memory of the stochastic motion. A close agreement between the diffusion coefficient obtained from the flux of temperature fluctuations and the test particle diffusion coefficient exists if there is space-time scale separation between temperature fluctuations and average. Numerical simulations confirm this property [1, 2].

The transport coefficients for a realistic model of the EC, which includes the existence of large scale potential fluctuations, is obtained. This approach is in some sense independent and complementary to the self-consistent simulations. It can determines the regimes of transport as functions of turbulence statistical parameters, not as function of the gradients.

The diffusion coefficients are determined using a semi-analytical method, the decorrelation trajectory method [2, 3]. The later is based on a set of smooth trajectories determined by the Eulerian correlation of the turbulence.

1. Turbulence model

Low frequency turbulence (ITG, TEP, ETG) is characterized by similar shape of the spectum. It has two (symmetrical) maxima in the poloidal wave number $k_\theta = \pm k_0$ and zero amplitude around $k_\theta=0$. The main difference consists in the typical wave numbers: $k_\theta\rho_i<1$ for ITG, $k_\theta\rho_i\sim1$ for TEP and $k_\theta\rho_e\sim1$ for ETG. The Eulerian correlation (EC) corresponding to these spectrum has zero integral in the poloidal direction with negative damains and sometimes oscillatory behaviour determined by the dominant wave number. Also, the potential drifts with the diamagnetic velocity V_* (of electron or of ions, depending on the type of turbulence). A simple model of the EC with these properties that also includes anisotropy is

$$E(x,y,z,t) = A\exp\left(-\frac{x^2}{2\lambda_x{}^2} - \frac{|z|}{\lambda_z} - \frac{t}{\tau_1}\right)\frac{\partial}{\partial y}\left[\exp\left(-\frac{(y-V_*t)^2}{2\lambda_y{}^2}\right)\frac{\sin[k_0(y-V_*t)]}{k_0}\right] \tag{1}$$

where A is the amplitude of potential fluctuations, k_0 is of the order of $1/\lambda_y$, τ_1 is the correlation time and λ_i are the correlation lengths. We have considered a multi-scale spectrum of the stochastic potential, modelled by the superposition of two functions (1), one with small correlation lengths λ_{xl}, λ_{yl} and the other with much larger correlation lengths λ_{xL}, λ_{yL}. The subscripts L for the large scale and l for the small scale were introduced in the parameters. We have previously shown that the transport does not strongly depend on the details of the EC and thus this simplified model is adequate for determining the transport regimes. The model is rather complex and contains 12 parameters that describe the turbulence plus the diamagnetic velocity V_* and the parallel velocity v_z. The potential motion with the diamagnetic velocity, electron parallel motion and the time variation of the potential represents decorrelation (detrapping) mechanisms which hinder electron motion on the contour lines of the potential.

The diffusion regimes are analyzed and the conditions when they appear are identified. A rich class of anomalous diffusion regimes appears when trajectory trapping is effective, i.e. when the combined action of the decorrelation mechanisms is weak enough. Trapping influences not only the values of the diffusion coefficients but also their scaling laws. We have shown that it is possible to obtain transport coefficients that are completely different of the sum of the transport coefficients produced separately by the large scale turbulence and respectively small scale turbulence even when the spectrum is composed by two well separated parts. We have investigated three physically relevant domains. First domain corresponds to electron heat transport in small scale turbulence. This is an important step, which permitted to evaluate the importance of electron trapping or eddying in the ETG type turbulence. We have shown that this effect can appear and that in these conditions the transport is strongly modified. The second study evaluates the influence of the large scale stochastic potential on electron heat transport. We have shown that the transport coefficient can strongly increase due to the combined action of the small and large scale turbulence. The third study evaluates the effects of small scale turbulence on ion transport.

2. Electron transport in ETG type turbulence

The results obtained for the special EC (1) are different from those for a decaying EC in the quasilinear case when there is no trapping. The differences appear in the poloidal transport coefficient, which has reversed dependence on the characteristic decorrelation time τ_d compared with a normal (decaying) EC. The diffusivity decreases with the increase of τ_d and the transport becomes sub diffusive in the limit of infinite τ_d (no decorrelation). On the contrary, in the nonlinear regime characterized by the existence of trajectory trapping, the poloidal transport is super diffusive if there is no decorrelation and for weak decorrelation (large τ_d) the diffusion coefficients are large and have linear dependence on $\tau_{d.}$ The cause of this behavior is the rotation of the turbulence with the diamagnetic velocity. The poloidal transport means actually the transport along the average velocity of the potential. This component of transport does not affect electron heat loss, but it provides (as shown below) the mechanism of interaction with the large scale potential fluctuations.

The radial transport is strongly reduced due to potential rotation and also due to trapping, which contribute to the limitation of electron radial excursions. Turbulence rotation with V_* is equivalent with an average potential xV_*, which modifies the contour lines of the potential and reduces their radial extension and trapping keeps electron on the contour lines.

3. Effects of the large scale turbulence on the electron transport

Essentially, the large scale determines local average velocities V_L, which are randomly oriented in different regions. Their influence on electron heat transport depends on the ratio of the average velocity V_L and the amplitude of the electric drift produced by the small scale stochastic potential V_l.

When $V_L > V_l$ there is no small scale electron trapping and V_L determines a decorrelation mechanism. The local transport coefficients depend on the value of V_L and also on its orientation. The average on the large scale of the local transport coefficients (essentially the average over the orientation of the large scale velocity) leads to a global radial diffusion coefficient, which is in most cases a decreasing function of V_L.

When $V_L < V_l$ and the other decorrelation mechanisms are very weak, electron trajectory small scale trapping is effective. In this case, V_L determines a strong modification of the diffusion, which becomes non-isotropic with the diffusion coefficient along the V_L much larger than in the perpendicular direction. This increased diffusion is produced by the splitting of the probability of electron displacements in two parts, one remaining at the initial radial position and the other moving radially with an average velocity V_r. The stationary part accounts for trapped trajectories and the moving one for the free electrons, which have V_r larger than the radial component of $\mathbf{V}_L$ such as to maintain the Eulerian flux.

The above results are obtain for $V_*=0$. In the presence of poloidal rotation velocity, the transport becomes much more complicated due to the contribution of V_* to the decorrelation from both small and large scale potential. Large variations of the diffusion coefficient appears for small changes of the parameters, which are not easy to understand. The anisotropy of the turbulence also influences the electron heat diffusion. Due to the complexity of the model and to the large number of parameters clear transport regimes appear only in particular conditions.

4. Effects of the small scale turbulence on ion transport

Essentially, the contribution of the small scale can be represented as a collisional process with the diffusion coefficient D_l determined by the decorrelation process induced by the large scale velocity. Due to the gyro average of the small scale, D_l is much smaller than ion diffusion produced by the large scale potential. However, in the nonlinear condition when ion large scale trapping is important, D_l determines a strong increase of the ion effective diffusion with a value that is much larger than D_l.

5. Conclusions

The main conclusion of this work is that the electron heat transport in multi-scale turbulence can be much larger that in small scale turbulence. The physical process responsible for increased transport is electron trajectory trapping combined with the existence of a large scale velocity. Our results show that the transport processes in two-scale turbulence are complex and that strong nonlinear effects appear in the presence of trapping. They suggest that simple well defined scaling in the global parameters of the plasma and extrapolations cannot be obtained. These results represent the first theoretical (semi-analytical) evaluation of the electron transport in multiple scale turbulence. Our test particle approach brings a complementary contribution to the understanding of the complex processes of electron transport, which is mainly obtained from large scale numerical simulations.

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