New Approaches to Numerical and Experimental Studies of Fast Ion Distributions in Magnetically Confined Plasmas

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I. Introduction. Ion velocity distribution functions determine the basic parameters of a reactor, such as fusion reaction rate in plasma and the integral power. Both beam injection and wave heating create significant populations of suprathermal ions making a considerable or even a major contribution to fusion reaction rates. Since analytical results concerning the ion distribution functions provide an extra physical insight and constitute a reliable basis for verification of numerical codes and for the analysis of experimental results, it is desirable to obtain exact solutions where possible.

Recent results [1-3] contribute to the improvement of the physical basis of neutral beam heating and current drive. Analytical and semianalytical solutions were obtained using a practical dimensionless form of Boltzmann's kinetic equation assuming spatial homogeneity, azimuthal symmetry, and Maxwellian distributions of target plasma species. In contrast with formerly considered simplified equations with truncated collision terms [4-6], the exact Landau–Boltzmann collision operator was used in [1-3], which conserves the number of particles, nullifies the collision term at statistical equilibrium, and describes the Maxwellization process naturally observed in correct solutions.

II. Overview of earlier approaches. A wide variety of modern theoretical and experimental studies related to plasma heating and non-inductive current drive rely on 1970s physics of suprathermal ions represented by [5,6]. The simplified collision operator from [6] was used in the work [7] dedicated to evaluation of the efficiency of radiofrequency heating in tokamaks. The same simplified collision term was used to study the interaction between lower hybrid waves and energetic ions in [8] and in wave-particle interaction models, in particular, in the modeling of α-particle driven TAE in ITER-like plasma [9]. Analytical solutions [6] are used in Ph.D. theses [10,11] as a model of slowing-down of fast α-particles. In recent work [12] simplified steady-state solution [6] was used to describe fast ion distributions resulting from neutral beam injection heating. In [13,14] simplified analytical solutions [5,6] were mentioned as realistic equilibrium slowing-down distribution functions.

In [15] a simplified collision operator similar to [5,6] was used in an equation including the effects of orbit losses. In [16] various aspects of fast ion behavior are investigated by comparing the experimental data with computed energetic ion distributions [5]. Solutions [6] were used for the interpretation of fast-particle diagnostic data in studies of fast-ion transport induced by energetic particle modes [17].

The majority of modern numerical codes dealing with beam heating and current drive employ [4-6]. The DBEAMS module in the BALDUR code [18] uses a simplified collision operator [5,6]. Discussions of current drive by neutral beam injection in [19-21] are also based on [5,6]. Numerical codes considered in [22] as a basis for the integrated modeling for ITER, such as ONETWO [23], ACCOME [24], ASTRA [25], are using analytical solutions [6]. ONETWO code was applied to calculate the beam driven current in [26] and to calculate fusion reactivity in [27]. In NBEAMS module [28] of the ITER systems code SUPERCODE calculation of the fast ion distribution function and the current driven by the fast ions is also based on [6]. Solutions from [6] are also used in the analysis of current drive with neutral beams as a part of the physics basis for ARIES-ST (spherical torus) nuclear fusion power plant concept [29].

III. Comparison of analytical solutions. Previously known simplified approximate solutions [4-6], widely used, e.g. in [7-29], are inappropriate to describe the time evolution of high energy tails of the distribution and are physically inadequate at lower energies, where the Maxwellization process should be observable. Formulae given in [19-21] are not applicable to multi-ion-species plasma, do not describe the time evolution owing to the use of steady state solutions, assuming a delta-like rather than arbitrary angle distribution of the fast ion source, and not taking into account velocity diffusion effects. The formulae used in [19-21] are noticeably different from fast ion distribution functions in [4-6]. Stationary solutions of Boltzmann equation used in [19-21] as well as solutions in [5,6] were obtained using various simplified expressions for Coulomb collision operator. As shown in [2], it is preferable to use the exact collision term in order to preserve its physical properties.

Table I shows different expressions used for the Coulomb collision term in [5] and [6]. Let $\phi(\mathbf{r},\mathbf{v}) \equiv n_\alpha(\mathbf{r}) f_\alpha(\mathbf{v})$ [cm^{-6}s^3] be the phase space distribution function of test particles of species α. We are considering the kinetic equation, neglecting spatial inhomogeneity and electric field, with Landau-Boltzmann collision term and a monoenergetic source function. Bearing in mind plasma in magnetic field, we assume the test particle distribution function to be axially symmetric. Using spherical polar coordinates in test particle velocity space, let us introduce two new variables, namely dimensionless velocity magnitude $u = v/v_c$, where $v_c = \varepsilon v_{T_e}$, $\varepsilon = (m_e/m_\alpha)^{1/3}$, and pitch angle cosine $\zeta = \cos\vartheta$.

Table I. Expressions for the Coulomb collision term [2] in comparison with [5,6].

Function	Ref. [2]	$v_{T_i} \ll v \ll v_{T_e}$	Ref. [5]	Ref. [6]
$p(u)$	exact [2]	$\frac{2\varepsilon}{3\sqrt{\pi}}+\frac{\varepsilon Z^{(a)}}{2u^3}$	$\frac{2\varepsilon}{3\sqrt{\pi}}$ $+\frac{8}{9\sqrt{\pi}}\frac{1}{\varepsilon^2}\frac{m_e}{m_i}\frac{T_i}{T_e}\frac{1}{u^3}$	$\frac{2\varepsilon}{3\sqrt{\pi}}+\frac{\varepsilon Z^{(a)}}{2u^3}$
$q(u)$	exact [2]	$\frac{4u}{3\sqrt{\pi}}+\frac{Z^{(b)}}{u^2}$ $+\frac{4\varepsilon}{3\sqrt{\pi}}\frac{1}{u}-\frac{\varepsilon Z^{(a)}}{2u^4}$	$\frac{4u}{3\sqrt{\pi}}+\frac{1}{\varepsilon^3}\frac{m_e}{m_i}\frac{1}{u^2}$ $+\frac{8\varepsilon}{3\sqrt{\pi}}\frac{1}{u}$ $-\frac{16}{9\sqrt{\pi}}\frac{1}{\varepsilon^2}\frac{m_e}{m_i}\frac{T_i}{T_e}\frac{1}{u_0}\frac{1}{u^3}$	$\frac{4u}{3\sqrt{\pi}}+\frac{Z^{(b)}}{u^2}$
$r(u)$	exact [2]	$\frac{4}{\sqrt{\pi}}$	$\frac{4}{\sqrt{\pi}}+\frac{4\varepsilon}{3\sqrt{\pi}}\frac{1}{u^2}$ $+\frac{16}{9\sqrt{\pi}}\frac{1}{\varepsilon^2}\frac{m_e}{m_i}\frac{T_i}{T_e}\frac{1}{u^5}$	$\frac{4}{\sqrt{\pi}}$
$w(u)$	exact [2]	$\frac{Z^{(eff)}}{2u^3}+\frac{2\varepsilon}{3\sqrt{\pi}}\frac{1}{u^2}$	$\frac{m_i}{2m_\alpha}\frac{Z^{(eff)}}{u^3}$	$\frac{Z^{(eff)}}{2u^3}$
$f(u)$	exact [2]	$-\frac{S_0\tau_s}{2\pi v_c^3}\frac{1}{u^2}\delta(u-u_0)Z(\zeta)$	$-S\tau_s u_0\delta(u-u_0)Z(\zeta)$	$-\frac{S_0\tau_s}{v_c^3}\frac{1}{u^2}\delta(u-u_0)\delta(\zeta-\zeta_0)$

The working form of the steady state equation is

$$p(u)\frac{\partial^2\phi}{\partial u^2}+q(u)\frac{\partial\phi}{\partial u}+r(u)\phi+w(u)\frac{\partial}{\partial\zeta}\left(1-\zeta^2\right)\frac{\partial\phi}{\partial\zeta}=f(u)\,. \tag{1}$$

The dimensionless functions $p(u)$, $q(u)$, $r(u)$, $w(u)$, and $f(u)$ are given in Table I, where $Z^{(a)}=\frac{m_\alpha}{n_eT_e}\sum_i\frac{Z_i^2n_iT_i}{m_i}$, $Z^{(b)}=\frac{m_\alpha}{n_e}\sum_i\frac{Z_i^2n_i}{m_i}$, $Z^{(eff)}=\frac{1}{n_e}\sum_i Z_i^2n_i$, the summation is over background plasma ions, $\tau_s=\left(\frac{m_\alpha}{Z_\alpha e\omega_{pe}}\right)^2\frac{v_c^3}{\Lambda m_e}$, and $\omega_{pe}=\sqrt{\frac{4\pi n_e e^2}{m_e}}$. The exact formulae for $p(u)$, $q(u)$, $r(u)$, $w(u)$ are explained and used in [2].

The simplified collision operators used in earlier bibliography [4-6] do not conserve the number of particles and do not nullify the collision term at statistical equilibrium. The formulae for fast ion current density given in [19-21] are not applicable to multi-ion-species plasmas; they do not describe the time evolution owing to the use of steady-state solutions, assuming a delta-like rather than arbitrary angle distribution of the fast ion source, and not taking into account velocity diffusion effects.

Semi-analytical stationary and nonstationary solutions with Coulomb collision term and a monoenergetic source function were described in [2,3]. New results improve the physical basis of neutral beam heating and current drive, clarify the discrepancies between analytical formulae in earlier bibliography, and extend the scope of semianalytical treatment with respect to [4-6] and [19-21].

IV. Conclusion. The new semianalytical results can be used in numerical modeling, for verification of solutions in more complex models, and in experimental data analysis, especially concerning nuclear processes and suprathermal particle diagnostics such as advanced neutral particle analysis systems [30,31].

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