Low-threshold absolute two-plasmon decay instability in second harmonic ECRH experiments at toroidal devices

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Introduction - Electron cyclotron resonance heating (ECRH) at power level of up to 1 MW in a single microwave beam is widely used nowadays in tokamak experiments and is considered for application in ITER for neoclassical tearing mode island control. As having been revealed by theoretical analysis a couple of decades ago [1] – [3] parametric decay instabilities (PDI) which may accompany ECR fundamental harmonic ordinary (O) mode and 2nd harmonic extraordinary (X) mode heating experiments are believed to be deeply suppressed by convective losses of daughter waves both along the magnetic field and in radial direction. Thus, the EC wave propagation and absorption are thought to be well described by linear theory and predictable in detail. However, during the last decade many observations have been made evidencing presence of anomalous phenomena which accompany ECRH experiments at toroidal devices. An eloquent example of these phenomena is the backscattering effect correlated to the MHD mode rotation observed recently in the 200 – 600 kW level 2nd harmonic ECRH experiment at Textor tokamak [4]. An explanation of this effect utilizing induced backscattering PDI low-threshold onset was proposed in [5] – [8]. The threshold lowering of this direct process according to [5] – [8] occurs due to excitation of trapped ion Bernstein waves possible due to the actual Textor plasma density profile possessing the local maximum in the O-point of the magnetic island [9]. Being direct and quite natural this explanation is not unique because the backscattering signal could be produced as a result of a secondary nonlinear process accompanying a primary low-threshold PDI of a different nature. A hint to this primary instability is provided by [4, 10] demonstrating the most intensive backscattering at plasma density in the magnetic island slightly exceeding the upper hybrid (UH) resonance value for half a pump frequency. In the present paper the effects of the parametric decay of the 2nd harmonic X-mode wave into two short wavelength UH plasmons propagating in opposite directions is considered. We demonstrate the possibility of the 3D localization of both the UH daughter waves. Similar to the mechanism of the IB wave trapping in radial direction [5], the radial localization of the UH waves can be achieved in a vicinity of the density profile local maximum often observed in ECRH experiments at toroidal devices (at the discharge axis for the peaked profile, at the edge for the hollow density profile or at the O-point of the magnetic island). On the other hand, when the pump power is high enough, the UH waves, propagating oppositely, can be trapped in the poloidal and toroidal directions due to the finite size of the microwave beam. The 3D localization of the UH waves leads to excitation of the absolute PDI. Being derived explicitly the threshold of this absolute PDI is drastically smaller than that provided by the standard theory [1] – [3].

Physical model - To elucidate the physics of the absolute PDI we analyze the most simple but nevertheless relevant to the experiment three wave interaction model in which the X-mode pump wave propagates almost perpendicular to the magnetic field $\vec{H} = H \cdot \vec{e}_z$ in the density inhomogenuity direction along unit vector $\vec{e}_x$ with its polarization vector being mostly directed along the poloidal direction ϑ, which almost coincides with unit vector $\vec{e}_y = \vec{e}_z \times \vec{e}_x$ direction. We represent a wide microwave beam of the X-mode pump wave propagating from the launching antenna inwards plasma along major radius in the tokamak mid-plane as

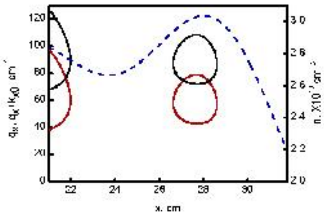

Fig. 1: (a, left and bottom axes) – 1D dispersion curves q_x (red curves) and q_x+k_{x0} (black curves); (b, right and bottom axes) – density profile with the local max. corresponding to O-point of m. island (q=2); $\omega = \omega_0/2$, $2\omega_{ce}(x_{ECR}) = \omega_0$, T_e=500eV, x_{ECR}=-28cm, q_y=0, k=l=13.

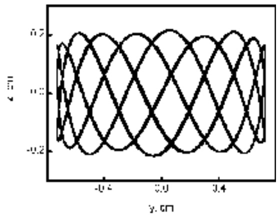

Fig. 2: The UH wave ray trajectory path demonstrating both its 2D trapping and the applicability of an adiabatic approximation. The same parameters as in fig. 1; P_0=600kW.

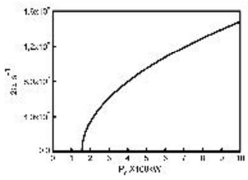

Fig. 3: The dependence of the growth rate on the power of the pump wave for same parameters as in fig. 1, 2; s=p=1, w=1cm.

$E_\vartheta = \dfrac{a_i(y,z)}{2}\exp(ik_0 x - i\omega_0 t) + c.c.$ where c.c. is complex conjugation, $a_i = \sqrt{\dfrac{8P_0}{cw^2}}\exp\left[-\dfrac{y^2+z^2}{2w^2}\right]$ is the amplitude, P_0 is the pump wave power and w is the beam waist. The basic set of integral equations describing the X-mode pumping wave decay into two UH waves, propagating in opposite directions, reads as

$$\hat{D}(\vec{r},\vec{r}';\omega)\{\phi_1(\vec{r}')\} = 4\pi\rho_1(\vec{r};\omega)\{a_i(\vec{r}),\phi_2(\vec{r})\};\ \hat{D}(\vec{r},\vec{r}';\omega_0-\omega)\{\phi_2(\vec{r}')\} = 4\pi\rho_2(\vec{r};\omega_0-\omega)\{a_i^*(\vec{r}),\phi_1(\vec{r})\} \quad (1)$$

The integral operators $\hat{D}$ in (1) are defined in weakly inhomogeneous plasma using the dispersion relation of the UH wave $D = l_T^2 q_\perp^4 + \varepsilon(x) q_\perp^2 + \omega^2 g^2/c^2 + \eta q_z^2$ where $l_T^2 = -3\omega_{pe}^2\upsilon_{te}^2/\left(2(\Omega^2-\omega_{ce}^2)(\Omega^2-4\omega_{ce}^2)\right)$, $q_\perp^2 = q_x^2 + q_y^2$, $g = \omega_{ce}/\Omega \cdot \omega_{pe}^2/\left(\Omega^2-\omega_{ce}^2\right)$, $\eta = 1-\omega_{pe}^2/\Omega^2$, $\Omega = [\omega, \omega_0-\omega]$. The nonlinear induced charge densities $\rho_{1,2}(\vec{r};\Omega)$ arising in r.h.s. of (1) and describing the UH wave and the fast X-mode EC wave coupling are given by an expression $\rho_j(\vec{r};\Omega_j) \simeq -\dfrac{k_0 u_{0x}}{4\pi\omega_0} q_{xj} q_{xp} \varphi_p \left[1 + \dfrac{3}{2}\dfrac{\omega_0^2}{\Omega_j^2 - \omega_{ce}^2}\right]$, $u_{0x} = \dfrac{|e|}{m_e}\dfrac{|\omega_e|}{\omega_0^2-\omega_{ce}^2} E_y$.

The reduced UH waves equations - As was shown in [5] – [8] the PDI threshold decreases substantially when one of the daughter waves, IB wave in the particular case of these references, is trapped at least in x - direction. This is also possible for the UH wave if the turning point of its dispersion curve and the local maximum of the non-monotonous density profile x_m in the O-point of the magnetic island are close one to another and the convective losses along the magnetic field are small, *i.e.* at $q_z \ll q_\perp$. Thus, we seek a WKB solution of the system (1) in a vicinity of x_m and UHR

$$\phi_j^{(s)} = \frac{C_j(y,z,t)\exp\left(-i\int_{x_{j,1}^*}^{x} Q_s(\Omega_j,q_y,\xi)d\xi - iq_y y\right)}{\left[\int_{x_{j,1}^*}^{x_{j,2}^*} dx\left(\left|\upsilon_{g,s}^{(+)}(\Omega_j,x)\right|^{-1} + \left|\upsilon_{g,s}^{(-)}(\Omega_j,x)\right|^{-1}\right)\right]^{1/2}}\left[\frac{\exp\left(-i\int_{x_{j,1}^*}^{x}\kappa_s(\Omega_j,q_y,\xi)d\xi\right)}{\sqrt{\upsilon_{g,s}^{(+)}(\Omega_j,x)}} + \frac{\exp\left(i\int_{x_{j,1}^*}^{x}\kappa_s(\Omega_j,q_y,\xi)d\xi\right)}{\sqrt{\upsilon_{g,s}^{(-)}(\Omega_j,x)}}\right], \quad (2)$$

$s=k,l$, $j=(1,2)$, $Q_s=\left(q_{x,s}^{(+)}+q_{x,s}^{(-)}\right)/2$, $\kappa_s=q_{x,s}^{(+)}-q_{x,s}^{(-)}$, $\upsilon_{g,j}^{(\pm)}=\partial D/\partial q_x\cdot\left[\partial D/\partial\Omega_j\right]^{-1}\Big|_{q_x^{\pm}}$, $\Omega_j=(\omega,\omega_0-\omega)$, $x_{j,1}^*$ and $x_{j,2}^*$ are solutions of equation $\upsilon_{g,s}^{(+)}\left(\Omega_j,x_{j,(1,2)}^*\right)=0$, $C_j(y,z,t)$ are slowly varying amplitudes and $q_x^{\pm}$ is the radial component of the wave vector $q_x^{\pm}(\omega,q_y,x)=\omega_{ce}/\upsilon_{te}\sqrt{-\varepsilon\pm\sqrt{\varepsilon^2-2\upsilon_{te}^2/c^2-q_y^2\upsilon_{te}^2/\omega_{ce}^2}}$ which obey the Born-Zommerfeld quantization: $\int_{x_{k,1}^*}^{x_{k,2}^*}\kappa_k\left(\omega\left(q_y^{k,l}\right),q_y^{k,l},\xi\right)d\xi=\pi k,\int_{x_{l,1}^*}^{x_{l,2}^*}\kappa_l\left(\omega_0-\omega\left(q_y^{k,l}\right),q_y^{k,l},\xi\right)d\xi=\pi l$. Upon solving last equation we get $q_y^{k,l}$ and $\omega\left(q_y^{k,l}\right)$. The solution (2) describes the UH waves trapped in a vicinity of the density maximum for which the convective losses in x direction are suppressed in full. The 1D dispersion curves of these waves for the typical conditions of Textor experiments ($T_e=500eV$, $\omega_0=140GHz$, $R_0=175cm$, $\omega_0/2=\sqrt{\omega_{ce}^2(x_m)+\omega_{pe}^2(x_m)}$, $2\omega_{ce}\left(x_{ECR}\right)=\omega_0$, $x_{ECR}=-28cm$, $x_m=28cm$, $w=1cm$), aimed at neoclassical tearing mode control, with the actual density profile [9] are shown in fig.1. Then, we substitute (2) into (1), multiply the first line and second line in it by $\phi_1^{(k)*}$ and $\phi_2^{(l)*}$, and evaluate integration over x that yields

$$\begin{aligned}&\left[\frac{\partial}{\partial t}-U_k\frac{\partial}{\partial y}+i\Lambda_{yk}\frac{\partial^2}{\partial y^2}+i\Lambda_{zk}\frac{\partial^2}{\partial z^2}+\nu_{ei}\right]C_1(y,z,t)=v_k(y,z)C_2(y,z,t);\\&\left[\frac{\partial}{\partial t}+U_l\frac{\partial}{\partial y}-i\Lambda_{yl}\frac{\partial^2}{\partial y^2}-i\Lambda_{zl}\frac{\partial^2}{\partial z^2}+\nu_{ei}\right]C_2(y,z,t)=v_l^*(y,z)C_1(y,z,t)\end{aligned}\tag{3}$$

where $\Lambda_{yk}=\dfrac{\Delta_k\{D_{qq}\}}{\Delta_k\{D_\omega\}}$, $\Lambda_{zk}=\dfrac{\Delta_k\{\eta\}}{\Delta_k\{D_\omega\}}$, $\Delta_k\{f\}=\int\limits_{x_{k,1}^*}^{x_{k,2}^*}\left(\dfrac{f^+(x)}{\upsilon_{g,k}^{(+)}(\omega)}+\dfrac{f^-(x)}{\upsilon_{g,k}^{(-)}(\omega)}\right)dx\left[\int\limits_{x_{k,1}^*}^{x_{k,2}^*}\left(\dfrac{1}{\upsilon_{g,k}^{(+)}(\omega)}+\dfrac{1}{\upsilon_{g,k}^{(-)}(\omega)}\right)dx\right]^{-1}$, $U_k=2q_y^{k,l}\Lambda_{yk}$, $D_{qq}^{\pm}=\partial D(\omega,q_x,x)/\partial\left(q_x^2\right)\Big|_{q_{x,k}^{(\pm)}}$, $D_\omega^{\pm}=\partial D(\omega,q_x,x)/\partial\omega\Big|_{q_{x,k}^{(\pm)}}$, ν_{ei} is electron-ion collision frequency and $v_k(y,z)$ is overlapping integral, which defines parametric coupling of two plasmons:

$$v_k=-i\frac{\omega_{ce}^2}{\omega_0^2-\omega_{ce}^2}\frac{a_i(y,z)}{2H_0\Delta_k(D_\omega)}\left(1+\frac{3}{2}\frac{\omega_0^2}{\omega_{pe}^2}\right)\left[\int_{x_{k,1}^*}^{x_{k,2}^*}dx\frac{1}{\left|\upsilon_{g,k}^{(+)}(\omega,x)\right|}+\frac{1}{\left|\upsilon_{g,k}^{(-)}(\omega,x)\right|}\right]^{-1/2}\left[\int_{x_{l,1}^*}^{x_{l,2}^*}dx\frac{1}{\left|\upsilon_{g,k}^{(+)}(\omega,x)\right|}+\frac{1}{\left|\upsilon_{g,k}^{(-)}(\omega,x)\right|}\right]^{-1/2}\times$$

$$\int\limits_{\max(x_{k,1}^*,x_{l,1}^*)}^{\min(x_{k,2}^*,x_{l,2}^*)}dx\left[\frac{q_{x,l}^{(-)}(\omega_0-\omega,x)q_{x,k}^{(+)}(\omega,x)e^{\left[ik_0x-i\int^x\left\{q_{x,l}^{(-)}(\omega_0-\omega)-q_{x,k}^{(+)}(\omega)\right\}ds\right]}}{\sqrt{\upsilon_{g,l}^{(-)}(\omega_0-\omega,x)}\sqrt{\upsilon_{g,k}^{(+)}(\omega,x)}}+\frac{q_{x,l}^{(+)}(\omega_0-\omega,x)q_{x,k}^{(-)}(\omega,x)e^{\left[ik_0x-i\int^x\left\{q_{x,l}^{(+)}(\omega_0-\omega)-q_{x,k}^{(-)}(\omega)\right\}ds\right]}}{\sqrt{\upsilon_{g,l}^{(+)}(\omega_0-\omega,x)}\sqrt{\upsilon_{g,k}^{(-)}(\omega,x)}}\right]$$

In the case of similar modes, *i.e.* at $k,l>>|k-l|$, we get $\Psi_k\simeq\Psi_l$ where $\Psi_k=\{U_k,\Lambda_{yk},\Lambda_{zk},\Xi_k,\gamma_k\}$. As was it shown for 1D case in [12, 13], the daughter waves generated via the parametric decay of the finite-size pump wave beam and propagating in opposite directions can be nonlinearly localized by the beam. Below, we demonstrate both analytically and numerically the 2D daughter waves localization provided by the two-dimensional finite-size beam of the pump wave. For this purpose we seek a WKB solution of (3) in a form $\propto\exp\left[\Omega t+iS(y,z)\right]$ that yields the Eikonal equation $\tilde{D}=(\Omega+\nu_{ei})^2+\left[U_k(\partial S/\partial y)+\Lambda_{yk}(\partial S/\partial y)^2+\Lambda_{zk}(\partial S/\partial z)^2\right]^2-\left|v_k(y,z)\right|^2=0$. For typical Textor experimental conditions the first term in the square brackets in Eikonal equation is negligible. To approximate its solution we use the ray-tracing procedure yielding the equations for the UH wave's front trajectory path

$$y''=-F(y,z),z''=-\Lambda_{zk}/\Lambda_{yk}\cdot F(y,z),F(y,z)=e^{\left(-(y/w)^2-(z/w)^2\right)}\left[e^{\left(-(y/w)^2-(z/w)^2\right)}-(\Omega+\nu_{ei})^2/\left|v_k(0,0)\right|^2\right]^{-1/2}\tag{4}$$

where the second derivatives are introduced over the dimensionless ray trajectory length. The system of the coupled equations (4) at $(\Omega+\nu_{ei})^2/|v_k(0,0)|^2<1$ describes the 2D finite behavior of the ray trajectory as is demonstrated in figure 2 for the Textor parameters (the same parameters as in figure 1, $U_k=0$, $\Lambda_{zk}/\Lambda_{yk}=17.35>>1$). The wave is shown to propagate along the magnetic field much faster than in y direction that makes possible the approximate analytical description of the solution to (4) and its quantization by the adiabatic invariance method. Further we assume an artificial "rectangle" pump beam: $|v_k(y,z)|^2=|v_k(0,0)|^2\cdot H(y+w)H(w-y)H(z+w)H(w-z)$ with $H(...)$ being Heaviside function. The consecutive quantization procedure yields $2\Omega=-2\nu_{ei}+\pi^2\!/_{2w^2}\sqrt{|v_k(0,0)|^2-\left[s^2\Lambda_{yk}+p^2\Lambda_{zk}\right]^2}$. Setting $\Omega=0$ we obtain for the most dangerous fundamental mode $p=s=1$ excitation the threshold

$$|v_k(0,0)|^2\left\{P_0^{th}\right\}=(\pi/2w)^4\left(\Lambda_{yk}+\Lambda_{zk}\right)^2+\nu_{ei}^2 \quad (5)$$

The dependence of the growth rate ($s=p=1$) given in (5) on the pump wave power is illustrated in figure 3 for the typical Textor parameters. As we can see the threshold of the PDI is $P_0^{th}=160kW$ and the growth rate at the power range of the pump wave $200\div600kW$ used in Textor is high enough $2\Omega\geq\omega_{ci}$ to make this parametric decay very dangerous.

Conclusions - We analyzed the parametric decay of the 2nd harmonic X-mode wave into two short wave-length UH plasmons which propagate in opposite directions. The possibility of the 3D localization of both the UH decay waves due to local maximum of the density in the O-point of the island (in the radial direction) and due to the finite-size pump wave beam (along the magnetic surface) was shown to lead to the excitation of the absolute PDI. We derived explicitly for "rectangle" pump beam the growth rate of the PDI and the threshold of absolute PDI which for the typical Textor parameters is in the power range of a couple hundreds kW that drastically smaller than that provided by the standard theory [1] – [3].

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