Non-Monotonic Frequency Sweeping of Fast Particle Driven Instabilities

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I. Introduction Alfvénic waves driven unstable by energetic particles in magnetic confinement devices often evolve into a strongly nonlinear regime, characterized by bursting mode amplitudes and significant frequency sweeping [1, 2]. Such events are intimately associated with the formation of long-living structures in the fast particle distribution function, so called *holes* and *clumps*, comprised of resonant particles moving coherently in phase space. As these entities evolve due to dissipation, the frequencies of the observed signals may diverge from the initial resonance.

The occurence, formation and temporal evolution of coherent phase space structures has been investigated theoretically in the 1D electrostatic bump-on-tail problem [3, 4]. It has been shown that holes and clumps form spontaneously in the near threshold limit when the kinetic drive γ_L from the resonant particles just exceeds the dissipative damping γ_d from the thermal plasma. Moreover, fast particle collisions and sources are important for the wave evolution: Both velocity space diffusion and Krook-type collisions tend to suppress holes and clumps, while drag enhances the hole/clump formation and gives rise to up/down asymmetric frequency sweeping that differs for holes and clumps.

Most previous investigations were limited to short range frequency sweeping phenomena, where the linear mode structure of the initial perturbation is preserved. In contrast, long range frequency sweeping involves significant changes in the mode amplitude and profile, which alters the ensuing sweeping rates. In this contribution, we generalize the long range formalism in [5] by accounting for fast particle sources and collisions and allowing the wave field to evolve in more general plasma equilibria. In particular, we focus on non-monotonic, so called *hooked*, frequency sweeping of holes, which can only occur when the fast particle collision operator contains drag. The investigation is performed by transforming to the moving reference frame of an isolated hole, thus enabling an efficient adiabatic description of the corresponding resonant particles.

II. Adiabatic Model We investigate a low amplitude, electrostatic perturbation, with a prescribed period λ, in a 1D bump-on-tail configuration. The model contains three plasma species: A population of static ions; a linear fluid background of cold electrons, subject to weak collisions; a low density population of energetic electrons, which are treated kinetically, including the effects of sources and collisional relaxation. The gradient of the fast electron equilibrium distribution function $F_0(v)$ provides a linear growth rate γ_L, while the dissipation in the background species, due to the infrequent collisions, damps the wave linearly at a rate γ_d. With this setup, it has been shown that holes and clumps form spontaneously in the near threshold limit $0 < \gamma_L - \gamma_d \ll 1$, see e.g. [4].

We focus our attention on the evolution of already established holes and clumps. In this *adiabatic* regime, the wave evolution is much slower than the bounce motion of electrons trapped in the wave field. The perturbation of interest is then a slowly evolving, BGK-like [6] wave with a time dependent frequency, which we choose to represent in terms of the wave potential energy $U\left(x - s\left(t\right); t\right)$. Here, U is periodic in its rapidly varying first argument, describing oscillations at the wave carrier frequency $\omega \sim \omega_p$, and slowly changing with respect to its second argument, which describes the evolution of the wave amplitude and structure. Moreover, the wave phase velocity $\dot{s}\left(t\right)$ changes slowly. To be specific, we focus on the case

$$d/dt\left[\ln U\ ,\ \ln \dot{s}\right] \ll \omega_B \ll \omega\ , \tag{1}$$

where ω_B denotes the bounce frequency of the particles trapped in the wave field.

The ordering (1) simplifies the treatment of the fast electrons, whose motion is governed by the wave frame Hamiltonian

$$\mathcal{H} = \frac{\left(p - m_e \dot{s}\left(t\right)\right)^2}{2m_e} + U\left(z \equiv x - s\left(t\right); t\right)\ . \tag{2}$$

At any moment, the (z, p) phase space portrait of $\mathcal{H}$ defines a separatrix (cf. Figure 1) centered around $p = m_e \dot{s}$, which can be viewed as a rigid boundary: As the phase velocity of the wave changes, the trapped electrons are convected along while the passing electrons are forced to jump over the moving separatrix. This natural separation of the fast electrons allows us to treat trapped and passing particles individually: The passing electron distribution function is assumed to be given by the equilibrium F_0. For the trapped electrons, on the other hand, we adopt the adiabatic invariant (cf. Figure 1)

$$J = \oint \sqrt{\frac{2}{m_e}\left(\mathcal{E} - U\left(z\right)\right)}\, dz \tag{3}$$

as an action variable. The new Hamiltonian then becomes independent of the corresponding canonical angle θ, which implies that the trapped electron distribution f is a slowly evolving function of merely J. It is then advantageous to average the trapped electron kinetic equation over the bounce motion of the trapped electrons, yielding

$$\frac{\partial\,\overline{\delta f}}{\partial t} + \beta\,\overline{\delta f} = -\rho\left(\dot{s}\right)\left[\ddot{s} + \frac{\alpha^2}{k}\right] + m_e \frac{\nu^3}{k^2}\frac{\partial}{\partial J}\left[J \frac{dJ}{d\mathcal{E}}\frac{\partial\,\overline{\delta f}}{\partial J}\right]\ . \tag{4}$$

Here, the trapped electrons are represented by their bounce averaged perturbed distribution $\overline{\delta f} \equiv \langle f \rangle - F_0\left(\dot{s}\right)$, with $\langle \ldots \rangle$ the bounce average, and the sources and collisions are modeled with a combination of Krook collisions, drag (slowing-down) and velocity space diffusion. The parameters β, α and ν are the corresponding collision frequencies, and $k = 2\pi/\lambda$ is the wave number. The function ρ accounts for the velocity profile of the equilibrium slope, cf. Figure 1. Note that $\overline{\delta f}$ must vanish at separatrix.

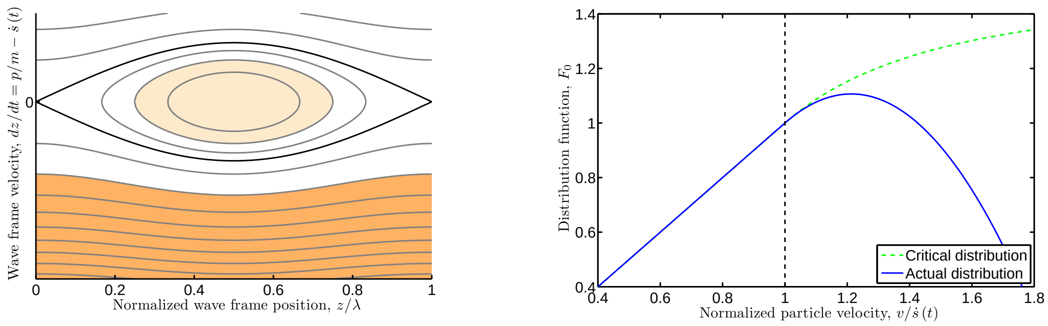

Figure 1: Left: Separatrix structure in wave frame phase space with coordinates (z, p). The colored areas represent trapped and passing particle actions. Right: The blue line portrays the fast particle "bump-on-tail" distribution used throughout this investigation. The green line is a critical distribution that keeps the wave amplitude constant throughout the evolution.

Moreover, knowing $\overline{\delta f}$ and the phase velocity $\dot{s}$, the ordering (1) enables one to conveniently solve the adiabatic Poisson equation [5],

$$\frac{\partial^2 U}{\partial z^2} + \frac{\omega_p^2}{\dot{s}^2} U = \frac{e^2}{\epsilon_0} \left[\int_0^\lambda \int \overline{\delta f}\, dv dz - \int \overline{\delta f}\, dv \right] , \tag{5}$$

as a boundary value problem for U.

Finally, the set of equations in the adiabatic model is closed by noting that in the adiabatic limit, the power dissipated in the cold electron background must balance the power released from the fast electrons during the mode evolution. Thus, a power balance condition can be written as [4]

$$m_e \, \dot{s} \left(\ddot{s} + \frac{\alpha^2}{k} \right) \int_0^\lambda \int \overline{\delta f} \, dv \, dz = \frac{2\gamma_d \lambda}{m_e \dot{s}^2} \int_0^\lambda U^2 (z) \, dz \ , \tag{6}$$

which constitutes a third relation between $\dot{s}$, U and $\overline{\delta f}$.

III. Hooked Frequency Sweeping In Figure 2, we present graphs displaying non-monotonic frequency sweeping of holes in the equilibrium distribution presented in Figure 1. The results are obtained by means of the adiabatic model, i.e. by solving self-consistently equations (4), (5) and (6), with (normalized) collisional parameters $\widetilde{\alpha} = 0.4$, $\widetilde{\beta} = 0, 2$ and $\widetilde{\nu} = 0$. We see that hooked frequency sweeping results with and without Krook collisions when $\widetilde{\alpha} \neq 0$, although the effect is more pronounced when $\widetilde{\beta} \neq 0$. Without Krook, the sweeping reversal is explained by looking at equation (6): As the mode

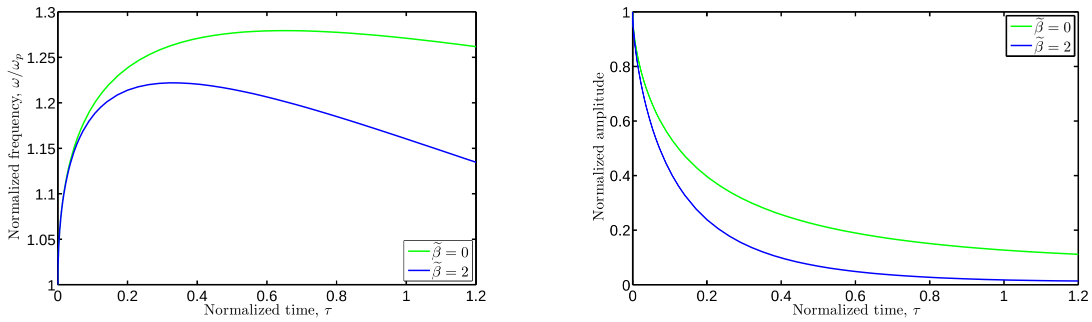

Figure 2: Two different cases of non-monotonic frequency sweeping when $\alpha \neq 0$. The blue line is for $\beta \neq 0$. The green line demonstrates the occurence of hooks due to the varying equilibrium slope alone, i.e. when $\beta = 0$.

initially sweeps towards higher frequency, the amplitude decreases. For the hole, this is due to the chosen fast electron equilibrium distribution function being sub-critical (cf. Figure 1) above the linear resonance. The corresponding amplitude evolution is shown in the plot to the right in Figure 2, where the green line depicts the case $\widetilde{\beta} = 0$. Eventually, the sweeping reaches a point where the right hand side balances the α^2/k-term in (6). At this stage, $\ddot{s} = 0$, and the sweeping reverses.

With Krook, the reversal point is reached earlier in the evolution, and the concomitant down-sweep is faster than with $\widetilde{\beta} = 0$. The explanation can once again be found by looking at the amplitude evolution: Krook collisions tend to decrease the perturbed fast electron distribution function, i.e. the hole depth, exponentially in time. Through the Poisson equation (5), this results in a quicker reduction of the wave amplitude, as seen in Figure 2. Therefore, the reversal occurs earlier in the evolution, and at lower frequency. In fact, the effect of velocity space diffusion is very similar to Krook collisions, so we would expect the same behavior with nonzero $\widetilde{\nu}$. It is worth noting that in the limit $t \to 0$, both curves in Figure 1 tend to the short range, square root result $\omega \propto \sqrt{t}$, confirming the picture that collisions do not significantly alter the initial frequency sweeping rate.

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