Simulation of fast ions interaction with ICRF wave in realistic tokamak magnetic field

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Introduction

Resonance electromagnetic radiofrequency wave absorption is one of the main plasma heating techniques in modern tokamaks. In simulations the absorption is typically described in terms of quasi linear theory, which based on the assumption of shortness of the resonance interaction time compared with bounce period. However, it is not a general case for realistic tokamak equilibrium. In the present work the wave-particle resonance interaction is studied numerically by direct integration of full particle orbits in realistic tokamak equilibrium in the presence of RF wave field. Most significant discrepancies between numerical results and predictions of the theory are found for the energetic ions with potato orbits, i.e. for the case typical for on-axis minority heating. Then for accurate description of the RF wave absorption and evolution of the RF heated ion distribution function a combination of analytical predictions with numerically calculated diffusion coefficients is suggested.

Analytical approach

Theory fundamentals [1, 2] are derived in case of the collisionless cyclotron damping. The condition of single-particle resonance is $\omega - k_{\|}V_{\|} = \omega_{ic}$. On each passage through the resonance the ion receives a kick in the perpendicular component of the particle velocity $V_\perp$. By integrating motion equation

$$m\dot{\vec{V}} = q[\vec{V},\vec{B}] + q\vec{E} + q[\vec{V},\vec{B}_1] \tag{1}$$

one can analytically calculate a change in $V_\perp$. Integration is performed along for unperturbed particle trajectories [3, 4]. Equation for the change in perpendicular component of the velocity

$$\dot{U} + i\Omega U = \frac{Ze}{m}\left(E_+ \exp\left(i\left(\int \vec{k}\vec{r} - \omega t\right)\right) + E_- \exp\left(-i\left(\int \vec{k}\vec{r} - \omega t\right)\right)\right) \tag{2}$$

where $U = V_x + iV_y$, $E_\pm$ are left- and right-hand components of the wave electric field.

Solution of this equation is

$$U\exp(i\int_{-\infty}^{t}\Omega dt') = U(-\infty) + \sum (E_+ J_{n-1}(k_\perp\rho)\int_{-\infty}^{t}\exp(i\Psi(t'))dt' + E_- J_{n+1}(k_\perp\rho)\int_{-\infty}^{t}\exp(-i\Psi(t'))dt') \qquad (3)$$

$$\Psi(t) = \int_{-\infty}^{t}(n\Omega + k_{||}V_{||} - \omega)dt' = \int_{-\infty}^{t}\eta(t')dt'$$

where $t' = t - t_0$, t_0 is resonance moment. The change in perpendicular energy per transit of the resonance surface is

$$\frac{m}{2}\left\langle V(t)V^*(t) - V(-\infty)V^*(-\infty)\right\rangle = \delta(V_\perp^2) = (\Delta V_\perp)^2 + 2V_\perp\Delta V_\perp\cos\varphi \qquad (4)$$

$$\Delta V_\perp = \tfrac{q}{m} I_n (E_+ J_{n-1}(\frac{k_\perp V_\perp}{\omega_{ci}}) + E_- J_{n+1}(\frac{k_\perp V_\perp}{\omega_{ci}})) \qquad (5)$$

I_n is effective interaction time, φ is a phase shift between the Larmor rotation and wave field.

For estimation of I_n according to [4, 5] one has to expand the phase Ψ in a Taylor series in the vicinity of resonance and then to apply a stationary phase approximation in calculating the integral (3):

$$I_n = \int_{-\infty}^{t}\exp(\pm i(\Psi(t'))dt' = \int_{-\infty}^{t}\exp(\pm i(\Psi(t_0) + \frac{1}{2}\ddot{\Psi}(t_0)t'^2))dt' = \sqrt{\left|\frac{2\pi}{\dot{\eta}_i}\right|}\exp(\pm i\frac{\pi}{4}) \qquad (6)$$

For particles with turning points close to the resonance layer $V_{||}$ tends to zero and I_n tends to infinity. Taking next terms in Taylor expansion in Ψ we obtain

$$\Psi(t) = \Psi(\theta) + \eta(\theta)(t-\theta) + \frac{1}{6}\ddot{\eta}(\theta)(t-\theta)^3 \Rightarrow I_n = \exp(\pm i\Psi(\theta))\frac{2\pi}{(\ddot{\eta}_\theta/2)^{1/3}} Ai(\frac{\eta_\theta}{(\ddot{\eta}_\theta/2)^{1/3}}) \qquad (7)$$

where $\dot{\eta}(\theta) = 0$ and Ai(x) is Airy function. Temporal derivatives for η are given by

$$\dot{\eta}_i = (nV_{||}\omega_{ci} - k_{||}\frac{V_\perp^2}{2})\frac{1}{B}\frac{dB}{ds} \text{ and } \ddot{\eta} = -n\Omega\frac{V_\perp^2}{2}(\frac{1}{B}\frac{dB}{ds})^2 + (nV_{||}\Omega - k_{||}\frac{V_\perp^2}{2})\frac{1}{B}(\vec{V}\nabla)(\vec{b}\nabla B)$$

$$\eta(\theta) = -\frac{\dot{\eta}_i^2}{2\ddot{\eta}_i} \quad \ddot{\eta}(\theta) = \ddot{\eta}_i$$

Then the resonance interaction time is taken [4] as $I_n = \min(\sqrt{|2\pi/\dot{\eta}_i|}, Ai(0)\cdot 2\pi/\sqrt[3]{0.5\ddot{\eta}_i})$.

This theory is built on the assumption of sufficiently short resonance time. Only in this case Taylor expansion and stationary phase approximation are correct. However, if the resonance time (wave-particle interaction time) is comparable to the bounce-period τ_b, the analytical theory described above is inapplicable. Furthermore, the analytical description in this case is hardly possible. The direct numerical simulations of the particle orbits in realistic tokamak magnetic fields in the presence of ICRF wave becomes the only way to get quantitative characteristics of the interaction.

Algorithm and calculations

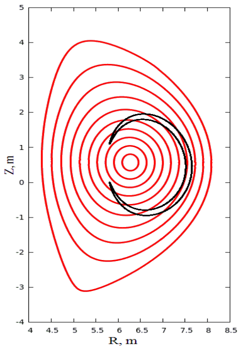

Fig. 1. Magnetic field configuration (red) and particle trajectory (black).

In our numerical code full particle orbit in the tokamak equilibrium is advanced in time using 4-th order Runge-Kutta algorithm. Then the changes of their energy and magnetic moment due to interaction with RF field are calculated. The magnetic field configuration and typical trajectory of the particle calculated for such field are shown in fig. 1. The magnetic field in the form $B = B_0 \frac{R_0}{R} + \frac{1}{R}[\nabla\psi \times \vec{e}_\varphi]$ is calculated using equilibrium data for poloidal flux function $\Psi(R_i, Z_i)$, interpolated by 2D cubic spline. Fast magnetosonic wave is set in a simplified model form for easy comparison with analytical results. The electric field with circular polarization is taken in the form $\vec{E}_\pm = \vec{a}\cos(\vec{k}\vec{R} - \omega t + \varphi) \pm \vec{b}\sin(\vec{k}\vec{R} - \omega t + \varphi)$. For calculations we take a ratio of E_+/E_- = 0.179 [2]. The magnetic field is $\vec{B}_1 = \frac{1}{\omega}[\vec{k}, \vec{E}]$. The wave vector is determined by $k_{||} = n_\varphi / R, k_\perp = \omega_{ci} / V_A$.

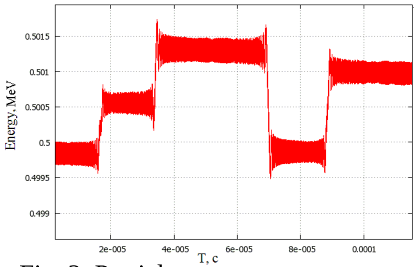

Fig. 2. Particle energy.

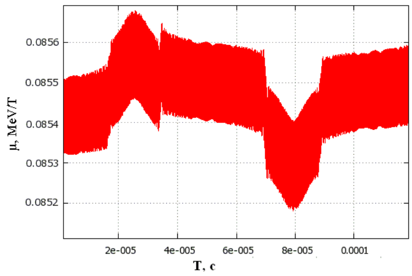

Fig. 3. Particle magnetic moment.

With the coordinates of the particle and the energy specified, the evolution of the energy (fig. 2), magnetic momentum (fig. 3) and change in $V_\perp$ can be calculated. Their stepwise changes are observed in the vicinity of wave-particle resonances. By considering a single resonance event, an average magnetic momentum before and after resonance interaction are calculated and the resulting kick in perpendicular velocity is determined:

$$\Delta V_\perp = \frac{1}{mV_\perp} B\Delta\mu = \frac{1}{mV_\perp} B(\mu_{av1} - \mu_{av2}) \quad (8)$$

However, the change in $V_\perp$ depends on the phase shift between the Larmor rotation and wave field for each particle entering the resonance. So the calculations for ensemble of the particles with same initial conditions but different phase of the Larmor rotation was carried out. Then, averaging of the calculated increments of $V_\perp$ given by equation (5) and E over particle ensemble yielded numerical results for comparison with analytical ones.

The calculations have shown that results of our numerical calculations are in good agreement with predictions of Stix theory, firstly, under conditions of relatively low energy particles and, secondly, for the case when turning points of trapped particles are well

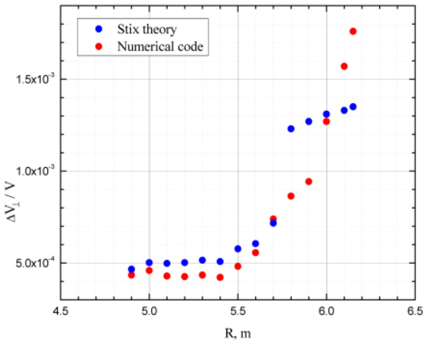

Fig. 4. Calculation for different locations of turning point. E_{in}=5 keV, R_{in}=4,8 m, Z_{in}=0,0 m. R_{res}=6,2 m.

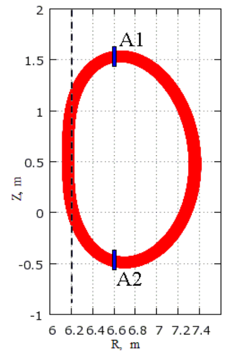

Fig. 5. Potato orbit.

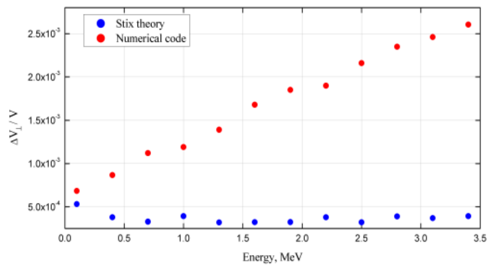

Fig. 6. Calculation of $\Delta V_{\perp}/V$ for different E_{in}.

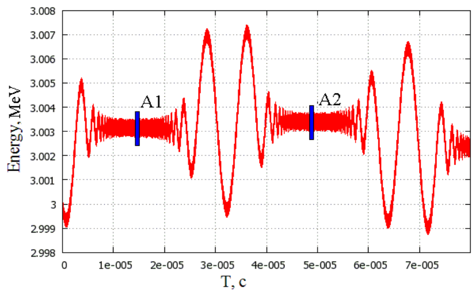

Fig. 7. Change in energy for one particle for on-axis ICR heating. E_{in}=3 MeV. Points A1 and A2 are from fig. 5.

separated from the resonance layer. However, with approaching of turning point to the resonance zone, a discrepancy in results is observed (fig. 4).

The significant discrepancy is accentuated for the high energy particles with "potato" orbits (fig. 5), i. e. for an on-axis ICRF-heating (fig. 6). In this case, bounce period τ_b becomes comparable with the interaction time I_n (fig. 7), whereas stationary phase method applicability requires $\tau_b << I_n$. Furthermore, it is shown in fig. 7 that one cannot determine the resonance as good as for the turning points away from the resonance area, because there is double resonance between points A1 and A2 (turning points are very close to each other and lie at the resonance layer).

Conclusions

The numerical code for calculation of the ion trajectories, total and perpendicular energy in the tokamak magnetic field with presence of ICRF wave has been developed. Under the condition of local resonance good agreement between numerical simulation results and theoretical predictions of Stix model was found. Significant deviations for high energy ions interacting with ICRF wave in on-axis heating scenario are clearly demonstrated. Further development of the numerical code is planned for calculation of the transport coefficients for kinetic modeling. By combining the Stix theory and the numerical calculation results it is possible to describe accurately the wave-particle interaction in the whole computation domain.

References

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