Influence of axial magnetic field on the dust acoustic wave in the cylindrical device

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The nonlinear structures, which represent the plasma states far from thermodynamic equilibrium, are either spontaneously created in laboratory and space plasmas or externally launched in laboratory plasmas under controlled conditions. The presence of charged dust grains introduces new features to the nonlinear structures, which are otherwise absent in the usual e-i plasma. In the past few decades, the propagation of nonlinear dust acoustic wave, in dusty plasmas with an unbounded planar geometry has been extensively studied theoretically [1]. Although, the one dimensional geometry may not be a realistic situation in laboratory devices and in space, but many of those studies are limited to one dimensional geometry [2-5]. The solitary waves in unmagnetized dusty plasmas without the dissipation and geometry distortion effects can be described by the Korteweg-de Vries (KdV) equation or Kadomtsev- Petviashvili (KP) equation [6-13]. Since most of the dusty plasmas in laboratory and space environments are confined in an external magnetic field, it is of practical interest to examine the properties of dusty plasma modes in a magnetoplasma. In dusty plasmas, if the dissipation is weak at the characteristic dynamical time scale of the system the balance between nonlinear and dispersion effects can result in the formation of symmetrical solitary waves. Also shock waves will be propagated in this system if dissipation effect is strong. Thus solitary waves and shock waves (oscillatory and monotonic types) can be produced in dusty plasmas.

With assumption, *the dust grain radius* $a <<$ *the electron gyroradius* ρ_e, the charging characteristics are not significantly influenced by the existence of an external magnetic field, since for $a << \rho_e$, the curvature effect of the trajectory of an electron (ion) impinging on a dust grain of radius a can be neglected. Also in the low-frequency regime, electrons and ions are assumed in a Boltzmann distribution due to the fact that the mode frequency is much lower than the electron and ion frequencies. In this letter, we have extended Ref. [14] by using the axial magnetic field and the dust temperature. The nonlinear dust acoustic waves in dusty plasmas with the combined effects of the transverse perturbation, and axial magnetic field are studied in the cylindrical coordinates. Using the perturbation method, a cylindrical nonlinear equation that describes the dust magneto acoustic wave is deduced for the first time. The inhomogeneous cylindrical Kadomtsev- Petviashvili (ICKP) equation is found and solved numerically.

Let us consider a fully ionized, collisionless dusty plasma consisting of a mixture of electron/ion (with Boltzmann distribution function), negatively charged dust particles in the presence of an external static magnetic field $\mathbf{B} = B_0\hat{\mathbf{z}}$. To study the magnetized DA solitary waves in nonplanar cylindrical geometry, we assume that the solitary wave propagates in a cylindrical geometry filled with hot magnetized dusty plasma. So the hydrodynamic equation for dust particles and Poisson's equation, can be written in cylindrical geometry as

$$\frac{\partial n_d}{\partial t} + \frac{n_d u_d}{r} + \frac{\partial (n_d u_d)}{\partial r} + \frac{1}{r}\frac{\partial (n_d v_d)}{\partial \theta} = 0\,, \tag{1}$$

$$\frac{\partial u_d}{\partial t} + u_d\frac{\partial u_d}{\partial r} + \frac{v_d}{r}\frac{\partial u_d}{\partial \theta} - \frac{{v_d}^2}{r} = \frac{\partial \phi}{\partial r} - \omega_c v_d - \frac{T_d}{ZT_i}\frac{1}{n_d}\frac{\partial n_d}{\partial r}\,, \tag{2}$$

$$\frac{\partial v_d}{\partial t} + u_d\frac{\partial v_d}{\partial r} + \frac{v_d}{r}\frac{\partial v_d}{\partial \theta} + \frac{v_d u_d}{r} = \frac{1}{r}\frac{\partial \phi}{\partial \theta} + \omega_c u_d - \frac{T_d}{ZT_i n_d}\frac{1}{r}\frac{\partial n_d}{\partial \theta}\,, \tag{3}$$

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} = n_d + \frac{1}{\mu - 1}\exp(\sigma\phi) - \frac{\mu}{\mu - 1}\exp(-\phi)\,, \tag{4}$$

where (r,θ) are radial and angle coordinates, (u_d, v_d) are the dust fluid velocity in r and θ directions, and n_d ,ϕ represent the dust density number and the electrostatic potential, respectively. The variables $t, r, n_d, (u_d, v_d)$, ϕ, are normalized to the dust plasma frequency $\omega_{pd} = \sqrt{4\pi n_{d0} Z^2 e^2 / m_d}$, Debye length λ_D, unperturbed equilibrium dust density n_{d0}, effective dust acoustic velocity $C_d = Zk_B T_i / m_d$ and $k_B T_i / e$, respectively. Here we have denoted $\mu = n_{i0} / n_{e0}$, $\mu_i = n_{i0} / Zn_{d0} = \mu / (\mu - 1)$, $\mu_e = n_{e0} / Zn_{d0} = 1/(\mu - 1)$, $Zn_{d0} + n_{e0} - n_{i0} = 0$, and $\sigma_i = T_i / T_e$. The cyclotron frequency of dust particles $\omega_c = qB_0 / m_d c$ is normalized by ω_{pd}.

In order to investigate dust acoustic wave in magnetized plasmas, we employ the standard reductive perturbation technique to obtain the nonlinear differential equation. The independent variables can be stretched as $\xi = \varepsilon^{1/2}(r - V_0 t)$, $\eta = \varepsilon^{-1/2}\theta$ and $\tau = \varepsilon^{3/2} t$, where ε is a small parameter and V_0 is the wave velocity. In our case, we assume weak magnetic field, so the normalized cyclotron frequency is a finite quantity of the order of the small parameter ε. Also, dependent variables are expanded as

$$n_d = 1 + \varepsilon n_1 + \varepsilon^2 n_2 + \varepsilon^3 n_3 + \cdots\,, \tag{5}$$

$$u_d = \varepsilon u_1 + \varepsilon^2 u_2 + \varepsilon^3 u_3 + \cdots\,, \tag{6}$$

$$v_d = \varepsilon^{3/2} v_1 + \varepsilon^{5/2} v_2 + \cdots, \tag{7}$$

$$\phi = \varepsilon \phi_1 + \varepsilon^2 \phi_2 + \varepsilon^3 \phi_3 + \cdots. \tag{8}$$

Substituting Eqs. (5)-(8) into Eqs. (1)-(4) and collecting the terms in different powers of ε, to lowest order in ε, we obtain

$$n_1 = u_1 / V_0 = -\phi_1 / (V_0^2 - T_d / ZT_i), \tag{9}$$

$$-V_0 \partial v_1 / \partial \xi = [1/V_0 \tau (1 - T_d / ZT_i V_0^2)] \partial \phi_1 / \partial \eta + \omega_c u_1. \tag{10}$$

In the next higher order of ε, we obtain

$$\frac{\partial n_1}{\partial \tau} + \frac{u_1}{V_0 \tau} + \frac{\partial}{\partial \xi}(n_1 u_1) + \frac{1}{V_0 \tau} \frac{\partial v_1}{\partial \eta} = V_0 \frac{\partial n_2}{\partial \xi} - \frac{\partial u_2}{\partial \xi}, \tag{11}$$

$$\frac{\partial u_1}{\partial \tau} + u_1 \frac{\partial u_1}{\partial \xi} + \omega_c v_1 - \frac{T_d}{ZT_i} n_1 \frac{\partial n_1}{\partial \xi} = \frac{\partial \phi_2}{\partial \xi} + V_0 \frac{\partial u_2}{\partial \xi} - \frac{T_d}{ZT_i} \frac{\partial n_2}{\partial \xi}, \tag{12}$$

$$\frac{\partial v_1}{\partial \tau} + u_1 \frac{\partial v_1}{\partial \xi} - \frac{T_d}{ZT_i V_0 \tau} n_1 \frac{\partial n_1}{\partial \eta} = \frac{1}{V_0 \tau} \frac{\partial \phi_2}{\partial \eta} + V_0 \frac{\partial v_2}{\partial \xi} - \frac{T_d}{ZT_i V_0 \tau} \frac{\partial n_2}{\partial \eta} + \omega_c u_2, \tag{13}$$

$$\frac{\partial^2 \phi_1}{\partial \xi^2} - \left(\frac{\sigma^2 - \mu}{\mu - 1} \right) \frac{\phi_1^2}{2} = n_2 + \left(\frac{\sigma + \mu}{\mu - 1} \right) \phi_2. \tag{14}$$

Now, using Eqs. (9)-(14) and eliminating variables n_2, u_2, v_1, u_1, one can obtain the inhomogeneous cylindrical Kadomtsev- Petviashvili (ICKP) equation,

$$\frac{\partial}{\partial \xi} \left[\frac{\partial \phi_1}{\partial \tau} + A \phi_1 \frac{\partial \phi_1}{\partial \xi} + B \frac{\partial^3 \phi_1}{\partial \xi^3} + \frac{\phi_1}{2\tau} \right] + \frac{1}{2V_0 \tau^2} \frac{\partial^2 \phi_1}{\partial \eta^2} = \frac{\omega_c^2}{2V_0} \phi_1, \tag{15}$$

where $A = (\mu - \sigma^2) V_0^3 / 2(\mu - 1) - 3/2V_0 + T_d V_0 / 2ZT_i (V_0^2 - T_d / ZT_i)^2$ and $B = V_0^3 / 2$. If the wave propagate without magnetic field and the transverse perturbation, the right hand side and the last term in the left hand side disappear, so the ICKP equation reduce to the ordinary cylindrical KdV equation, which it has an exact solitary wave solution. The terms $(1/2\tau)\partial \phi_1 / \partial \xi$ and $(1/2V_0 \tau^2) \partial^2 \phi_1 / \partial \eta^2$, can be canceled if we define new variable $\chi = \xi - V_0 \eta^2 \tau / 2$, and $\phi_1 = \phi_1(\chi, \tau)$. Then the ICKP equation (15) is reduced to the inhomogeneous KdV equation,

$$\frac{\partial \phi_1}{\partial \tau} + A \phi_1 \frac{\partial \phi_1}{\partial \chi} + B \frac{\partial^3 \phi_1}{\partial \chi^3} = \frac{\omega_c^2}{2V_0} \int \phi_1 d\chi. \tag{16}$$

Without magnetic field (D=0), it is well known that the KdV equation has a solitary wave solution

$$\phi_1 = \frac{3u_0}{A} \mathrm{sec}\,h^2 \left[\sqrt{\frac{u_0}{4B}} [\xi - (0.5 V_0 \eta^2 + u_0) \tau)] \right], \tag{17}$$

where u_0 is a constant represent wave velocity. It is clear that the amplitude and wave velocity of solitary wave (17) are uniquely determined by the parameters of the system and only depend on the initial conditions.

For solving the inhomogeneous nonlinear equation (16), we have used the Adomian decomposition method (numerical solution). The result shows for the weak magnetic field ($\omega_c / \omega_{pd} \sim \varepsilon$), the shape of solitary wave (width and amplitude) and the symmetry of the wave-packet change. Also, the results show the cylindrical wave will slightly deform as time goes on.

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