Fokker-Planck model for pitch-angle scattering induced loss of fast ions from the tokamak plasmas

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1. Introduction

Precise modelling of spatial and velocity distributions of fast ion loss in tokamaks is important for predicting the heat load associated with charged fusion products and beam ions escaping from the confining magnetic configuration in future fusion reactors. Further, such modelling enables the identification and interpretation of the loss mechanisms of fast ions in present day tokamak plasmas [1-5]. Typically relevant simulations were based on Monte-Carlo approaches [6,7] or on simplified models of spatial distributions only [8,9] and allow for a qualitative rather than quantitative information on loss distributions. A detailed predictive modelling of fast ion loss distributions requires the development of new approaches or at least a substantial improvement of the existing methods [4, 5]. The purpose of the present paper is to establish a technique for assessing the loss distributions of energetic ions in tokamaks using the Fokker-Planck (FP) approach.

2. Fokker-Planck equation in the constant-of-motion space

Our study is based on the drift FP equation for fast ions in the phase space of motion invariants, $\boldsymbol{c}$, and of angular coordinates, $\boldsymbol{\theta}$, determining the particle position on the orbit. In case of axisymmetric tokamak considered here, such a position is specified by the poloidal angular coordinate, ϑ, only. Therefore sufficient is the FP treatment in a 4D phase space $\mathbf{x}=\{\boldsymbol{c},\vartheta\}$, i.e.

$$\dot{\vartheta}\partial_{\vartheta} f = L_{\mathbf{x}} f + S(\mathbf{x}), \quad L_{\mathbf{x}} = \nabla_{\mathbf{x}} \cdot \left(\mathbf{d}_{\mathbf{x}} + \vec{\mathbf{D}}_{\mathbf{x}} \cdot \nabla_{\mathbf{x}}\right) \tag{1}$$

where $\mathbf{d}_{\mathbf{x}}$ and $\vec{\mathbf{D}}_{\mathbf{x}}$ describe convective and diffusive collisional transport of fast ions associated correspondingly with the slowing down and pitch-angle scattering and S($\mathbf{x}$) is the source term.

2.1 <u>FP description of confined ions</u> Due to the smallness of the collisional rates of the slowing down, ν_S, and of the pitch-angle scattering, $\nu_\perp$, as compared to the frequency of poloidal motion, $\dot{\vartheta}$, the distribution function of confined ions, $f(\mathbf{x})$, can be represented as a superposition of the dominant part, $f_0(\boldsymbol{c})$, which is independent on angular coordinate and of a small oscillating part, $f_1(\boldsymbol{c},\vartheta)$, varying periodically with ϑ. Note that $f_0(\boldsymbol{c})$ satisfactorily describes the ions with confined orbits and is determined by bounce averaged FP equation in 3D constant-of-motion (COM) space [9, 10]

$$0 = \langle L_{\mathbf{x}} \rangle f_0 + \langle S \rangle, \quad \langle ... \rangle = \oint d\vartheta \sqrt{g_{\mathbf{x}}}(...) \equiv \Omega(\mathbf{c}) \oint d\vartheta (...) / \dot{\vartheta} \tag{2}$$

where $\sqrt{g_{\mathbf{x}}}$ is the Jacobian of transformation from Eulerian coordinates (**r**,**v**) to Lagrangian coordinates **x**, $\Omega \equiv \sqrt{g_{\mathbf{x}}}\dot{\vartheta}$ is ϑ-independent value due to an evident relationship $\nabla_{\mathbf{x}} \cdot \dot{\mathbf{x}} = 0$. Distribution function $f_0(\boldsymbol{c})$ can be obtained by solving a boundary value problem in the COM space. Following [9, 10] we can use the following set of COM variables (energy E, normalised magnetic moment λ and maximum radial coordinate on the particle orbit $r_{\max}$). Evidently that, in the case of confined orbits, $r_{\max}$ can not exceed the radius of the tokamak first wall, r_{fw}. Moreover $f_0(\boldsymbol{c})$ should satisfy the following boundary condition

$$f_0\left(E, \lambda, r_{\max} = r_{\text{fw}}\right) = 0 . \tag{3}$$

Knowledge of the $f_0(\boldsymbol{c})$ allows to find the oscillating part of distribution function $f_1(\boldsymbol{c},\vartheta)$ by performing the ϑ-integration of Eq. (1), i.e.

$$\Omega f_1 = \widehat{L}_{\mathbf{x}} f_0 + \widehat{S}(\mathbf{x}), \quad \widehat{(...)} = \int_0^{\vartheta} d\vartheta \sqrt{g_{\mathbf{x}}}\,(...), \tag{4}$$

where poloidal variable ϑ is determined by the relationship $r = r_{\max} - \left(r_{\max} - r_{\min}\right)\sin^2 \vartheta/2$ with r_{min} the minimum radial coordinate on the particle orbit. Note that generally the oscillating part of distribution function $f_1\left(\vartheta; E, \lambda, r_{\max} = r_{\text{fw}}\right) \neq 0$ and is dominant for marginally confined ions with $r_{\max} \rightarrow r_{\text{fw}}$. Evidently that the radial flux of fast ions lost to the first wall is determined by those with the unconfined orbits, i.e. with $r_{\max} > r_{\text{fw}}$.

2.2 <u>FP description of unconfined ions</u> To examine the distribution function of lost ions we use a set of COM variables $\hat{\mathbf{c}} = \left\{E, \xi_{\text{fw}}, \chi_{\text{fw}}\right\}$, where ξ_{fw} and χ_{fw} are pitch and poloidal angle at the first wall. Correspondingly the cyclic poloidal variable $\hat{\vartheta}$ is determined by the relationship $r = r_{\text{fw}} - \left(r_{\text{fw}} - r_{\min}\right)\sin^2 \hat{\vartheta}/2$ and fast ions, escaping to the first wall as a result of collisional convection and diffusion transport, are described by 4D FP equation (1) in variables $\hat{\mathbf{x}} = \left\{\hat{\mathbf{c}}, \hat{\vartheta}\right\} \equiv \left\{E, \xi_{\text{fw}}, \chi_{\text{fw}}; \hat{\vartheta}\right\}$. It is important that diffusion and convection in poloidal angle χ_{fw} significantly dominate those in pitch angle ξ_{fw} as well as in the energy. The reason for that is localisation of lost particles in rather narrow range of poloidal angles [5], $0 < \chi_{\text{fw}} < \chi_{\text{loss}} \sim 0.1 - 0.3 \ll \pi$, where

$$\chi_{loss} = \left(\frac{4D_r \tau_b}{a^2}\frac{V_\chi^2}{V_d^2}\right)^{1/4}, \tag{5}$$

D_{r} is the radial diffusion coefficient of lost ions; V_χ and V_d are correspondingly poloidal and drift components of particle velocity. Domination of convective and diffusive transport in angle coordinate $\hat{x}^3 \equiv \chi_{\text{fw}}$ makes possible to neglect the collisional fluxes in $\hat{x}^2 \equiv \xi_{\text{fw}}$ and in $\hat{x}^1 \equiv E$ and to use a reduced 2D FP equation of a following form

$$\dot{\hat{\vartheta}}\partial_{\hat{\vartheta}} f = g_{\hat{\mathbf{x}}}^{-1/2}\partial\hat{x}^3 g_{\hat{\mathbf{x}}}^{1/2}\left(\hat{d}^3 + \hat{D}^{33}\partial\hat{x}^3\right) f . \tag{6}$$

Source term $S(\hat{\mathbf{x}})$ is not accounted for in Eq. (6) because in the case of unconfined ions it is associated with the collisionless first orbit losses and not with the collisional convective and diffusive ones. Finally Eq. (6) should be appended by the following initial and boundary conditions

$$f\left(\hat{\vartheta},\chi_{\rm fw};E,\xi_{\rm fw}\right)\Big|_{\hat{\vartheta}=0}=0,\quad f\left(\hat{\vartheta},\chi_{\rm fw};E,\xi_{\rm fw}\right)\Big|_{\chi_{\rm fw}=0}=f_1\left(E,\lambda,r_{\max}=a;\vartheta\right). \tag{7}$$

In Eq. (7) the function $f_1(E,\lambda,r_{\max}=a;\vartheta)$ is determined by Eq. (3) and corresponds to the marginally confined fast ions, variables $\{E,\xi_{\rm fw}\}$ in present 2D FP treatment are the parameters. Note that for marginally confined particles $\hat{\vartheta}=\vartheta$ and a normalised magnetic moment λ can be expressed as a function of COM variable $\xi_{\rm fw}$ by the following relationship

$$\lambda=\left(1-\xi_{\rm fw}^2\right)R\left(r=a,\chi=0\right)/R_c\equiv\left(1-\xi_{\rm fw}^2\right)\left(1+a/R_c\right), \tag{8}$$

where $R(r,\chi)$ is the major radius, a and R_c correspond to the minor and major plasma radii. Radial flux $\Gamma(E,\xi_{\rm fw},\chi_{\rm fw})$ of fast ions lost to the first wall is determined by the following expression

$$\Gamma\left(\chi_{\rm fw},E,\xi_{\rm fw}\right)=f\left(\hat{\vartheta}=2\pi,\chi_{\rm fw};E,\xi_{\rm fw}\right)V_d^r\left(\chi_{\rm fw},E,\xi_{\rm fw}\right), \tag{9}$$

where V_d^r is the particle radial velocity at the first wall. Thus solution of Eq. (6) under conditions (7) allows examination of spatial and velocity distributions of collisional loss via simple formula (9).

2.3 Results of numerical modeling Here we represent the results of numerical evaluation of the poloidal and pitch-angle distributions of a convective-diffusive collisional loss of 130 *keV*

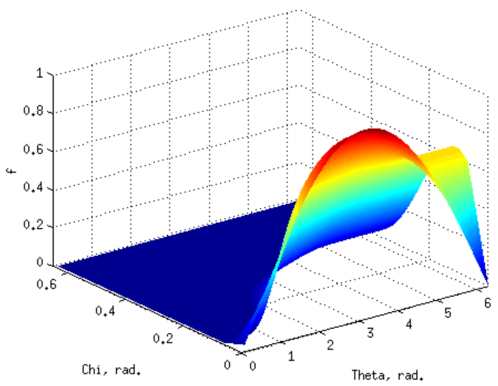

Fig. 1: Fast ion distribution function of co-circulating lost deuterons with E = 130 keV and ξ_{FW}*=0.5 vs polidal angular variable* $\hat{\vartheta}$ *and poloidal angle* χ_{FW} *at the first wall.*

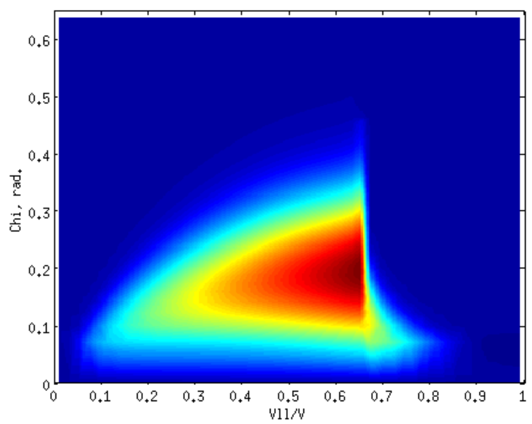

Fig. 2: Contours of the convection-diffusion collisional flux of co-circulating lost deuterons in the plane spanned by the pitch angle cosine ξ_{FW} *and poloidal angle* χ_{FW} *at the first wall.*

deuterons to the first wall of JET-like tokamak plasma with a = 0.95 m and R_c = 2.95 m, $n_e(0) = n_i(0) = 0.7\cdot10^{14}\ m^{-3}$, $n_e(a) = n_i(a) = 0.2\cdot10^{14}\ m^{-3}$, $T_e(0) = T_i(0) = 5.0\ keV$, $T_e(a) = T_i(a) = 1.0\ keV$. We use also model magnetic configuration with Shafranov shift 0.2 m, elongation $k(0) = 1.3$, $k(a) = 1.7$, triangularity 0.15 and plasma current I = 2.5MA. To simplify simulation we suppose that $f_1\left(E,\lambda,r_{\max}=a;\vartheta\right)\propto\sin\left(\vartheta/2\right)$. Fig. 1 displays the calculated distribution function of lost 130 keV deuterons with pitch-angle cosine ξ_{FW}=0.5 as dependent on the polidal angular variable $\hat{\vartheta}$ and poloidal angle χ_{FW}. As expected, $f(\hat{\vartheta},\ \chi_{FW})$ is localised in rather narrow range of poloidal angles at the wall $0 < \chi_{FW} < 0.3$. Fig. 2 represents the distribution of lost co-circulating deuterons over the pitch angle cosine ξ_{FW} and poloidal angle χ_{FW} at the first wall. The maximum loss is observed at $\chi_{FW} = 0.2$ for marginally trapped ions with $\xi_{FW} = 0.65$.

3. Conclusions

We demonstrate that in drift approximation the distribution function of fast ions lost from the axisymmetric tokamak plasmas as a result of collisional convection-diffusion transport can be treated by 1D in COM space and 1D in poloidal angular coordinate Fokker-Planck kinetic equation. Solution of this equation allows direct evaluation of the spatial and velocity distributions of the flux of lost ions to the tokamak first wall. Modeled collisional loss of fast deuterons in JET-like tokamak are found to be localized in rather narrow range of poloidal angles ($0 < \chi_{FW} < 20°$) below the plasma midplane. It should be pointed out that solution of the boundary value problem for lost fast ions allows to extend our Fokker-Planck code FIDIT [9, 11] oriented at present time predominantly on the description of confined fast ions (only spatial and velocity distributions of circulating-trapped cone loss and velocity distributions of total loss of fast ions are calculated) also to detailed description of lost ions (including loss distribution over the tokamak first wall). Finally we note that approach developed should be useful for the verification of Monte-Carlo models used for the simulation of fast ion loss from toroidal plasmas as well.

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