Convective magnetohydrodynamical modeling and simulation of electric arcs

P. Delmont[1], M. Torrilhon[1]

[1] *MathCCES, RWTH Aachen, Germany*

Electric arc model

When an electric potential is applied across a highly resistive gaseous medium, an electric discharge may occur and create a plasma column, which is called an electric arc. Recently, Torrilhon and Wright [4] developed an electric arc model based on the equations of magneto-hydrodynamics (MHD). They studied the non-convective limit of this model in detail [3, 4]. In this work, we study the full convective arc model. We perform axisymmetric electric arc simulations.

The MHD equations can be written as

$$\partial_t \mathbf{u} + \nabla \cdot \mathbf{F} = \mathbf{S} \tag{1}$$

with $\mathbf{u} = (\rho, \rho\mathbf{v}, \mathbf{B}, E_{tot})$, $F = \left(\rho\mathbf{v}, \rho\mathbf{v}\mathbf{v} + p_{\text{tot}}\mathbf{I} - \frac{1}{\mu_0}\mathbf{B}\mathbf{B}, \mathbf{B}\mathbf{v} - \mathbf{v}\mathbf{B}, (E_{\text{tot}} + p_{\text{tot}})\mathbf{v} + \frac{1}{\mu_0}(\mathbf{B}\cdot\mathbf{v})\mathbf{B}\right)$ and $\mathbf{S} = \left(0, \mathbf{0}, \hat{\partial}\mathbf{B}, \frac{1}{\mu_0}\mathbf{B}\cdot\hat{\partial}B + \rho c_v \hat{\partial} T\right)$.

Here we implicitly introduced the non-convective system (from [2, 3, 4]), given by

$$\left(\hat{\partial}\mathbf{B}, \rho c_v \hat{\partial} T\right) = \left(\nabla \times \frac{\mathbf{j}}{\sigma(T)}, \lambda \nabla^2 T + \frac{1}{\sigma(T)}\mathbf{j}^2\right). \tag{2}$$

These equations express the conservation of mass, momentum and energy, completed by Faradays law. They are written in an appropriate nondimensional conservative form. ρ denotes the mass density, $\mathbf{v}$ is the plasma velocity and $E_{tot} = \varepsilon + \frac{1}{2}\rho v^2 + \frac{1}{2\mu_0}B^2$ is the total energy (with internal energy ε). Here, p is the thermal pressure, γ is the ratio of specific heat and $\mathbf{B}$ is the magnetic field and $p_{tot} = p + \frac{1}{2\mu_0}B^2$ is the total pressure. c_v is the specific heat and λ denotes the heat conductivity. T is the temperature and $\sigma(T)$ refers to the electric conductivity. Finally, $\mathbf{j} = \frac{1}{\mu_0}\nabla \times \mathbf{B}$ is the current density. The system is closed by an ideal gas equation of state $\varepsilon = \frac{1}{\gamma - 1}p = c_v \rho T$. Essential to the model is the purely temperature dependent electric conductivity which captures the highly resistive gas as well as the conductive plasma. Around the ionization temperature $\overline{T}$, the electric conductivity jumps with several orders of magnitude. Following [4], we define

$$\sigma(T) \equiv \sigma_{\min} + \frac{\sigma_{\max} - \sigma_{\min}}{1 - \text{erf}\left(\frac{T_0 - \overline{T}}{T_\varepsilon}\right)} \left(\text{erf}\left(\frac{T - \overline{T}}{T_\varepsilon}\right) - \text{erf}\left(\frac{T_0 - \overline{T}}{T_\varepsilon}\right)\right). \tag{3}$$

Finally, we will work in cylindrical coordinates and assume axisymmetry. We will assume that $\mathbf{v} = (v_r, 0, v_z)$ and $\mathbf{j} = (j_r, 0, j_z)$, such that $\mathbf{B} = (0, B, 0)$. In this case, the MHD equations (1) simplify to a five dimensional hyperbolic system with parabolic source terms.

In electric arc simulations, one often couples a Maxwell solver to a Navier-Stokes solver. This splits the system in an electromagnetic part on the one hand, and a thermal and convective part on the other hand. Amongst some other problems, this leads to incorrect characteristic speeds, since the electromagnetic contribution is ignored here. Therefore, we will alternatively split the system into ideal magnetohydrodynamics, and the parabolic thermal non-convective system. The first system, ideal MHD, is a non-stiff hyperbolic system of conservation laws. Well-known solvers exist for ideal MHD, and in the simulations presented, we use an explicit local Lax-Friedrich scheme with MINMOD limiter. Also the time step Δt is calculated from the CFL condition of the hyperbolic system. The parabolic non-convective part contains the source terms which drive the system. The temperature dependence of the electric resistivity introduces stiffness to the system. Another numerical challenge is that the ignition happens on very short time scales at an a priori unknown time. These considerations suggest the use of an implicit solver with adaptive time integrator. Therefore, as in [2], we solve the parabolic part of the system by the RADAU solver [1]. This is a fifth order implicit Runge-Kutta scheme with adaptive time integrator. It is appropriate for banded Jacobian structures, which typically appear when writing out a system of ODEs as a PDE system. We perform the necessary number of RADAU steps to advance the system over a time Δt.

Simulation results

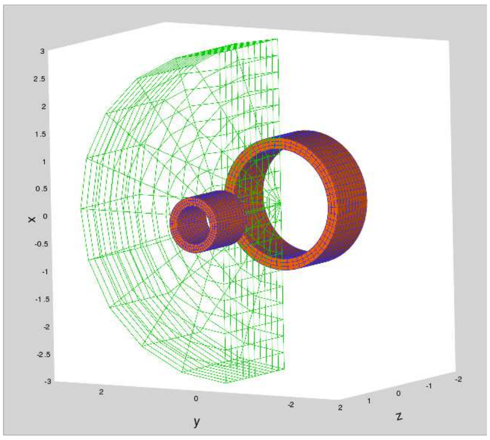

Figure 1: The computational domain.

We will perform 3D axisymmetric electric arc simulations on a cylindrical domain with cylindrical hollow contacts at both sides of the domain, as shown in figure 1. The dimensions of the numerical domain are $[r,z] \in [0m, 0.5m] \times [-0.1m, 0.1m]$, and hollow cylindrical contacts are placed at and $r \in [0.15m, 0.20m]$ on the lower wall and at $r \in [0.10m, 0.15m]$ at the upper wall. Initially, a uniform gas ($\rho = 34.3\frac{kg}{m^3}$) at rest fills the domain at $300K$. We apply an electric potential ($240kV$) across the chamber. This potential induces an electric field, and Ohmically heats the gas. It also dictates the initial conditions for the magnetic field. When the potential is large enough, a discharge occurs and an electric arc is formed. We consider constant $\gamma = 1.057$ and $c_v = 10^3 \frac{J}{Kkg}$. Also the thermal conductivity $\lambda = 8.25 \cdot 10^2 \frac{W}{mK}$ is taken constant. The parameters for the electric conductivity are taken as $\sigma_{\min} = 1.65 \cdot 10^{-3} \frac{A}{Vm}$, $\sigma_{\max} = 1.65 \cdot 10^4 \frac{A}{Vm}$, $\overline{T} = 12000K$ and $T_\varepsilon = 3000K$. The boundary conditions are given by $T = 300K$, and $\mathbf{j}$ follows from the applied electric field.

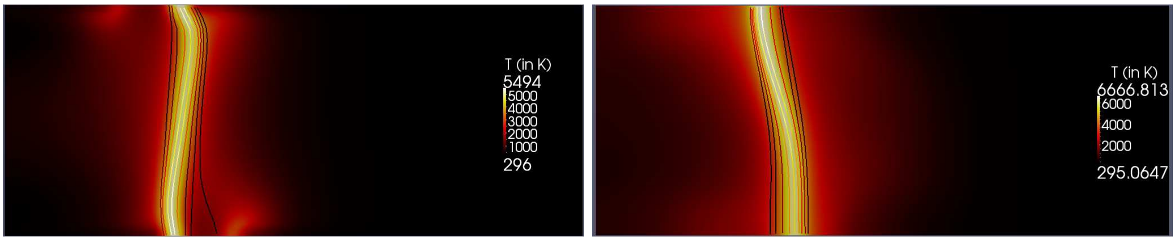

Figure 2: Shown are temperature profiles and streamlines for current during the electric arc ignition in the non-convective and the convective case, respectively. Due to cooling at the boundary the ignition happens later in the convective case.

Figure 2 shows a non-convective and a convective electric arc ignition simulation at when $I_{\text{arc}} = 200kA$. For the non-convective case this happens at $t = 1.3010^{-3}s$. The convection allows the gas to cool down faster, since warmer gas is convected near the wall. For the convective case the ignition happens much later and at $t = 4.552\ 10^{-3}s$ a total current $I_{\text{arc}} = 200kA$ is reached. Soon after, at $t = 4.553\ 10^{-3}s$ the total current $I_{\text{arc}} = 5MA$. This perfectly illustrates the range of time scales involved, and the need for an addaptive time integrator.

Once a critical current through the arc is achieved, we keep this current constant. This changes the boundary conditions for $\mathbf{B}$: on the inside of the contact, we set $B = 0$, and at the outside of the contact we set $B(r, \pm Z) = \frac{I_{\text{arc}}}{2\pi r}$ and $B(R,z) = \frac{I_{\text{arc}}}{2\pi R}$. We will switch to this current-driven situation for two different currents: $I_{\text{arc},1} = 200kA$ and $I_{\text{arc},2} = 5MA$. The situation in these cases is essentially different. In the first case, the plasma is thermally dominated and the Lorentz force is negligible compared to the thermal pressure, while in the second case the plasma is magnetically dominated, as can be seen in figure 3. In the thermally dominated case, the arc is pushed outwards, while in the magnetically dominated case, the arc is pushed inwards, as can be seen in figure 4. It should be noted that the traditional splitting of the system into Maxwell's equations

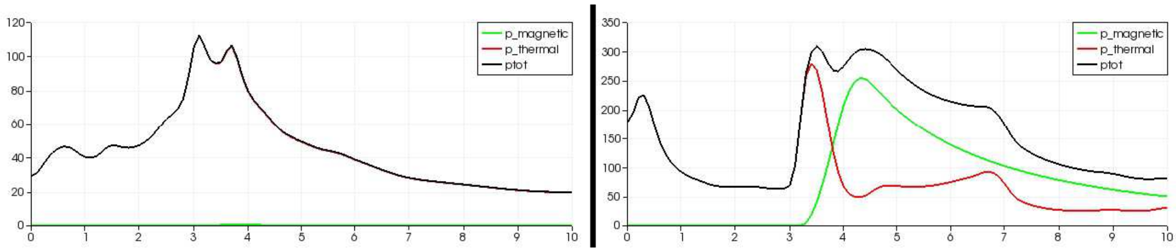

Figure 3: The pressure profile at $Z = 0$ during ignition at $t = 4.552\ 10^{-3}s$ and $t = 4.553\ 10^{-3}s$. One sees that the ignition happens on small time scales. At high currents, the traditional splitting in the Navier-Stokes equations and the Maxwell equations will lead to incorrect characteristic speeds.

on the one hand, and Navier-Stokes equations on the other hand, will lead to incorrect characteristic speeds. It should be noted that for more realistic equations of state this magnetically dominated behaviour appears at lower currents. These simulations are a work in progress, and will be presented in a future publication.

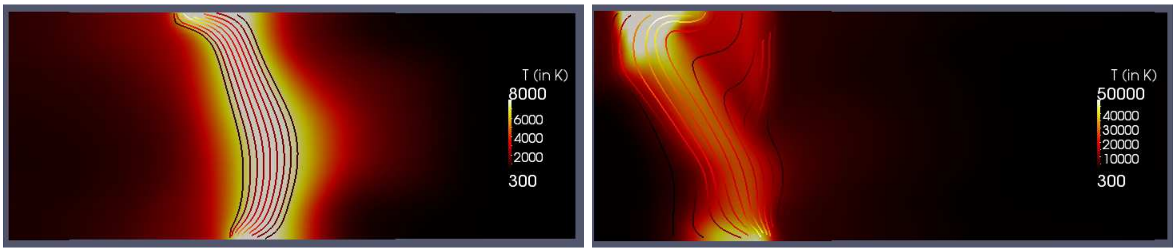

Figure 4: A thermally and a magnetically dominated electric arc at constant total current. In the magnetically dominated case, the Lorentz force pushes the electric arc inwards.

References

[1] E. Hairer & G. Wanner, *Solving Ordinary Differential Equations: Stiff and differential-algebraic problems*, Springer series in Computational Mathematics, Vol. 14, 2nd editition, 1996

[2] P. Delmont & M. Torrilhon, *Magnetohydrodynamical modeling and simulations of electric arc extinction in a network*, Proc. ECCOMAS, 2012, submitted

[3] M. Torrilhon, *Stability of reduced electric arc models*, Proc. ICIAM, 2007

[4] M. Torrilhon & D. Wright, *Modeling of Electric arcs: A Study of the Non-Convective Case with Strong Coupling*, Submitted.