Hydrodynamic stability of self-similar supersonic heat waves related to inertial confinement fusion

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Hydrodynamic instabilities are a key issue in laser-driven inertial confinement fusion where a sufficiently uniform implosion of a spherical pellet is expected to achieve thermonuclear burn. Little attention has been devoted to the early shell-irradiation stage of the pellet implosion compared to the subsequent shell-acceleration phase during which the ablative Rayleigh–Taylor instability occurs. Our ongoing project aims at obtaining a better description of the hydrodynamic stability of this early shell irradiation flow. Using self-similar ablative heat-wave solutions of gas dynamics equations with nonlinear heat conduction [1] as mean flows and devising, in this simplified setting, a spectrally accurate numerical method to compute both these solutions and their time-dependent linear perturbations, has allowed us to fulfill this goal [2]. Here we propose to extend this approach to the supersonic heat-wave regime which is relevant to the very early irradiation phase of the shell, prior to its ablation. In doing so, we also wish to obtain a detailed description of the isothermal-shock-wave-and-thermal-precursor structure present in ablative flows in order to remove the restriction of the perfect shock-wave front approximation of this structure which was used in [1, 2].

Mean flow

The flows we consider are those resulting from applying time-dependent external heat flux and pressure at the free boundary of a semi-infinite slab of a compressible, inviscid, heat-conducting fluid with a polytropic equation of state $p = \rho RT$, $\mathscr{E} = C_v T$, $C_v = R/(\gamma-1)$, and a nonlinear conductivity of the form $\kappa = \chi\,(\overline{\rho}/\rho_0)^{\mathrm{m}}\,(\overline{T}/T_0)^{\mathrm{n}}$, $\mathrm{m} \leq 0$, $\mathrm{n} \neq 1$, where ρ_0 and T_0 are some standard density and temperature. The corresponding equations of motion, written in terms of the Lagrangian coordinate m, such that $dm = \overline{\rho}\,dx$, come as

$$\left.\begin{array}{r}\partial_t\overline{\rho} + \overline{\rho}^2\,\partial_m\overline{v}_x = 0, \quad \partial_t\overline{v}_x + \partial_m\overline{p} = 0,\\ \partial_t\left(\overline{v}_x^2/2 + \overline{\mathscr{E}}\right) + \partial_m\left(\overline{p}\,\overline{v}_x + \overline{\varphi}_x\right) = 0, \quad \text{with} \quad \overline{\varphi}_x = -\kappa\overline{\rho}\,\partial_m\overline{T},\end{array}\right\} \quad \text{for } m \geq 0, \qquad (1)$$

where the symbols used have their usual meanings. Assuming the fluid to be initially homogeneous, cold and at rest, self-similar solutions to this system of equations may be obtained upon considering specific time-power laws for the external heat-flux and pressure [3]. Such solutions cover the supersonic heat-wave regime which prevails during the early shell irradiation, where a supersonic heat wave preceding an isothermal shock wave penetrates the otherwise undisturbed shell. Self-similar solutions to (1) come as [1]

$$\overline{G}(\xi)=\overline{\rho}(m,t),\ \overline{V}(\xi)=t^{\alpha-1}\overline{v}_x(m,t),\ \overline{\Theta}(\xi)=t^{2(\alpha-1)}\overline{T}(m,t),\ \overline{\Phi}(\xi)=t^{3(\alpha-1)}\overline{\varphi}_x(m,t),\quad (2)$$

where $\xi = mt^{-\alpha}$, $\alpha = (2\mathrm{n}-1)/(2\mathrm{n}-2)$, and are solutions to a nonlinear eigenvalue problem which consists in a four-dimensional system of first-order ODEs, of the form

$$d_\xi\overline{\mathbf{Y}}=\mathscr{F}(\xi,\overline{\mathbf{Y}}),\quad \text{where}\quad \overline{\mathbf{Y}}(\xi)=(\overline{G},\overline{V},\overline{\Theta},\overline{\Phi})^{\mathsf{T}},\quad (3)$$

with boundary conditions

$$\overline{G}=1,\quad \overline{V}=0,\quad \overline{\Theta}=0,\quad \text{as}\quad \xi\to+\infty,\qquad \overline{P}=\overline{G\Theta}=\mathscr{B}_p,\quad \overline{\Phi}=\mathscr{B}_\varphi,\quad \text{at}\quad \xi=0.\quad (4)$$

Sought solutions present an isothermal shock-wave front, say at $\xi=\xi_{\mathrm{s}}$, which corresponds to the essential singularity (sonic hyperplane) [4] of system (3), and a heat-wave front at $\xi = \xi_{\mathrm{tf}} > \xi_{\mathrm{s}}$, for which this system degenerates into a three-dimensional system. This thermal-front singularity is characterised by the following root-branch behaviors of the flow reduced variables (*e. g.* see [4])

$$\overline{G}\sim 1+\overline{G}_{\mathrm{tf}}\breve{\xi},\quad \overline{V}\sim\overline{V}_{\mathrm{tf}}\breve{\xi},\quad \overline{\Theta}\sim\overline{\Theta}_{\mathrm{tf}}\breve{\xi},\quad \text{as}\quad \breve{\xi}=(1-\xi/\xi_{\mathrm{tf}})^{1/\mathrm{n}}\to 0^+.\quad (5)$$

The sonic-hyperplane singularity is circumvented by applying the isothermal shock-wave jump relations while the thermal-front singularity is handled by performing the change of variables $\xi\to\breve{\xi}$, $\overline{\mathbf{Y}}(\xi)=\breve{\overline{\mathbf{Y}}}(\breve{\xi})$ in the front vicinity, and retaining there the corresponding formulations of (3) and (4).

A numerical procedure, based on the multi-domain Chebyshev spectral method of [1], has been devised to compute solutions to the boundary value problem (BVP) made of eqs. (3) and (4). This procedure makes use of this BVP $\breve{\xi}$-formulation for sub-domains in the thermal-front neighborhood, while keeping its ξ-formulation for the rest of the flow. Using the shock-front and thermal-front locations, ξ_{s} and ξ_{tf}, as input parameters, the procedure applies a relaxation process in each computational sub-domain, starting from an initial guess obtained *(i)* by the leading-order approximation (5) within the thermal-front sub-domain, and *(ii)* by a finite-difference shooting method for the other sub-domains. Spectral accurate results over the whole interval $0\le\xi\le\xi_{\mathrm{tf}}$, and which are free from the approximation errors made in (5), are thus obtained.

This numerical procedure is capable of treating a wide variety of flows (see Fig. 1), ranging from negligible ($\xi_{\mathrm{tf}}=0.75$) to significant ($\xi_{\mathrm{tf}}=3.07$) supersonic heat-waves, and thus spanning

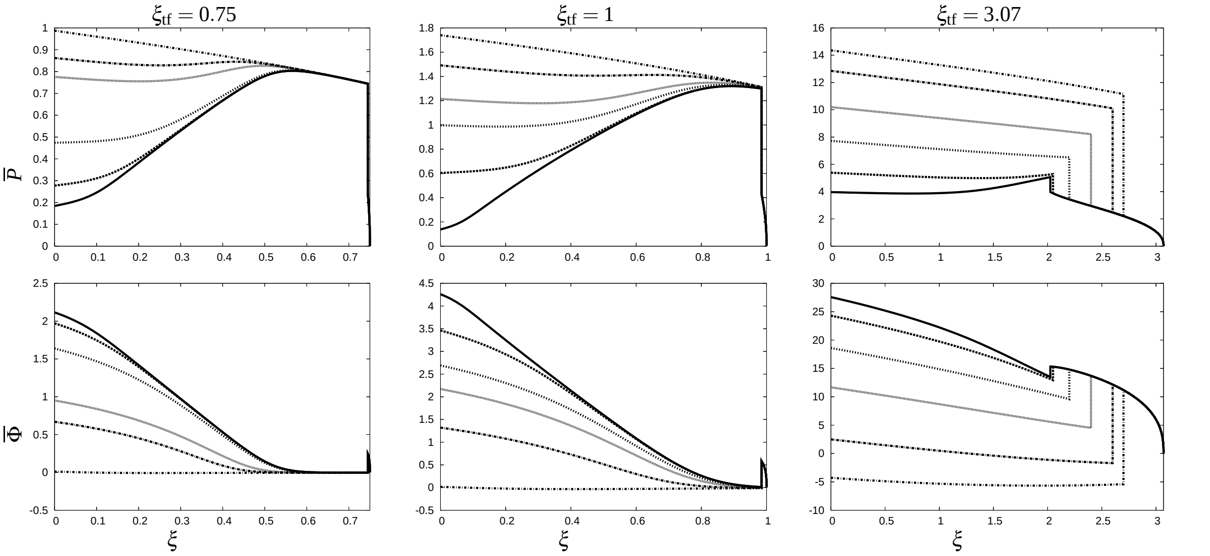

Figure 1: Pressure (top) and heat flux (bottom) reduced variables for $(\mathrm{m},\mathrm{n}) = (0, 5/2)$, three locations of the thermal-front, $\xi_{\mathrm{tf}} = 0.75$, 1 and 3.07, and different shock-front position values ξ_{s}.

the whole range of configurations between the ablative and the supersonic heat-wave regimes. This enables us to explore larger regions of the boundary-condition $(\mathscr{B}_\varphi, \mathscr{B}_p)$-parameter space than those allowed by the perfect shock-front approximation: see [1, Fig. 6]. This includes the capability of recovering solutions previously obtained within this perfect shock-front approximation (compare the curves $\xi_{\mathrm{tf}} = 0.75$ and $\xi_{\mathrm{tf}} = 1.0$ of Fig. 2 with those $\xi_{\mathrm{s}} = 0.75$ and $\xi_{\mathrm{s}} = 1.0$ of [1, Fig. 6]) but also, for a fixed shock-front location, of describing flows at higher (lower) external heat fluxes (pressures) than previously allowed (curve $\xi_{\mathrm{tf}} = 1.0$ of Fig. 2 and $\xi_{\mathrm{s}} = 1.0$ of [1, Fig. 6]). Variations of the shock-front location ξ_{s} for a given thermal-front abscissa ξ_{tf} (Fig. 1) highlight the thickening of the supersonic heat-wave extent when increasing the external heat-flux and decreasing the external pressure. For the smallest values of the thermal-front abscissa ξ_{tf} ($\xi_{\mathrm{tf}} = 0.75$ and $\xi_{\mathrm{tf}} = 1.0$ in Fig. 1), the corresponding flows range from the isothermal shock-wave propagation regime (lowest $\mathscr{B}_\varphi$, highest $\mathscr{B}_p$) to the ablative heat-wave regime (highest $\mathscr{B}_\varphi$, lowest $\mathscr{B}_p$). For the largest value of ξ_{tf} ($\xi_{\mathrm{tf}} = 3.07$ in Fig. 1), the flows belong to the supersonic heat-wave regime.

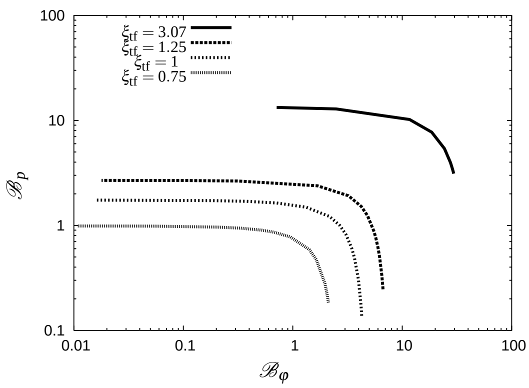

Figure 2: Lines of constant ξ_{tf} in the $(\mathscr{B}_\varphi, \mathscr{B}_p)$-space for $(\mathrm{m},\mathrm{n}) = (0, 5/2)$.

Linear perturbations

Following [2], the linear stability of these self-similar flows is investigated using an Eulerian description—in the (m,y,z)-coordinate system—of the flow linear perturbations, say ρ, v_x, $v_\perp (= v_y e_y + v_z e_z)$, p, T, the relevant system of PDEs reading then

$$\begin{aligned}
&\partial_t \rho + \overline{\rho}\left(\partial_m \overline{\rho}\, v_x + \overline{\rho}\, \partial_m v_x + \partial_m \overline{v}_x\, \rho + \nabla_\perp \cdot \overrightarrow{v_\perp}\right) = 0,\\
&\partial_t v_x + \overline{\rho}\, \partial_m \overline{v}_x\, v_x + \partial_m p - \partial_m \overline{p}\, \rho/\overline{\rho} = 0,\\
&\partial_t \overrightarrow{v_\perp} + \nabla_\perp p/\overline{\rho} = 0,\\
&C_v\left(\partial_t T + \overline{\rho}\, \partial_m \overline{T}\, v_x\right) + \overline{\rho}\, \partial_m \overline{v}_x\, T + \overline{p}\, \partial_m v_x + \partial_m \varphi_x + \left(\overline{p} \nabla_\perp \cdot \overrightarrow{v_\perp} - \partial_m \overline{\varphi}_x \rho + \nabla_\perp \cdot \overrightarrow{\varphi_\perp}\right)/\overline{\rho} = 0,
\end{aligned} \tag{6}$$

where $\nabla_\perp = (\partial_y, \partial_z)^{\mathsf{T}}$. In practice [2], the yz-Fourier transform of this system ξ-formulation is considered so that perturbation variables are replaced by their Fourier components of transverse-wavenumber modulus $k_\perp$: see eq. (7) below. The presence of the mean-flow reduced-variable ξ-derivatives as coefficients of the perturbation PDEs and the singular behavior (5) of these derivatives at the thermal-front $\xi = \xi_{\mathrm{tf}}$ call for a specific treatment there. This treatment consists in the change of variables $\xi \to \breve{\xi}$, $\widehat{Q} \to \breve{\widehat{Q}}$, for any of the dependent variables Q introduced in (2), as summarized by the relation

$$q(m,y,z,t) \xrightarrow{\mathcal{F}_{yz}} \widehat{Q}(\xi, k_\perp, t) = \breve{\xi}^{\,1-\mathrm{n}}\, \breve{\widehat{Q}}(\breve{\xi}, k_\perp, t). \tag{7}$$

The resulting formulations of (6) and of the linear-perturbation boundary conditions at the thermal-front, guarantee that the new perturbation variables $\breve{\widehat{Q}}$ remain finite there.

The numerical method that we are still currently developing for computing solutions to (6) in the present case, builds up upon the multi-domain Chebyshev spectral method devised in the perfect-shock-front approximation case [2], but implements a double formulation of the perturbation PDEs: the previous $(\xi, \widehat{Q})$-formulation for the flow region downstream to the isothermal shock-front ($0 \leq \xi \leq \xi_{\mathrm{s}}$) and the new $(\breve{\xi}, \breve{\widehat{Q}})$-formulation for the supersonic heat-wave region ($\xi_{\mathrm{s}} \leq \xi \leq \xi_{\mathrm{tf}}$).

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