Controlling chaos in wave-particle interaction

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We analyse the behaviour of a relativistic particle moving under the combined action of a uniform magnetic field and a stationary electrostatic wave given as a series of pulses. We apply to our system a method of chaos control that creates invariant tori in phase space, reducing and confining the chaotic trajectories to small regions. Besides regularizing the system, we show that this method of control is able to improve regular particle acceleration.

I. Introduction

Wave-particle interaction is basically a non linear process that presents regular e chaotic motion in its phase space. As a general rule, any system becomes more and more chaotic as we increase the perturbation applied to it. However, systems that present wave-particle interaction usually become chaotic for small values of the amplitude of the wave. Such a characteristic makes it important to control chaos in the system to have regular behaviour for larger wave amplitudes.

Some years ago, a method based in perturbation theory and Lie algebra was proposed in Ref. [1] to control chaos in conservative systems that can be described by an integrable Hamiltonian plus a small perturbation. This method consists in creating invariant tori in the whole phase space to remove global chaos.

The method developed in Ref. [1] has been used mainly as a way to control chaotic transport in Hamiltonian systems such as in magnetized plasmas [2-4] and fusion devices [2]. However, we will show that the control of chaos is also useful if one is concerned with regular particle acceleration. Periodic and quasi-periodic trajectories are recovered when chaos is reduced in phase space. Since these trajectories are the ones responsible for regular acceleration, it is possible to improve the process by controlling chaos in the system.

We analyse the regular acceleration experienced by a relativistic particle moving under the combined action of a uniform magnetic field and a stationary electrostatic wave. We show that the initial energy of the particle can be lower in the controlled system and even so its final energy will be higher than in the system without control. Thus we see that the particle is more accelerated when we control chaos in the system.

II. Description of the system and method of control

Consider a relativistic particle with charge q, mass m, and canonical momentum $\mathbf{p}$ moving under the combined action of a uniform magnetic field $\mathbf{B} = B_0\hat{\mathbf{z}}$ and a stationary electrostatic wave of wave vector k, period T, and amplitude $\varepsilon/2$ lying along the x axis. The transversal dynamics of this system is described by the dimensionless Hamiltonian (1) written in action-angle variables:

$$H = \sqrt{1+2I} + \frac{\varepsilon}{2}\cos\left(k\sqrt{2I}\sin\theta\right)\sum_{n=-\infty}^{+\infty}\delta\left(t-nT\right) \tag{1}$$

where $x = \sqrt{2I}\sin\theta$ and $p_x = \sqrt{2I}\cos\theta$. Besides, we chose to work with a pulsed system that is represented by the periodic collection of delta functions in Hamiltonian (1).

The method of control proposed in Ref. [1] and described in Ref. [3, 4] is valid for Hamiltonian systems that can be written as an integrable Hamiltonian H_0 plus a small perturbation εV, with $\varepsilon << 1$. This method aims to find a control term $f(\varepsilon V)$ such that the controlled Hamiltonian $H_f = H_0 + \varepsilon V + f(\varepsilon V)$ is integrable or presents a more regular behaviour than the original one. Moreover, the control term must fit another important condition: for energetical purposes, the amplitude of $f(\varepsilon V)$ must be much smaller than the amplitude of εV. For example, if εV is of order ε, than $f(\varepsilon V)$ should be of order ε^2.

Following the procedure described in Ref. [1, 3, 4], we calculate the control term $f(\varepsilon V)$ for Hamiltonian (1) as (see Ref. [5] for the details about the calculation of $f(\varepsilon V)$):

$$f(\varepsilon V) = \frac{\varepsilon^2}{8}\cos\left(2k\sqrt{2I}\sin\theta + \pi\right)\sum_{n=-\infty}^{+\infty}\delta\left(t-nT\right) \tag{2}$$

and we see that $f(\varepsilon V)$ corresponds simply to a stationary electrostatic wave of wave vector $2k$, period T, amplitude $\varepsilon^2/8$ and phase π lying along the x axis.

III. Results

The Hamiltonian of the system with the addition of the control term (2) is given by:

$$H = \sqrt{1+2I} + \left[\frac{\varepsilon}{2}\cos\left(k\sqrt{2I}\sin\theta\right) + \frac{\varepsilon^2}{8}\cos\left(2k\sqrt{2I}\sin\theta + \pi\right)\right]\sum_{n=-\infty}^{+\infty}\delta\left(t-nT\right) \qquad (3)$$

Figure 1 contains the phase space of the original system in Panel (a) and the phase space of the controlled dynamics in Panel (b). The figure was constructed for $T = 2\pi(1+1/15)$, $k = 2.2$ and $\varepsilon = 0.22$. In Panel (a), we have a situation of global chaos and almost all the phase space is filled by chaotic trajectories. Just the region of low action I and some islands remain regular. When we add the control term to the system, it creates invariant tori in phase space that prevent chaos to spread to large regions as can be seen in Panel (b). In this figure, chaos has destroyed just the separatrix of the islands and the invariant tori that are still present in phase space confine the chaotic trajectories to small regions around the islands of a chain.

For the parameters used in Figure 1, the amplitude of $f(\varepsilon V)$ is just 5.5% of the amplitude of εV. Nevertheless, the control term is able to reduce chaos in the system in a very efficient way. Another important feature of $f(\varepsilon V)$ is that it does not alter the main structures of islands in phase space as can be seen comparing both panels in Figure 1.

Considering the islands of the system, we observe that the value of the action I increases when the electrostatic wave transfers energy to the particle and therefore it is possible to coherently accelerate the particle. In Figure 1.(a), chaos has destroyed the most external trajectories of the islands centered at $I \approx 1.78$ and $\theta \approx 0;\ \pi$ and just the internal trajectories survive. In Figure 1.(b), the addition of the control term to the system recovers the most external trajectories of these islands, improving the process of regular acceleration.

Numerical calculations tell us that in the system without control, the maximum energy of the particle in those islands is $E_{\max} \approx 2.34$, while its minimum energy is $E_{\min} \approx 1.93$. In the controlled dynamics, the maximum energy of the particle in the islands is $E_{\max} \approx 2.41$ and its minimum energy is $E_{\min} \approx 1.86$. Thus, the minimum energy of the particle is 3.63% lower and its maximum energy is 2.99% higher in the controlled system than it would be in the system without control. It means that in the controlled dynamics we may begin with a lower initial energy and even so the particle will achieve a higher final energy.

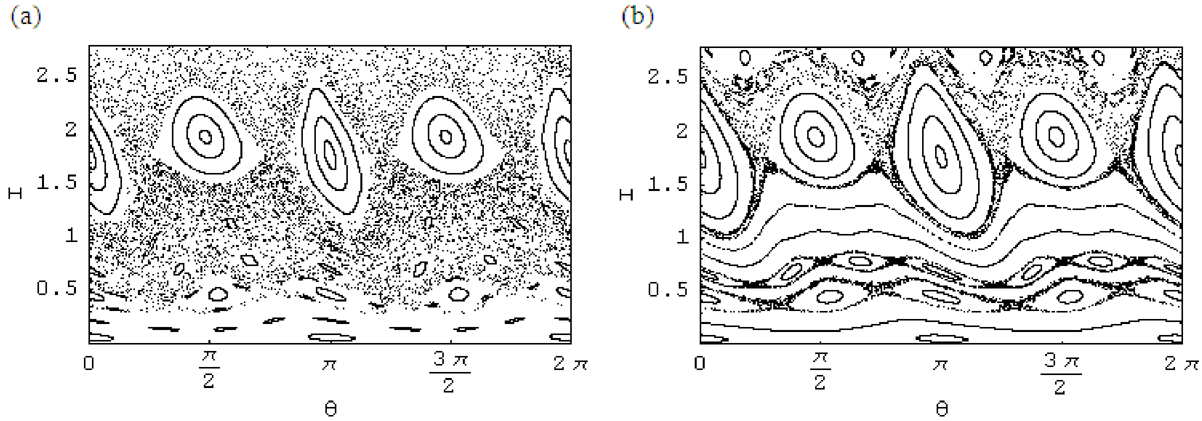

Figure 1: Phase spaces of the system for $T = 2\pi(1+1/15)$, $k = 2.2$ and $\varepsilon = 0.22$. Panel (a) illustrates the system without control while Panel (b) shows the system with the addition of the control term.

IV. Conclusions

We analysed the interaction between a magnetized relativistic particle and a stationary electrostatic wave given as a series of periodic pulses. We applied to the system a method of control of chaos developed in Ref. [1] for near-integrable Hamiltonians. The method consists in adding a control term to the system that creates invariant tori in the whole phase space, making the controlled dynamics more regular.

For the system under study, the control term is simply a second stationary electrostatic wave that should be added to the system. The amplitude of the control term is much smaller than the amplitude of the electrostatic wave originally applied to the system. Nevertheless, the control term is able to reduce and confine chaos in phase space very efficiently. Moreover, the addition of the control term to the system does not change its main structures of islands.

To conclude, we showed that the control of chaos improves the regular acceleration experienced by the particle in the islands of the system. In the controlled dynamics, the particle gets more accelerated than in the original system even with a lower initial energy.

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