Numerical modeling of seed island formation by forced magnetic reconnection

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Abstract: The seed island formation is studied numerically using two-fluid equations, and the island width is calculated for ASDEX Upgrade and reactor relevant parameters.

1. Introduction and theoretical model

Neoclassical tearing modes (NTMs) observed in experiments often grow up from small seed magnetic islands generated by triggers like sawteeth. In this paper two-fluid equations are utilized to study the seed island formation. The large aspect-ratio tokamak approximation is utilized. The magnetic field is defined as $\mathbf{B}=B_{0t}(\mathbf{e}_t-\mathbf{e}_\theta k_t/k_\theta)+\nabla\psi\times\mathbf{e}_t$, where ψ is the helical flux function, $k_\theta=m/r$ and $k_t=n/R$ are the wave vector in $\mathbf{e}_\theta$ (poloidal) and $\mathbf{e}_t$ (toroidal) direction, r and R are the minor and major radius, and the subscript 0 denotes an equilibrium quantity. The plasma velocity $\mathbf{v}=v_\parallel\mathbf{e}_\parallel+\nabla\phi\times\mathbf{e}_t$, where ϕ is the stream function.

The mass conservation equation, generalized Ohm's law, and equation of motion in the perpendicular (after taking $\mathbf{e}_t\cdot\nabla\times$) and parallel (to magnetic field) direction are utilized. Normalizing the length to the minor radius a, time t to the resistive time $\tau_R=a^2\mu_0/\eta$, ψ to aB_{0t}, $\mathbf{v}$ to a/τ_R, and electron density n_e to its value at r=0, one has [1]

$$\frac{dn_e}{dt}=d_1\nabla_\parallel j-\nabla_\parallel(n_e v_\parallel)+\nabla_\perp(D_\perp\nabla_\perp n_e)+S_n, \quad (1)$$

$$\frac{d\psi}{dt}=E_0-\eta j-\frac{\eta}{\nu_{ei}}\frac{dj}{dt}+\Omega\nabla_\parallel n_e, \quad (2)$$

$$\frac{dU}{dt}=-S^2\nabla_\parallel j+\mu\nabla_\perp^2 U+S_m, \quad (3)$$

$$\frac{dv_\parallel}{dt}=-C_s^2\nabla_\parallel P/n_e+\mu\nabla_\perp^2 v_\parallel, \quad (4)$$

where $d/dt=\partial/\partial t+\mathbf{v}_\perp\cdot\nabla_\perp$, $j=-\nabla_\perp^2\psi-2nB_{0t}/(mR)$ is the parallel plasma current density, and $U=-\nabla_\perp^2\phi$ is the plasma vorticity. $S=\tau_R/\tau_A$, $\tau_A=a/V_A$ is the toroidal Alfv́en time, η is the resistivity, E_0 the equilibrium electric field, S_n the particle source, and S_m the poloidal momentum source. $d_1=\omega_{ce}/\nu_{ei}$, ω_{ce} is the electron cyclotron frequency, $\Omega=\beta d_1$, $\beta=4\pi P/B_{0t}^2$, and $P=P_e$ is the electron pressure. C_s, ν_{ei}, μ, and $D_\perp$ are the normalized ion sound velocity, electron-ion collisional frequency, plasma viscosity, and perpendicular particle diffusivity. Constant electron temperature T_e and cold ion are assumed.

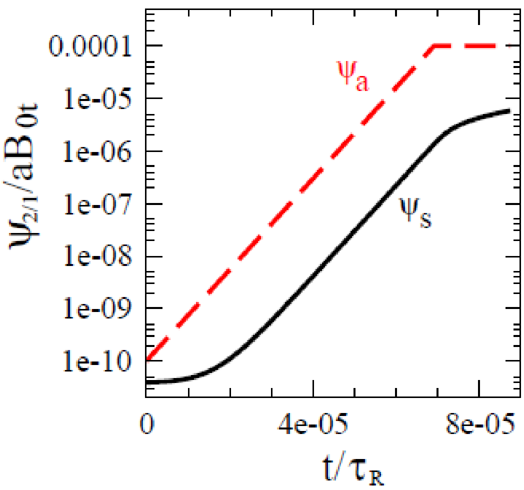
A monotonic profile for the safety factor q is assumed with the q=2 surface located at r_s=0.628a, and $L_q=q/(aq')=0.336$ at r_s. Only a single helicity, m/n=2/1, is considered. The plasma is stable against m/n=2/1 tearing modes if there is no externally applied trigger. The trigger for driving the seed island is taken into account by the following boundary condition,

$$\psi_a = \psi_{a0} exp[\gamma_f t + i(m\theta + n\varphi)] \quad (5)$$

for $\psi_a < \psi_{a,max}$, and ψ_a is kept constant in time after $\psi_a = \psi_{a,max}$ is reached, where $\psi_a = \psi_{m/n,a}/aB_{0t}$ is the normalized m/n component of ψ at r=a, γ_f is the growth rate of the trigger, and θ and φ are the poloidal and toroidal angle.

2. Numerical results and summary

Figure 1 demonstrates an example of the time evolution of the ψ_a with m/n=2/1 (dashed curve), which grows exponentially with $\gamma_f = 2\times10^5/\tau_R$ from $\psi_{a0}=10^{-10}$ until $\psi_{a,max}=10^{-4}$ is reached at $t=6.92\times10^{-5}\tau_R$, and afterwards it is kept constant in time. The solid curve shows the induced m/n=2/1 component of the normalized helical flux, $\psi_s=\psi_{2/1}/aB_{0t}$, at the q=2 surface, obtained from the reduced MHD approximation for $S=1.97\times10^8$, $\mu=0.21(a^2/\tau_R)$ and η_s=5.74, where η_s is the normalized plasma resistivity at r_s. One can see that there is a linear phase in which the mode growth rate is the same as that of the trigger.



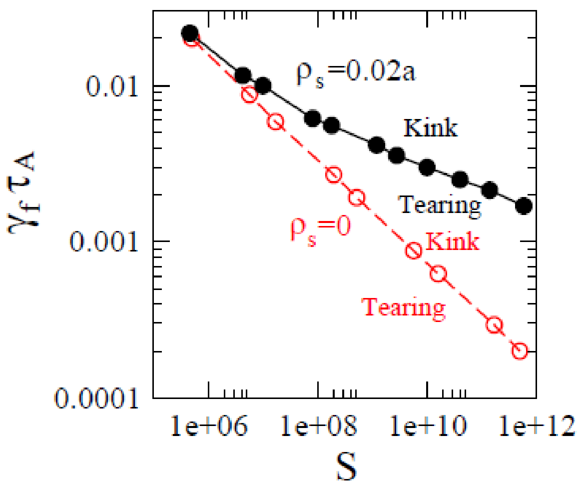
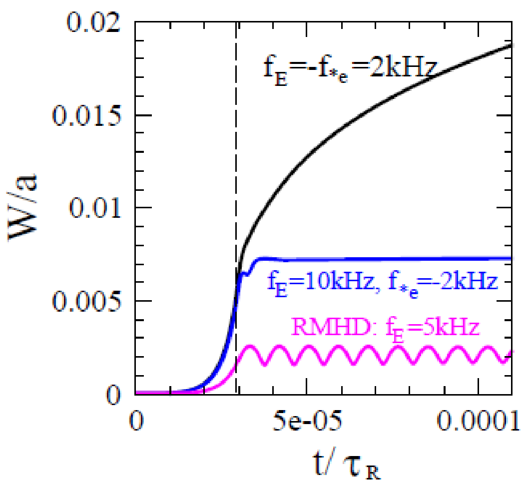


Figure 1 (left) Time evolution of the external perturbation ψ_a (dashed curve) and the induced m/n=2/1 component of the normalized helical flux, $\psi_s=\psi_{2/1}/aB_{0t}$, at the q=2 surface (solid curve). Figure 2 (right) Boundary between the driven tearing and driven kink mode in the S and $\gamma_f\tau_A$ plane for ρ_s=0.02a (solid curve) and 0 (dashed curve) in the linear phase.

Depending on plasma parameters and the trigger's growth rate, in the linear phase the driven mode can be of either a tearing or kink mode, as can be identified from the radial profile of $\psi_{2/1}$, which has the same sign across r_s for tearing modes but changes sign for kink modes. In figure 2 the boundary between the driven tearing and kink mode is shown in the S and $\gamma_f\tau_A$ plane by the dashed curve for ρ_s=0. The kink mode regime is found for $\tau_A\gamma_f>cS_s^{-1/3}$, being consistent with analytical theory [2], where $S_s=S\eta_0/\eta_s$ is the value of S at q=2 surface. From the dashed curve one finds c=1.02 and $c_1=c/(k_{\|}')^{2/3}$ =0.88. The solid curve is for $\rho_s=(T_e/m_i)^{1/2}/\omega_{ci}=a(\Omega d_1)^{1/2}/S=0.02a$ (neglecting the equilibrium electron density gradient, the ion parallel velocity and the electron inertia). The finite ρ_s effect extends the tearing mode regime to larger values of γ_f. Once the trigger's growth is slowed down in the nonlinear phase, the driven mode becomes a tearing type even if it is a kink type in the linear phase.

For ASDEX Upgrade parameters with B_{0t}=2T, a=0.5m, R=1.7m, T_e=2keV and $n_e=3\times10^{19}m^{-3}$ at r_s, and the local electron diamagnetic drift frequency f_{*e}=-2kHz, one has $S_s=2.65\times10^8$, $\Omega=9.43\times10^4$, $C_s=2.05\times10^7(a/\tau_R)$, $d_1=3.12\times10^7$, $\nu_{ei}=2.2\times10^4$/s, and $\rho_s/a=0.0065$. $\mu=18.8(a^2/\tau_R)$ and $D_\perp=\mu/5$ are assumed. Based on experimental data for type I ELMs, $1/\gamma_f=50\mu s$, $\psi_{a,max}=10^{-4}$, and after $\psi_a=10^{-4}$ is reached, it is kept constant for another 2ms [3].



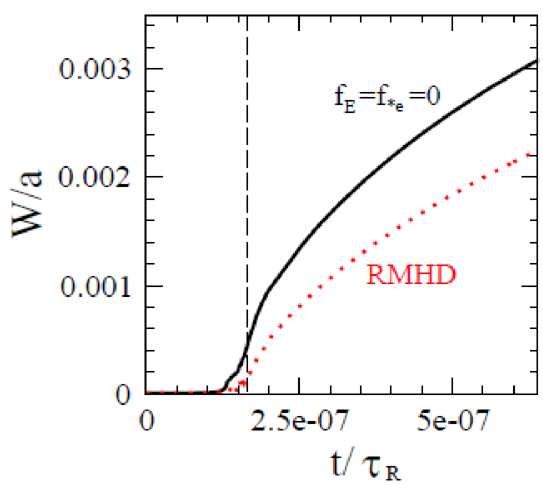


Figure 3 (left) Nonlinear growth of the m/n=2/1 island for f_E=2kHz and 10kHz for ASDEX Upgrade parameters. The island width obtained from the reduced MHD equations for f_E=5kHz is also shown. Figure 4 (right) Nonlinear growth of the m/n=2/1 island for $f_E=f_{*e}=0$ (solid curve) and reactor relevant parameters. The dotted curve is obtained from the reduced MHD equations.

The corresponding nonlinear growth of the m/n=2/1 island driven by the external field perturbation is shown in figure 3 for f_E=*2kHz* and *10kHz*, where f_E is the equilibrium plasma rotation frequency at r_s. f_E>0 refers to the rotation in the ion diamagnetic drift direction. For

f_E=*2kHz*, the driven mode frequency ($f=f_{*e}+f_E=0$) is the same as that of the trigger's, so that the resulting island is the largest. The island width reaches about 0.02a at $t=1.1\times10^{-4}\tau_R$, corresponding to the time that a constant $\psi_a=10^{-4}$ has lasted for 2ms. The vertical dashed line shows the time when ψ_a reaches $\psi_{a,max}$. For f_E=10kHz, being typical for ASDEX Upgrade discharges with neutral beam injection, the island width is only about 0.007a (0.35cm), since the relative frequency between the trigger and mode is $f=f_{*e}+f_E=8$*kHz*. The island width obtained from the reduced MHD equations for f_E=*5kHz* is much smaller. It is seen that the seed island generated by an (simulated) ELM is small for typical ASDEX Upgrade parameters when taking into account the frequency difference between the mode and trigger, being much smaller than the poloidal ion gyroradius, which is about a few centimetres.

Assuming for a fusion reactor with B_{0t}=6T, a=2m, R=6m, $\mu=0.2m^2/s$, $D_\perp=\mu/5$, T_e=10keV and $n_e=10^{20}m^{-3}$ at r_s, one has $S_s=1.94\times10^{10}$, $\Omega=1.8\times10^6$, $C_s=2.05\times10^9(a/\tau_R)$, $d_1=3.14\times10^8$, τ_R=4202s, $\mu=210(a^2/\tau_R)$, $\nu_{ei}=6.59\times10^3/s$, and ρ_s=0.0012a. In figure 4 the growth of the m/n=2/1 island, obtained from the two-fluid equations, is shown for $f_E=f_{*e}=0$ (solid curve). The induced seed island width reaches about 0.003a at $t=6.4\times10^{-7}\tau_R$, which corresponds to the time that the trigger perturbation $\psi_a=10^{-4}$ has lasted for 2ms. The vertical dashed line shows the time when ψ_a reaches $\psi_{a,max}=10^{-4}$. The dotted curve is obtained from the reduced MHD equations. When a frequency difference between the driven mode and trigger is taken into account, the island is found to be even smaller as expected. In the linear phase the driven modes are of the kink-like type for both cases. The smaller induced island compared to the ASDEX-Upgrade case is due to smaller plasma resistivity and ρ_s in this case.

In summary, (1) For typical ASDEX Upgrade parameters and a frequency difference 8 kHz between the trigger and the driven mode, the generated m/n=2/1 seed island is about 0.007a (0.35cm) for $\psi_{a,max}=10^{-4}$. (2) For a fusion reactor like ITER, if one neglects the frequency difference between the trigger and the driven mode and assumes that $\psi_{a,max}=10^{-4}$ lasts for 2ms, the generated m/n=2/1 seed island is about 0.003a (0.6cm), being much smaller than the poloidal ion gyroradius.

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