A Model for Electron Cyclotron Spectra at Down-Shifted Frequencies and Reconstruction of Superthermal Electron Velocity in Tokamak T-10

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1. Introduction. Recent observations of the downshifted electron cyclotron (EC) radiation spectra in tokamak T-10 simultaneously in the range of the first and second EC harmonics of thermal plasma EC emission suggested the presence of a fraction of superthermal electrons with energies in the range up to few hundreds of keV [1]. The observed spectra did not depend on the angles of detected waves and on their polarization. The characteristic feature of the experiments in tokamak T-10 is the smallness of the plasma density at the periphery (noticeably lower than such in large tokamaks) that allows the propagation of the 1st harmonic O-mode EC radiation at the periphery (with increasing density the allowable region for this radiation is expelled from plasma). Here we developed a model for quantitative interpretation of the above measurements in the case of the purely Ohmic discharge and formulated and solved an inverse problem for the reconstruction of the velocity distribution function (VDF) of superthermal electrons in tokamak T-10.

2. Physics model. We propose the following model to describe the observed ECR spectrum: (1) superthermal electrons are localized within 3 cm layer near the plasma edge (it is suggested by the experiment [1]); (2) almost vacuum limit for calculating the EC emission rate is used – Schott-Trubnikov formula [2] neglecting refraction, but with allowance for wave cut-off; (3) negligible absorption of ECR at down-shifted frequencies by the bulk plasma; (4) ECR intensity is homogeneous and isotropic in angles in-between the vacuum chamber metal wall and the cut-off zone due to multiple reflections of the radiation from the wall. The conditions of the proposed physics model allow us to apply the methods formerly developed and implemented in the numeric code CYNEQ [3] for ECR power losses in tokamak-reactors, because therein the radiation at high frequencies responsible for the EC power losses appears to be trapped in between the vacuum chamber wall and the inner region of high optical opacity. In this paper we do not consider mechanisms other than EC emission for the observed spectra.

The radiation spectral temperature, T_{rad}, is defined in the standard way in terms of the observed intensity of the outgoing EC radiation, I:

$$T_{rad}(\omega) = \frac{8\pi^3 c^2}{\omega^2} I(\omega) \quad (1)$$

The formula for the intensity of outgoing radiation, I, in the case of the neglect of mixing of the modes, caused by wave reflection from the wall of vacuum chamber, is as follows:

$$I(\omega) = I_X(\omega) + I_O(\omega), \quad (2)$$

$$I_{X,O}(\omega) = \frac{\int d\Omega_{\vec{k}} \int\limits_{V_{X,O}^{esc}(\omega)} dV\, q_{X,O}(\rho, \theta_{pol}, \omega, \theta_k)}{\int\limits_{(d\vec{S}\cdot\vec{n})\geq 0} d\Omega_{\vec{k}} \int\limits_{S_w} (d\vec{S}\cdot\vec{n})(1-R_w) + \int d\Omega_{\vec{k}} \int\limits_{V_{X,O}^{esc}(\omega)} dV\, \kappa_{X,O}(\rho, \theta_{pol}, \omega, \theta_k)}, \quad (3)$$

X and O denote, respectively, extraordinary and ordinary waves; ρ, effective radial coordinate (magnetic surface label); θ_{pol}, poloidal angle; $\vec{k}$, wave vector; $\vec{n} = \vec{k}/k$, wave direction; θ_k, pitch-angle of the wave vector $\vec{k}$; $q_{X,O}(\rho, \theta_{pol}, \omega, \theta_k)$ is power density of the ECR source; $\kappa_{X,O}(\rho, \theta_{pol}, \omega, \theta_k)$, ECR absorption coefficient; S_w, area of vacuum chamber's inner surface; R_w, reflection coefficient of EC radiation from the wall. $V_{X,O}^{esc}(\omega)$ is a part of the plasma volume where the ECR from superthermal electrons can propagate almost without absorption, with taking into account the wave cut-off. The power density of ECR source can be calculated in the vacuum limit, but with allowance for the wave cut-off:

$$q_{X,O}(\rho, \theta_{pol}, \omega, \theta_k) = \int d^3p\; F_e^{hot}(\rho, \theta_{pol}, p_{||}, p_\perp)\, \eta_{X,O}(\rho, \theta_{pol}, p_{||}, p_\perp, \omega, \theta_k), \quad (4)$$

$F_e^{hot}(\rho, \theta_{pol}, p_{||}, p_\perp)$ is the superthermal electron VDF, $p_\perp$ and $p_{||}$ are perpendicular and parallel to the magnetic field components of momentum, respectively; $\eta_{X,O}$ is the emissivity for a single electron, calculated using Schott-Trubnikov formula [2].

3. Reconstruction of supertermal electron VDF. To restore the VDF of superthermal electrons we solve an optimization problem that assumes determination of the function $F(\rho, \theta_{pol}, p_{||}, p_\perp)$ for which the spectrum, $T_{rad}^{calc}(\omega)$, calculated in the frame of the proposed model, is as close as possible to the experimental ECR spectrum, $T_{rad}^{exp}(\omega)$. We reduce the problem's dimensionality by averaging over the individual electrons trajectories which are calculated with allowance for two conservation laws, namely the adiabatic invariant for an electron in a non-uniform magnetic field and the conservation of electron total energy. Thus, the inverse problem for reconstruction of VDF corresponds to a search for the VDF, averaged over poloidal angle at a fixed magnetic surface (e.g., for trapped particles this means the averaging over banana trajectory).

The general form of the VDF's reconstruction algorithm is formulated as follows. The reduced phase-space $(p_{||0}, p_{\perp 0}, \rho)$ (where 0 index stands for the point of minimum magnetic field at the trajectory) is divided into the cells labeled with index N. For each cell we calculate the contribution $\tilde{T}_{rad}$ to the calculated spectra $T_{rad}^{calc}(\omega)$ by the formulas (1)-(4), which are applied to the cell and averaged over the electrons trajectory with $\int \frac{d\theta_{pol}}{2\pi} \to \frac{1}{\tau_b} \int \frac{ds}{|v_s|}$ operator.

Thus, the calculated spectrum is given by the formula:

$$T_{rad}^{calc}(\omega) = \sum_N \tilde{T}_{rad}(\omega, p_{||N}, p_{\perp N}, \rho_N) \cdot x_N, \tag{5}$$

$$x_N = F_e^{hot}(\rho_N, p_{||0N}, p_{\perp 0N}), \tag{6}$$

where x_N is the number of superthermal electrons in the N^{th} cell.

The optimization problem is formulated as follows:

$$\min f, \quad x_N \geq 0, \quad f = f(T_{rad}^{exp}(\omega), T_{rad}^{calc}(\omega)), \tag{7}$$

where f is the objective function defined as the deviation of the calculated spectrum, $T_{rad}^{calc}(\omega)$, from the experimental one, $T_{rad}^{exp}(\omega)$. Here we choose f equal to the square of the Euclidean norm (L-2 norm), and T_{rad} functions are considered as vectors in the discretized ω-space.

The results of the reconstruction of the superthermal electronVDF for the case of one of the observed spectra in the discharge № 36057 in tokamak T-10 (see curve №3 in Figure 2 for radiation spectral temperature in [1]) are presented in fig. 1. Main parameters of tokamak T-10 are as follows: major radius R_0=1.5 m, minor radius a=0.3 m, elongation k_{elong}=1, magnetic field B_0=2.48 T at R=R_0,. We take the value of the wall reflection coefficient R_w=0.6. The spectral region under interpretation includes the region lower than the 1st (38-60 GHz) and the 2nd harmonic cold resonance at low magnetic field side (78-120 GHz). The 1st harmonic thermal radiation range (60-78 GHz) and the 2nd harmonic thermal radiation range (> 120 GHz) are excluded (here there is a strong absorption of the ECR from superthermal electrons by the bulk plasma).

4. Conclusions. We formulated and solved an inverse problem for the reconstruction of the velocity distribution function (VDF) of superthermal electrons in parallel and perpendicular momenta, and in magnetic flux surfaces at the plasma edge. We consider the case when superthermal electrons are localized within 3 cm layer at the plasma edge as suggested by the experiments [1]. It is shown that in the Ohmic discharges in tokamak T-10 the superthermal electrons are the trapped particles at the low magnetic field side of toroid. Their mean kinetic energy lies in the range ~150-200 keV, and the density fraction is about ~10^{-4} relative to the main plasma at the edge.

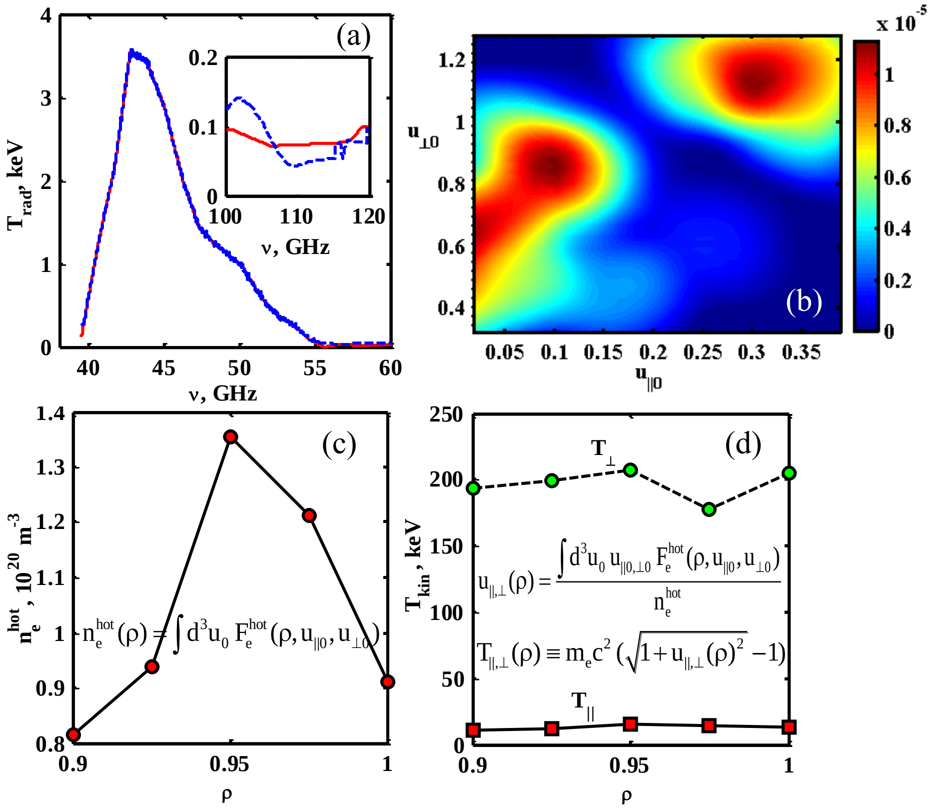

Fig. 1 (a) Radiation spectral temperatures: observed (red) (see curve №3 in Figure 2 [1]) and obtained as a result of solving the optimization problem (7) (blue). (b) The superthermal electron VDF, $x = F_e^{hot}(\rho, u_{\|0}, u_{\perp 0})$ ($u=p/m_e c$), for fixed magnetic surface ρ=0.9, reconstructed from the observed spectral intensity. (c) The profile of superthermal electron density. (d) The profile of superthermal electron mean kinetic energy. Mean kinetic energies of superthermal electrons obtained by spatial averaging are as follows: $<T_{kin}>=213$ keV, $<T_{kin\|}>=13$ keV, $<T_{kin\perp}>=196$ keV. The ratio of superthermal electron density to the main plasma density within the layer is $\delta_{hot}=1.8\ 10^{-4}$.

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