Dynamics of a Two-Dimensional Flow Subject to Electromagnetic Forces

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abstract. A novel experimental is presented to study the dynamics of two-dimensional (2D) structures of an electrolyte solution subject to electromagnetic forcing. A thin layer of potassium hydroxide is poured into a square-base container with a strong magnetic field achieved by permanent neodymium magnets inserted underneath the base. The set of electrodes of alternating polarity distributed along the perimeter of the container generates currents in opposite directions in the flow. Coherent primary vortices are thus generated by the $\vec{j} \times \vec{B}$ force and farther from the edge secondary vortices are observed driven by the primary ones and viscosity. We show that the gradient of the magnetic field leads to vortices which gives rise to 'jets' moving the fluid in a rectilinear fashion. The coupling of this *gradB* jet with the primary and secondary vortices leads to, (1), the onset of larger vortices by inverse cascade thus (2), the destruction of the small-scale vortices and (3) the modification of the jets directions. This occurs when the velocities of the jet and the corners vortices are in the same direction but not when they are opposite.

Experimental Setup. The container is illustrated in Fig. 1 having a square shape with a base dimension of 25×25 cm made of electrically non-conducting Plexiglas. We install 13 stainless-steel electrodes on each side separated by $L = 2$ cm and connected in parallel to a DC power supply (2.5 V, 1 A) with *alternating polarity* and constant voltage. The electrolyte used is a Potassium Hydroxide (KOH) solution with a concentration is 27% of water weight where the solution is at its maximum conductivity of approximately 550 Siemens/m [1]. The viscosity of the KOH solution is about that of water. The height of the solution is $h = 1$ cm making the ratio $h/L = 0.5$ which is in the range where three-dimensional effects are quite limited [2]. Some properties of the KOH solution in SI units are inserted in the table below.

Formation of Primary Vortices. One droplet is inserted close to the container edge and left to diffuse creating a spot with diameter about 2 cm, that is the distance between two electrodes. The current is then switched on and the evolution of the flow in front of the electrodes is recorded. In (b), taken 2 seconds after (a), the fluid motion is visible where in between the electrodes the dye is pushed away from the edge while outside it is pulled towards the edge. One can estimate the velocity of the flow to be about 1 cm/s. In (c) and (d), one can identify the bipolar vortex which starts to change shape as a consequence of the non-linear interaction among neighboring vortices. These results are rather similar to the experiment and numerical simulation in Ref. [3].

Self-generated Jets and Inverse Cascade In order to understand the origin of the diagonal jet, we perform an experiment where the dye initially

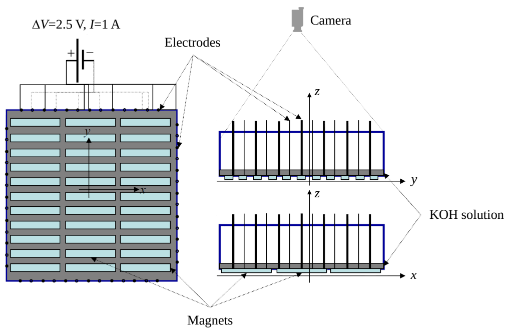

Figure 1: Illustration of the experimental setup with a top view and two side views of the container. All of the electrodes are connected to the power supply alternatively so the negatively charged electrodes (open circles) are shown to be connected with dotted lines whereas the connection of the positively charged electrodes (filled circles) is shown in solid lines. The magnets are installed in three 'drawers' inserted in the y-direction each containing 11 magnets. All the magnets have their North pole up the page. The x, y and z directions are illustrated.

covers the entire surface of the container. Fig. 2(a) is taken 10 s after the current is switched on. It shows that the jets are not yet detected and primary vortices are seen at the container edges. Fig. 2(b) and (c), taken 15 and 20 s respectively after the power is set on, shows the two parallel jets in the y-direction with the dye transported over almost the full container width. The absence of a magnetic field in the area where the jets are formed creates a strong magnetic field gradient of about $\partial_x B \simeq 1$ T/m. It is rather well-known in plasma physics that despite the explicit absence of the magnetic gradient in the magnetohydrodynamic equations, it plays nevertheless a primary role similar to the pressure gradient [4]. The pressure gradient term is thus eliminated and the resultant equation is that for vorticity in the z-direction (ω_z), where

$$\rho \partial_t \omega_z = -j_x \partial_x B_z - j_y \partial_y B_z. \tag{1}$$

The general form of ohm's law leads to a current $\vec{j} = \sigma(\vec{E} + \vec{v} \times \vec{B})$. However, since we are far from the electrodes, one may neglect the electric field contribution leading to

$$\partial_t \omega_z = \frac{1}{2\rho} \sigma u_x \partial_x B_z^2.$$

The role of the magnetic field gradient in producing vorticity is now obvious. The magnetic field dip in the x-direction leads to a gradient which is positive and negative driving vorticity in opposite directions, thus generating counter-rotating structures. Extending this analysis to several parallel magnets, a succession of opposite and counter-rotating vortices take place in the region of

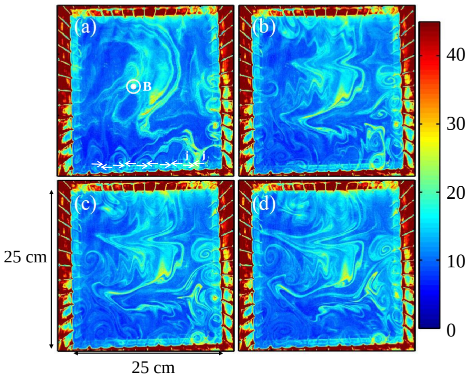

Figure 2: (a) shows the dye after it was manually diffused to fill the whole container. This snapshot is taken 10 s after turning on the current and the primary vortices are rather clearly seen near the edges of the container. In (b), taken at $t = 15$ s, the two jets are observed as they move the dye in the y-direction. This is even clearer in (c) and (d) taken at $t = 20$ and 25 s, where the interaction between these $gradB$-jets and the primary and secondary vortices is observed leading to the modification of both properties.

strong magnetic field gradient giving rise to a jet in the region of minimum magnetic field as observed experimentally.

The second phase in the diagonal jet generation is dominated by the $gradB$ jet interaction with secondary and primary vortices which exist on its side as it can be seen in Fig. 2. In order to understand the $gradB$ jet interaction with vortices, we perform an experiment where glass beads are used as tracers and the illumination is done using a laser sheet. We overlay the result of several contours in order to show the motion of the beads from one frame to another. This is shown in Fig. 3 where in (a) one of the two $gradB$ jets is seen heading from the top of the figure to its bottom along with the secondary vortices on the sides. Because of the jet interaction with the secondary vortices, these vortices merge into a larger vortex observed in Fig. 3(b). We deduce that the interaction of the $gradB$ jet with the surrounding vortices leads to (1) vortex merging at the edge and (2) modification of the $gradB$ jet direction from the y-direction to almost a diagonal direction. The jet is still observed in Fig. 3(d) taken 40 seconds later which reflects its persistence as a function of time.

Conclusion. We presented a study of two-dimensional coherent structures using conducting liquids subject to electromagnetic force. The electric field is generated between equally-spaced electrodes inserted at the edge of the container. The magnetic field is created by permanent neodymium bar magnets fixed below the container. When the current is turned on, coherent vortices are generated at the edge in agreement with the theoretical predictions. Since the flow is viscous, primary vortices lead to the creation of secondary vortices which

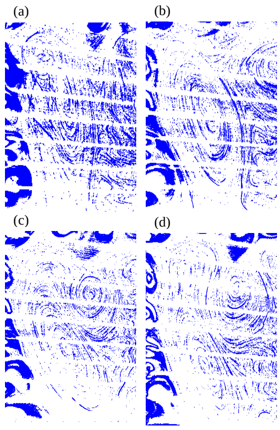

Figure 3: The formation of a diagonal jet as a function of time is illustrated in this figure as we overlay the contour plots of 50 images at half the light intensity maxima coming from the beads. In (a), $t = 10-20$ s primary and secondary vortices are visible as well as the *gradB* jet. In (b) $t = 30-40$ s, it shows the vortex merging and the onset of one large vortex. In (c), $t = 70-80$ s shows the appearance of another large vortex that was hardly visible in (b) to the extreme right to form two counter-rotating vortices and the jet now is moving in the diagonal direction. in (d), we overlay between $t = 110-120$ s after the power is switched on and it confirms (c). The white straight lines result from the shadow of the electrodes which intercept the laser sheet.

are farther from the container edge. The setup of the magnets created a region with a large gradient in the axial magnetic field. It is shown that this generates vorticity and thus lead to the onset of what we call *gradB* jets. The interaction of these jets with the primary and secondary vortices lead to a modification of the jet direction as well as vortex merging. Two conditions appear to be important in this study, (1), the direction of the jet with respect to the corner vortices and (2), no vortex merging is recorded without the perturbation of the *gradB* jets.

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