Free-boundary equilibrium transport simulations of ITER scenarios under control

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Introduction

In this paper, we report on our recent development and applications of coupled transport, free-boundary equilibrium (FBE) and magnetic control simulations of tokamak discharge evolution. The well established CRONOS transport code [1] now includes a recent free-boundary equilibrium solver, FREEBIE [2]. The coupling scheme takes particularly care of the consistency between equilibrium and transport and features also an implicit (self-consistent) mode. FREEBIE provides realistic magnetohydrodynamic (MHD) equilibria, which are compatible with the poloidal field (PF) system of the tokamak. Plasma dynamics is described by circuit equations (together with the transport equations). FBE is important e.g. for current ramp-up and ramp-down, when the PF coils must induce large currents while staying within the Volt-seconds limits, or for long (steady-state) high-performance phases, when the plasma shape must be maintained precisely. FBE are, as in reality, subject to horizontal and vertical displacement instabilities, which must be controlled by a properly designed controller. Here, we use the same controller framework as is used with DINA-CH [3, 4]. However, the controller interface is very general and enables coupling with other controllers, independent of the control parameters and actuators.

Transport – free-boundary equilibrium coupling

Although equilibrium and transport are tightly coupled through the geometry and the pressure and current profiles, their simulations are often carried out independently or loosely coupled. Codes that have advanced FBE capabilities often do not offer sufficient transport and/or heating and current drive source models. On the other hand, CRONOS, which is a state-of-the-art transport suite of codes, was, until recently, lacking a FBE module; FBE simulations were only possible using an explicit coupling with DINA-CH with the same small time steps as are necessary to follow the equilibrium evolution [3].

We have fully incorporated FREEBIE into CRONOS, taking advantage of the fact that both codes are implemented in Matlab. The coupling is schematically shown in Fig. 1. Two coupling modes are possible: explicit and implicit. In both cases, the equilibrium is solved until a convergence criterion on the averaged plasma current profile is satisfied. In the implicit mode, the transport equations are iterated during a single time step together with the equilibrium and an additional convergence criterion on ψ and on the toroidal flux scale is added. Therefore, self-consistency is provided between the transport and the equilibrium equations and no equilibrium quantities (such as the geometric coefficients) are considered constant any more during the transport equations time step. The convergence criterion is

$$\varepsilon^k < \tau \vee \varepsilon^{k-1} - \varepsilon^k < \tau' \qquad (1)$$

where k is the iteration index, τ and τ' are the requested tolerances and

$$\varepsilon^k \equiv \left\| \langle j \rangle_{\mathrm{eq}}^k - \langle j \rangle_{\mathrm{diff}}^k \right\| + \left\| \psi_{\mathrm{diff}}^k - \psi_{\mathrm{diff}}^{k-1} \right\| + \left| \rho_{\max}^k - \rho_{\max}^{k-1} \right| \qquad (2)$$

The last two terms in (2) are non-zero only in the implicit mode, $\rho_{\max}$ is the toroidal flux on the plasma boundary.

Crucial for the FBE – transport coupling are boundary conditions. Inconsistencies in the fluxes can lead to incorrect results. Typically applied prescribed total plasma current or loop voltage conditions cannot be used as these quantities are now solved consistently with the equilibrium. We have implemented two boundary conditions, which both seem to work properly. The first one is a predictor-corrector type. Assume there exist a difference between the boundary fluxes coming from the transport (current diffusion) and the equilibrium equations: $\Delta\psi = \psi_{\mathrm{b}}^{\mathrm{diff}} - \psi_{\mathrm{b}}^{\mathrm{equil}}$. We now make use of a general plasma inductance L_{i}:

$$\psi_{\mathrm{a}} - \psi_{\mathrm{b}} = L_{\mathrm{i}} I_{\mathrm{p}} \qquad (3)$$

to construct a plasma current predictor for the next transport solution:

$$I_{\mathrm{p}}^* = I_{\mathrm{p}} \left(1 + \Delta\psi / \left(\psi_{\mathrm{a}} - \psi_{\mathrm{b}} \right) \right), \qquad (4)$$

which leads to minimize $\Delta\psi$. The second boundary condition requires splitting of the plasma and external magnetic fluxes at the magnetic axis: $\psi_{\mathrm{a}} = \psi_{\mathrm{a}}^{\mathrm{ext}} + \psi_{\mathrm{a}}^{\mathrm{pl}}$ and using a general relation

$$I_{\mathrm{p}} = -C\psi', \quad \psi' = \left. \frac{\partial \psi}{\partial \rho} \right|_{\rho=\rho_{\max}} \qquad (5)$$

where C is a geometric constant dependent on the plasma shape, slowly varying with time. Finally, the discrete time derivative of (3) yields a Robin boundary condition:

$$\psi_{\mathrm{b}} + C L_{\mathrm{ext}} \psi' = \left(\tilde{\psi}_{\mathrm{b}} + \psi_{\mathrm{a}}^{\mathrm{ext}} - \tilde{\psi}_{\mathrm{a}}^{\mathrm{ext}} \right) - \tilde{L}_{\mathrm{ext}} \tilde{I}_{\mathrm{p}}, \qquad (6)$$

where tilde denotes the value at the previous time and $L_{\mathrm{ext}} = \psi_{\mathrm{a}}^{\mathrm{pl}} / I_{\mathrm{p}} - L_{\mathrm{i}}$.

A controller, represented by a black box in Fig. 1, is used to evolve the plasma according to prescribed waveforms and to stabilize the plasma position. The same controllers that are used for DINA-CH and CRONOS-DINA-CH simulations are used. The input control parameters are the coil currents, the plasma current centroid position, I_{p} and the gaps synthetic diagnostics. The output actuators are the PF coil voltages. There is principally no limit on the input and output variables; the controller interface, which can be called either from Matlab or from Simulink via a level-2 S-function, is completely independent of the simulation and can output any available control parameters. More actuators can also be input, e.g. heating and current drive sources for more detailed plasma profiles control.

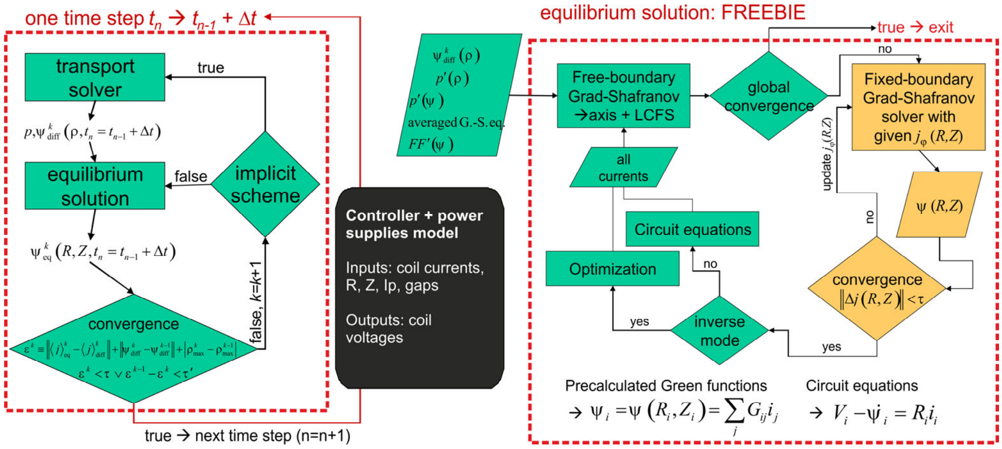

Fig. 1. The CRONOS – FREEBIE coupling scheme. Single time step is depicted on the left while the detail of the equilibrium solver is on the right.

Results for an ITER hybrid scenario

An ITER hybrid scenario, which has been developed using CRONOS fixed boundary simulations, is analyzed by means of the free-boundary equilibrium CRONOS simulations described above, using the explicit coupling scheme. In particular, the initial plasma evolution is simulated. Feed-forward currents are required for the (R, Z, I_p)-controller employed for this part of the scenario. We calculate these currents using the inverse mode of FREEBIE, in which the plasma shape and the current are prescribed and the circuit equations are substituted by an optimization routine—see Fig. 1. A suitable regularization term, which minimizes the currents in the PF coils and their derivatives, must be included in the optimization procedure. Otherwise, saturated coil currents and unreasonable voltage demands are calculated by FREEBIE. In Fig. 2 and 3, we show the results of a simulation with such feed-forward currents. During the first 8 seconds, the power supply model, which takes into account the voltage limits, is switched off. Excessive voltages, resulting primarily from the feed-forward currents, are applied and the plasma can be reasonably controlled. After 8 s, the controller cannot sustain the reference waveforms because of the voltage limits and coil saturations and the simulation finally stops converging at ~11.5 s.

By switching on the regularization term, which minimizes $\sum I^2_{\mathrm{PF\,coils}}$ and $\sum \Delta I_{\mathrm{PF\,coils}} / \Delta t$, and finding its suitable weight, it was possible to create more reasonable waveforms and feed-forward currents. Corresponding results are shown in Fig. 4 and 5. The voltage demands are now much lower and within the power supplies limits, except for the first ~50 ms, when the shell currents are building up. We compare results obtained with the Robin and the predictor-corrector boundary condition (b.c.). The time traces of the voltages and the control parameters are similar. The Robin b.c. yields somewhat more numerical noise (see e.g. the oscillations around 3.5 s). Very importantly, as shown in Fig. 5, the transport and equilibrium values of ψ on the axis and the boundary are consistent, independent of the b.c.

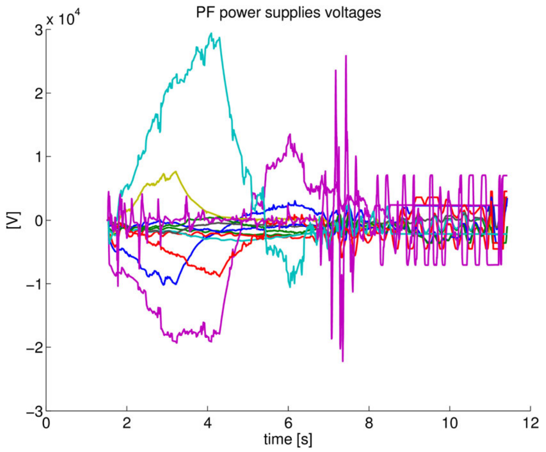

Fig. 2. Power supplies (CS, PF, VS) voltages from the full FBE CRONOS controlled simulation with non-optimized feed-forward currents.

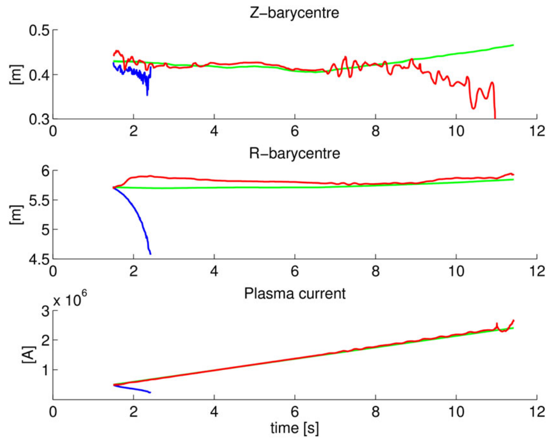

Fig. 3. Time traces of the plasma barycentre (current centroid) position and I_p: reference (green), controlled (red) and uncontrolled (blue) simulation.

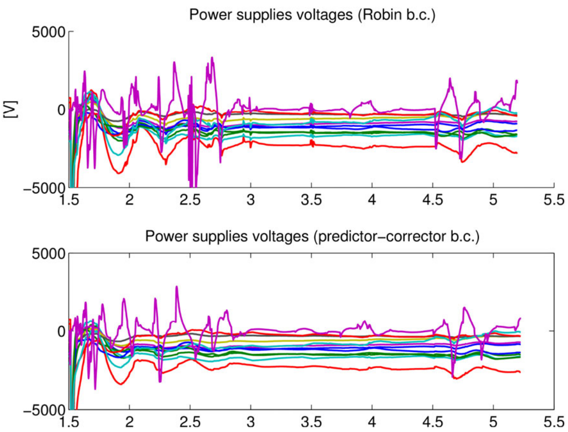

Fig. 4. Power supplies (CS, PF, VS) voltages from the full FBE CRONOS controlled simulation with better optimized feed-forward currents.

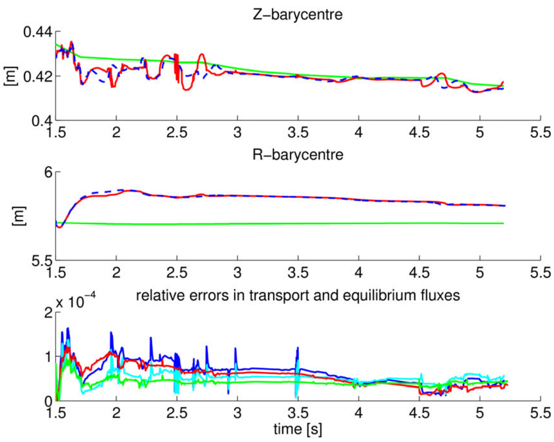

Fig. 5. Top and middle: time traces of the plasma barycentre: reference (green), Robin b.c. (red), predictor-corrector b.c. (dashed blue). Bottom: $\left|\psi_{a,b}^{diff} - \psi_{a,b}^{eq}\right| / \psi_{a,b}^{diff}$ for Robin (blue, cyan), and predictor-corrector (red, green) b.c.

Summary

CRONOS is now fully equipped for free-boundary equilibrium simulations using FREEBIE and an external controller. Both explicit and implicit schemes are implemented. These capabilities are demonstrated on a limited part of an ITER hybrid scenario, showing that with optimized waveforms it is possible to perform simulations in free-boundary regime with reasonable results and that equilibrium and transport are simulated consistently.

References

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