Deconvolution of collective scattering signal measured on Hall thruster plasma.

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1 Introduction

The plasma of a Hall thruster is created in a cylindrical channel between an external cathode and an interior anode. The electrons emitted from the cathode are trapped in the channel by a radial magnetic field and ionize the Xenon atoms injected at the anode. Then, the ions are accelerated by the electric field at a high velocity (around 15 km/s) and produce the thrust that would drive a satellite. Since the exhaust velocity of electrical thrusters is much greater than that of chemical thrusters, less propellant is necessary for a given displacement.

However, the efficiency of these thrusters is significantly reduced by electron transport towards the anode. The electron mobility can not be explained by classical diffusion and the transport is said to be anomalous. Wall conductivity or micro-turbulence are the two main phenomena that could explain this anomalous transport. Micro-turbulence at the millimeter scale has been observed in PIC code simulations, predicted by linear kinetic theory and observed experimentally by the collective light scattering technique [1]. Mainly, two modes have been discovered with the diagnostic: the axial mode parallel to the axial electric field and the azimuthal mode almost parallel to the $\vec{E} \times \vec{B}$ direction.

In most case, the width in the k-space of the density fluctuations is much larger than the one of the gaussian profile of the laser beam. Accordingly, the raw signal obtained with the collective scattering is assumed to be proportionnal to the dynamic form factor $S\left(\vec{k}, f\right)$. However, this is not valid for the azimuthal mode and the raw signal must be deconvoluted.

2 The collective light scattering diagnostic

Collective scattering is a non-invasive diagnostic used to investigate electron density fluctuations at scales much larger than the Debye length by means of a laser beam. It can be shown that the far scattered electric field is proportionnal to the spatial Fourier transform of the density $n(\vec{r}, t)$ weighted by the electric field scattered by a single electron. Based on the superheterodyne technique [2], both the amplitude and the phase informations of the field are measured with the sensor. The spectral power density is

$$P\left(\vec{k},f\right)=P_0\iiint d^3\vec{k}'\left|W\left(\vec{k}-\vec{k}'\right)\right|^2 S\left(\vec{k}',f\right) \quad (1)$$

with P_0 a quantity that depends on the diagnostic parameters and $\left|W\left(\vec{k}\right)\right|$ the Fourier transform of the measurement volume profile defined by

$$\left|W\left(\vec{k}\right)\right|^2=\frac{e^{-\frac{\|\vec{k}_\perp\|^2}{2\Delta k^2}}}{\left(\sqrt{2\pi\Delta k^2}\right)^2}\delta\left(k_z\right) \quad (2)$$

In our experiment, $\Delta k=\sqrt{2}/w_0\approx$416 rad/m is the standard deviation of the gaussian beam width in the $\vec{k}$ space [3] and $w_0\approx$3.4 mm the gaussian beam waist. k_z and $k_\perp$ are along and perpendicular to the beam axis, respectively. The norm k and the direction α of the observation wave vector $\vec{k}$ can be varied on the optical bench. In Fig. 1, the measurement volume and the wave vector are presented as well as the angle α between the x-axis that is in the direction of the electric field and the k vector. Notice that the y-axis is assimilated to be in the $\vec{E}\times\vec{B}$ direction.

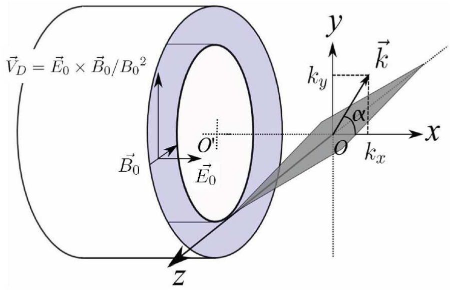

Figure 1: Sketch from Ref. [1] of the scattering volume (in gray) in front of the Hall thruster.

3 The collective light scattering diagnostic applied to the azimuthal fluctuations

Raw measurements of the form factor have shown that its dependence is exponential in k, gaussian in α and in f. As the convolution of an exponential or gaussian function remains of the same form, the dynamic form factor will be written

$$S(k,\alpha,f)=Ae^{-\rho k}e^{-\frac{(\alpha-\alpha_{max})^2}{2\Delta\alpha^2}}\frac{e^{-\frac{(f-f_D(\alpha,k))^2}{2\Delta f^2}}}{\sqrt{2\pi\Delta f^2}} \quad (3)$$

where A is the amplitude, ρ is the characteristic length of the exponential function, α_{max} and $\Delta\alpha$ the center and the standard deviation angles of the Gaussian function, respectively. The maximum $f_D\left(\alpha,k\right)$ of the f-gaussian function is the so-called dispersion relation of the $\vec{E}\times\vec{B}$ and Δf the standart deviation. In the jet frame, the former is predicated to be isotropic and to vary linearly with k on the explored interval. In the laboratory frame, a Doppler effect must be added due to the parameter v_p, the beam velocity assumed to be along the thruster axis. Thus, it can be written as

$$f_D\left(\alpha,k\right)=\frac{kv_p}{2\pi}\cos\left(\alpha\right)+\frac{kv_g}{2\pi}+f_{cst} \quad (4)$$

where the parameter v_g represents the group velocity and the frequency f_{cst} is the intercept of the linear dispersion relation.

After convolution, it can be shown [4] that the amplitude A_0 and the standart deviation $\Delta\alpha_0$ of the observed static form factor are linked to the unconvoluted one by

$$\begin{cases} A_0 = A\sqrt{\dfrac{\Delta\alpha^2}{\Delta\alpha^2+\frac{\Delta k^2}{k^2}}} \\ \Delta\alpha_0 = \sqrt{\Delta\alpha^2+\frac{\Delta k^2}{k^2}} \end{cases} \tag{5}$$

In addition, the angle α_0 at which the dispersion relation is observed is equal to

$$\alpha_0 = \frac{\alpha\Delta\alpha^2+\alpha_{max}\frac{\Delta k^2}{k^2}}{\Delta\alpha^2+\frac{\Delta k^2}{k^2}} \tag{6}$$

Equation (6) gives the true angle at which the dispersion relation is measured by collective scattering. It means that when measuring in the direction α, one is actually measuring in the direction α_0 which is pulled towards α_{max}.

4 Deconvolution of the collective scattering signal

Using Eqs. (5) and (6), the signal of the collective scattering can be deconvoluted. In Fig. 2(a), static form factors of the azimuthal mode are presented in function of α and for different k. The result of the deconvolution is that the amplitude is underestimated by collective scattering while the standard deviation is overestimated. For the different k values, the corrected standart deviation $\Delta\alpha_0$ varies less then 10% : $\Delta\alpha_0 = 74 \pm 6.3$ mrad. In Fig. 2(b), the experimental and corrected dispersion relations are presented in function of α and for different k. As expected from Eq. (6), the α intervals explored experimentally are shortened after deconvolution. To both, experimental and corrected points, a fit of the form

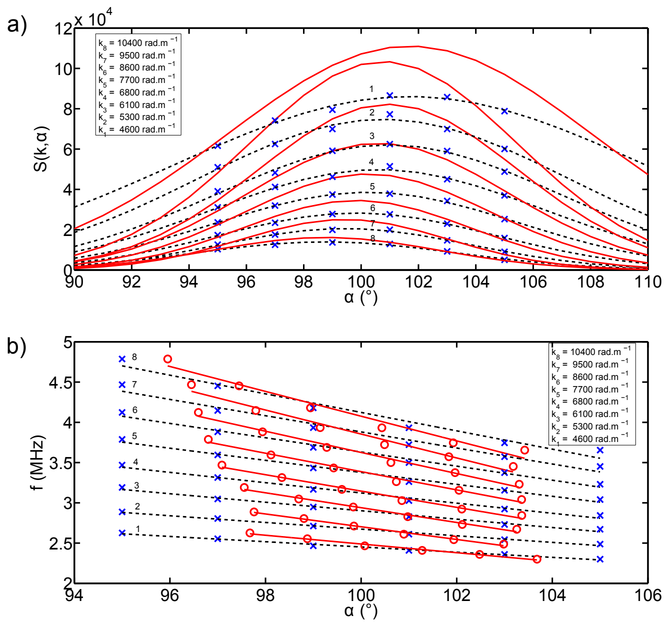

Figure 2: Crosses: Experimental points. Dashed lines: (a) Gaussian fit or (b) fit with Eq. (7) of the experimental points. Solid lines: (a) Corrected form factor or (b) dispersion relation.

$$f_{fit} = f_0(k) + \frac{kv_p}{2\pi}\cos(\alpha) \tag{7}$$

is applied to obtain the two parameters v_p and f_0. The parameter f_0 is seen to be shifted towards higher values on the order of 0.4 MHz after deconvolution. In Fig. 3, the variation of the corrected and experimental v_p is depicted in function of k. For the raw data, it is clear that v_p is not homogeneous and increases with k. On the other hand, after deconvolution, the trend of v_p is rather constant with a mean value of 5510 m/s and an standart deviation of 115 m/s. Thus, experimental results are compatible with the idea that in the jet frame the frequency of the azimuthal mode is independent of the orientation of the wave vector. However, the mean value is 5510 m/s about two times lower than the expected ion beam velocity. The reason is not yet clear but the localization of the scattered signal in front of the thruster could be an explanation. The source of the signal could be localized near the outer edges of the thruster. As the ion beam has a certain divergence, the ion velocity is not purely along the axis and the axial velocity would be lower on the edges. Comparative measurements with LIF are planed to investigate this point.

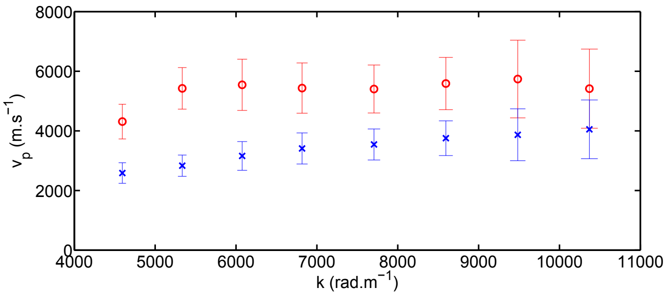

Figure 3: Crosses: v_p from the experimental points. Circles: v_p from the corrected points.

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