A model of radial-electric-field generation in the tokamak boundary plasma

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1. According to recent studies [1], the radial diffusion of ions plays an important role in the generation of radial electric fields. The stochastic motion of impurity particles induced by electrostatic turbulent fluctuations in the boundary region of a tokamak plasma was analyzed in [2, 3, 4]. There it was shown that the diffusion coefficient is rather high for heavy impurities (such as the C^+ ions considered there), and argued that the resulting expulsion of heavy ions from the turbulent region leads to charge separation. In our model, the heavy ions undergoing stochastic motion move outwards of the turbulent region. This region we represent in the form of a stripe of width $2L$, $(-L \le x \le +L)$, in the poloidal section. One may say that the heavy ions experience a “turbulent drag force” which pushes them out of the turbulent stripe. Outside the stripe this turbulent drag force is absent, leading to depletion and accumulation of heavy ions at the inner and outer periphery of the turbulent region, respectively. Such heavy-ion depletion and accumulation is also furthered by collisions with neutrals outside the interval $(-L, L)$, impairing ion motion. Thus, charge separation takes place and a radial electric field $E_x(x,t)$ is gradually generated. We write the total heavy-ion drift velocity in the form $V(x,t) = U(x) + \mu E_x(x,t)$, with $\vec{U}(x)$ the stochastic part due to the (stationary) turbulent fluctuations and $\mu E_x(x,t)$ the “frictional” part determined by collisions with the neutrals. The turbulent fluctuations force are assumed to force the heavy ions to move only in the positive x-direction (radially outward), i.e.,

$\vec{U}(x) = \{U(x) \geq 0,0,0\}$ and $U(x) = 0$ if $|x| > L$. We moreover assume $E_x < 0$ and that the coefficient μ is related to the ion mobility ν by $\mu = Ze\nu$. The plasma electrons and ions are considered to be immobile. They only take part in the formation of the turbulent structure of stationary hills and valleys [2, 3]. We use basic equations of the form

$$\frac{\partial n}{\partial t} = -\frac{\partial}{\partial x}\left\{nV(x,t) - D(x)\frac{\partial n}{\partial x}\right\}, \qquad (1)$$

$$\frac{\partial E_x}{\partial x} = \frac{e}{\varepsilon_0}\{Z\,n(x,t) - \overline{n}_0\}, \qquad \overline{n}_0 = n_{e0} - n_{i0}, \qquad (2)$$

where $n(x,t)$ and Z are the density and charge number of the heavy ions, n_{e0} and n_{i0} are the electron and plasma-ion densities, respectively. Equation (1) is the Fokker-Planck equation, with $D(x)$ the heavy-ion collisional diffusion coefficient [6]. For the stochastic velocity $U(x)$ we use the estimate $U(x) = \varepsilon(x)/B_0$ [2, 3], where B_0 is the toroidal magnetic field and $\varepsilon(x) = \sqrt{\left\langle \left[E_{turb}(x,t)\right]^2 \right\rangle_t}$ is the mean-root square time-averaged turbulent electric field with $\varepsilon(x) = 0$ (and hence $U(x) = 0$) for $|x| > L$. We assume the diffusion coefficient D to be constant and use for it the expression given in [6]. Equations (1) and (2) can be reduced to one equation for the radial electric field:

$$\frac{\partial E_x}{\partial x} = -\{U(x) + \mu E_x\}\left\{\frac{\partial E_x}{\partial x} + \frac{e}{\varepsilon_0}\overline{n}_0\right\} + D\frac{\partial^2 E_x}{\partial x^2}. \qquad (3)$$

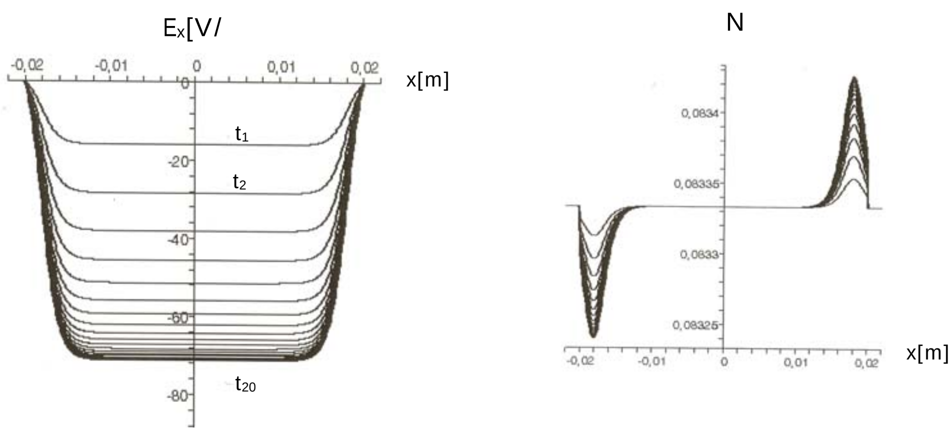

Fig. 2

Further, the following parameter values are used: $D = 10 m^2 s^{-1}$, $Z = 12$, $T = 10^4 K$, $\bar{n}_0 = 10^{15} m^{-3}$, and $L = 0.02\, m$. For the mobility ν we use Einstein's relation $\nu = D / kT$, with k Boltzmann's constant. The maximum value of the ion velocity, localized in the interval $(-L, L)$, is chosen to be $U_{\max} = 10^4 m/s$. In Fig.1 the time evolution of the radial electric field with the initial condition $E_x(x,0) = 0$ and the boundary condition $E(\pm 1.1 \cdot L, t) = 0$ is shown for $t_1 = 1 \cdot 10^{-10} s$, $t_2 = 2 \cdot 10^{-10} s, \ldots\; t_{20} = 20 \cdot 10^{-10} s$. From Fig.1 it is seen that the radial electric field approaches a stationary maximum value. The latter result is in accordance with the general theory of the Fokker-Planck equation [5]. Figure 2 illustrates the charge separation described above. The perturbation of the initially uniform heavy-ion density $N = n / \bar{n}_0$ is localized at the peripheries of the turbulent region. In view of the simplicity of our model, verification of our results requires discussion based on a more realistic model (e.g., on the turbulent potential given by the Hasegawa-Wakatani system of equations).

2. It is almost generally accepted that an ergodic layer generated by auxiliary coils on tokamaks can mitigate the ELM instability by creating a radial electric field [1, 7]. The ergodic layer forms a slab of chaotic magnetic field lines close to the tokamak edge. In that region both the ergodic layer and the turbulent electrostatic potential considered above will influence particle dynamics. Therefore, in what follows we would like to investigate in which conditions the effect of the electrostatic perturbation in the system predominates. We use a system of field lines characterized by the radially dependent safety factor q in cylindrical configuration, parametrized by coordinates (r, θ, z). We assume that the density perturbation along magnetic field lines is constant (to enable 2D simulation), and that the radial and polar wave numbers are similar: $\varphi = \varphi_0 \cos(k_r r) \cos[k_\theta r_d (\theta - z q_d^{-1} R_0^{-1})]$. For the model of the ergodic layer, we use the simplest case of two overlapping rows of magnetic islands with the Hamiltonian in which the vector potential can be expressed using the toroidal flux ψ: $A_\theta = \psi$,

$$H = \frac{p_r^2 + (p_\theta - eA_\theta)^2 + (p_z - eA_z)^2}{2m} + e\varphi,$$

$$-A_z = R_0^{-1} \int q^{-1} d\psi + L R_0^{-1} \psi [\cos(m_1 \theta - n_1 z R_0^{-1}) + \cos(m_2 \theta - n_2 z R_0^{-1})]. \qquad (4)$$

We have observed three main regimes for protons (and heavier, e.g. carbon, ions) with total energy of 20 eV. First, the effect of the electrostatic perturbation is detectable at approximately 0.03 V. The diffusion caused by the perturbation is of the same magnitude as the one caused by a very strong ergodic layer ($l \approx 0.001$ m). Further, at approximately 1-2 V the potential disrupts Poincaré sections, and changes the stability of trajectories. Last, above 5-20 V the system is dominated by the electrostatic perturbation: Poincaré sections contain confined trajectories, and there is no change in the phase diagrams of diffusion based on the ergodic layer. The results obtained in the same way for electrons suggest that the effect of the electrostatic perturbation is much smaller (compared with the effect of the ergodic layer) at the same amplitudes.

Fig. 3: Poincaré sections for 20eV ions at potential amplitude 0V (left), 0.03V (middle) and 5V (right).

An exception appears when the potential energy of electrostatic perturbation is comparable with the electrons' temperature. Since the plasma temperature is rather low at the edge, the mentioned effect could be of some importance. The difference between the effects of the potential on light and heavy particles thus represents also a case for further study.

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1. V. Rozhansky et al., Nucl. Fusion **50**, 034005 (2010).
2. L. Krlin, J. Stöckel and V. Svoboda, Plasma Phys. Control. Fusion **41**, 339 (1999).
3. L. Krlin, J. Stöckel, V. Svoboda and M. Tendler, Physica Scripta **T84**, 221 (2000).
4. L. Krlin, R. Paprok, V. Svoboda, Eur. Phys. J. D **48,** 95 (2008).
5. H. Risken, *The Fokker-Planck Equation,* (Springer-Verlag, Berlin 1984), 133-137.
6. E.M. Lifshitz and L.P. Pitaevskii, *Physical Kinetics* (Pergamon Press, Oxford, 1981), 40-42.
7. T.E. Evans et al., Nucl. Fusion **48**, 024002 (2008).